TQFTs are in the Swampland

Jonathan J. Heckman

University of Pennsylvania

Based On:

hep-th/2601.09781 w/ C. Cummings

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hep-th/2601.09781 w/ C. Cummings

as well as many collaborations w/

V. Chakrabhavi, M. Cvetič, A. Çavuşoğlu,

A. Debray, M. Dierigl, J. Kuntz, M. Hübner,

J. McNamara, S. Meynet, M. Montero, Murdia

A. Sharon, C. Vafa, I. Valenzuela

Background

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QFT very useful in many physical systems

- Many degrees of freedom
- Some notion of locality

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QFT very useful in many physical systems

- Many degrees of freedom
- Some notion of locality

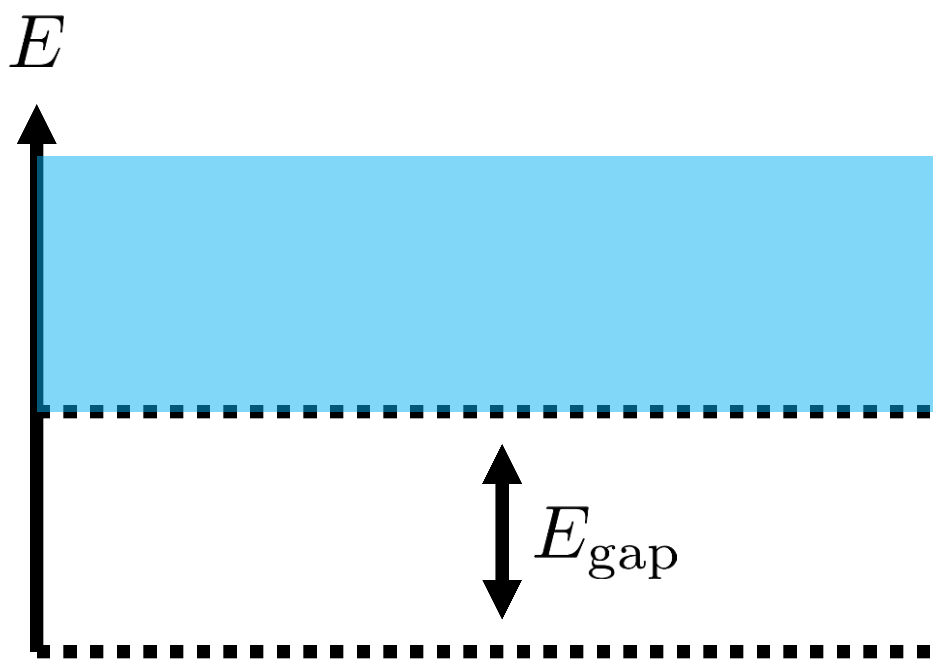
TQFTs a special case: also very useful!

- Systems with a gapped spectrum (deep IR)
- Generalized global symmetries:

  $\mathcal{U} = \int [da] e^{iS[a, \Phi_{\mathrm{bkgnd}}]}$

Motivating Question

TQFTs and gapped spectra: How big can E_{gap} be?



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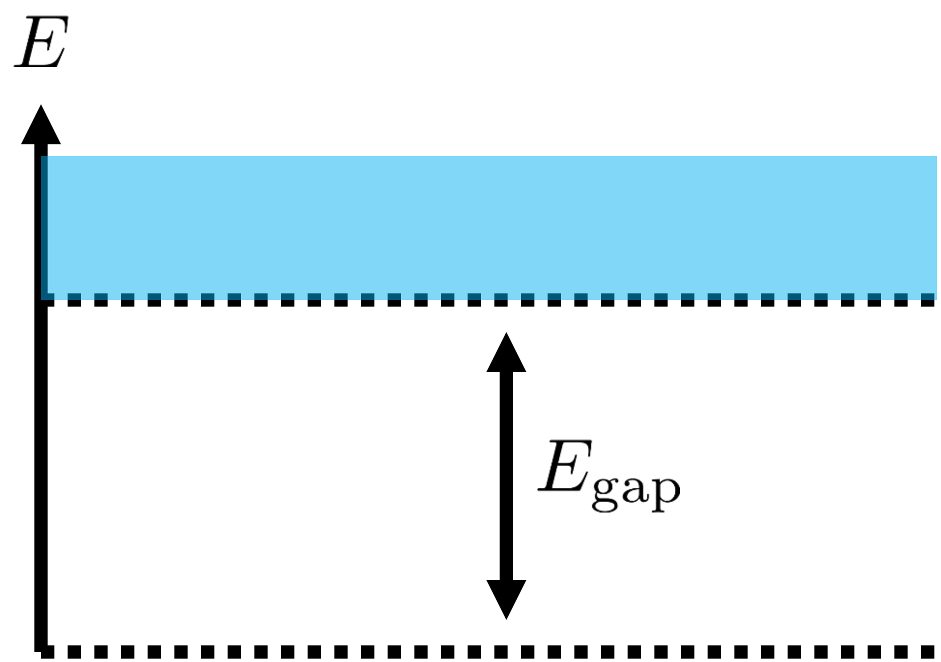

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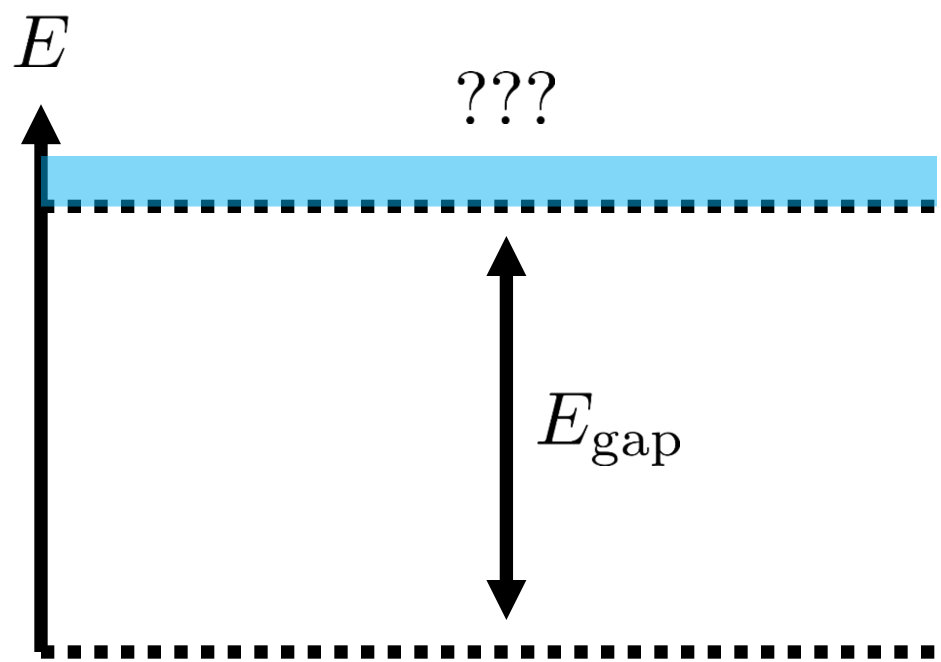
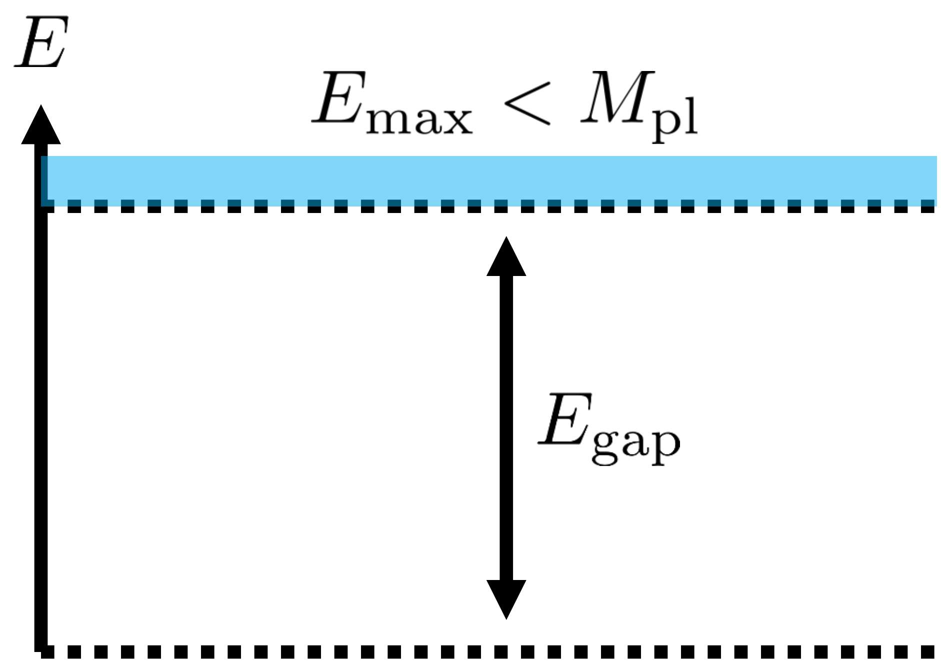


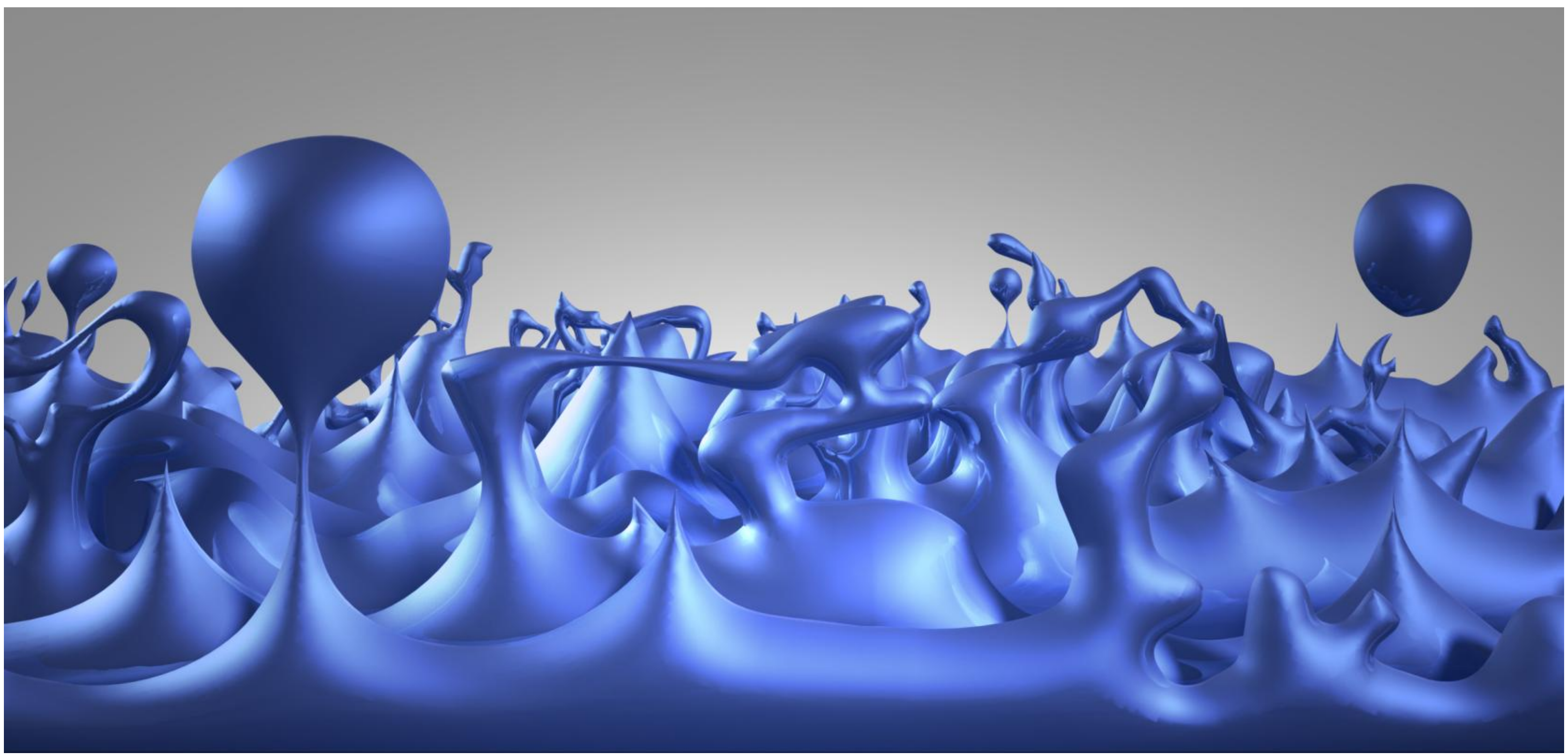
General Claim

Gravity sets an upper bound



The Planck Scale: $E = M_{\rm pl}$

Vague Lore: Topology Breakdown...



The Planck Scale: $E = M_{\rm pl}$

Vague Lore: Topology Breakdown...

Broader Claim

(Implicit Theme: TFTs $\leftrightarrow$ Global Symmetries)

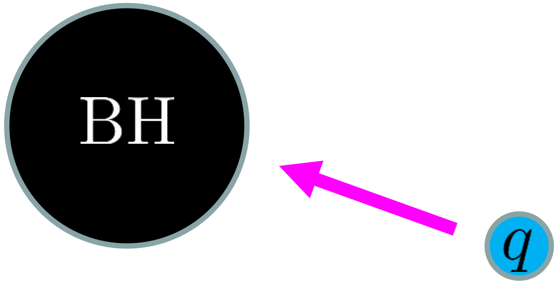
No Global Symmetries in Quantum Gravity

Global Symmetries of QFT “Emerge in IR”

Another Motivation

Global symmetries very helpful in QFT

Lore: No global symmetries in quantum gravity

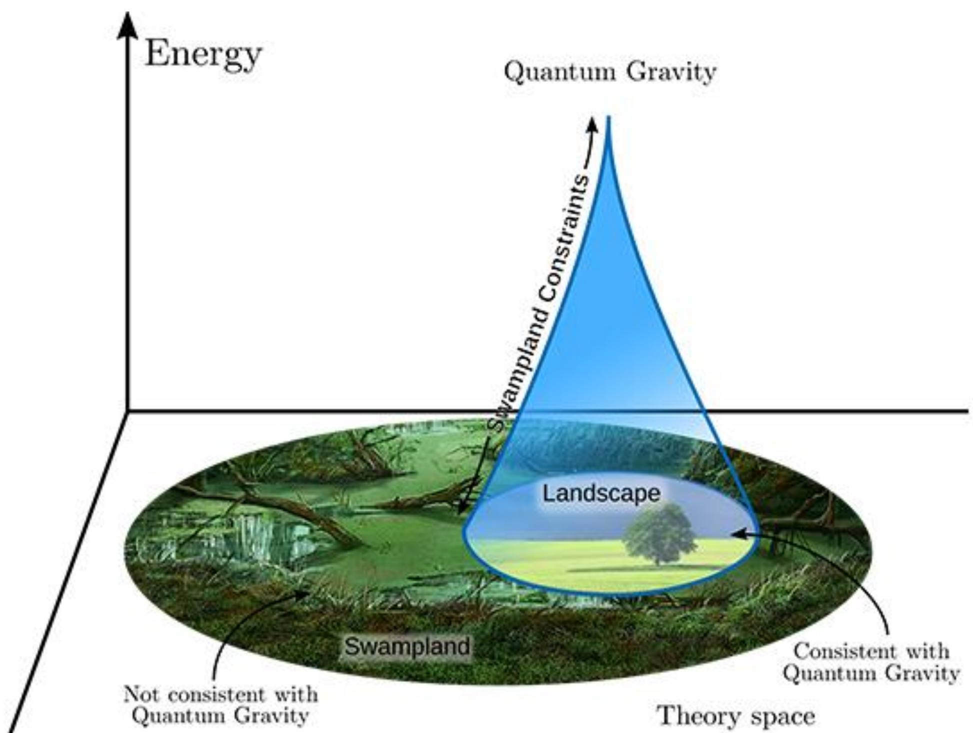


Puzzle: What happened to the global symmetries???

Swampland

Vafa et al... '05 $\to \infty$

Gravity Imposes Non-Trivial Constraints on QFTs



Swampland

Vafa et al... '05 $\rightarrow \infty$

Gravity Imposes Non-Trivial Constraints on QFTs

Summary / Punchlines

Use AdS/CFT to show:

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Summary / Punchlines

Use AdS/CFT to show:

- TQFTs couple to gravity non-pert: $\sim e^{-(M_{\mathrm{pl}}/E)^{\nu}}$
- TQFTs couple to gravity pert: $\sim (E/M_{\mathrm{pl}})^{\nu}$
- In UV, TQFT fields "have kinetic terms"

Outline

- Top Down Perspective
- Holography & TQFTs vs Gravity
- Conclusions / Future

Top Down Perspective

Top Down Perspective

(Start in UV w/ Gravity and then decouple it)

UV Motivated Approaches

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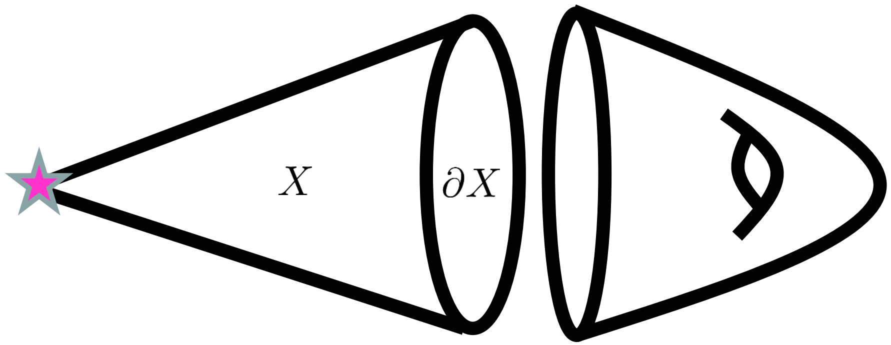
String Compactification



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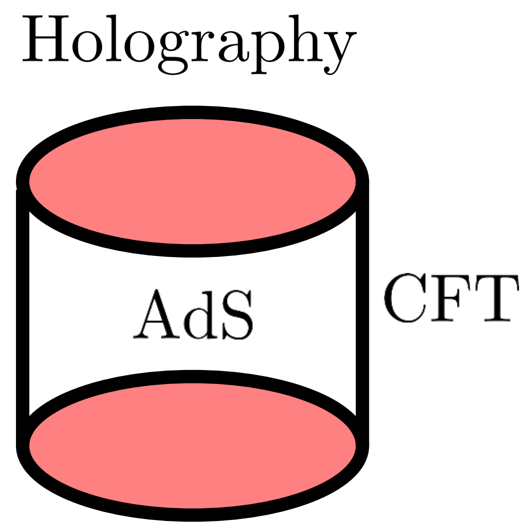
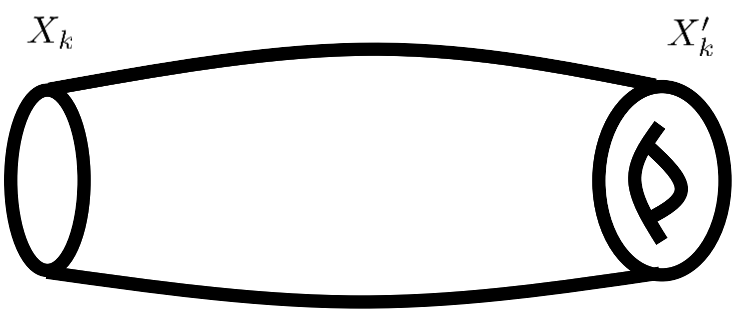


UV Motivated Approaches

Cobordisms



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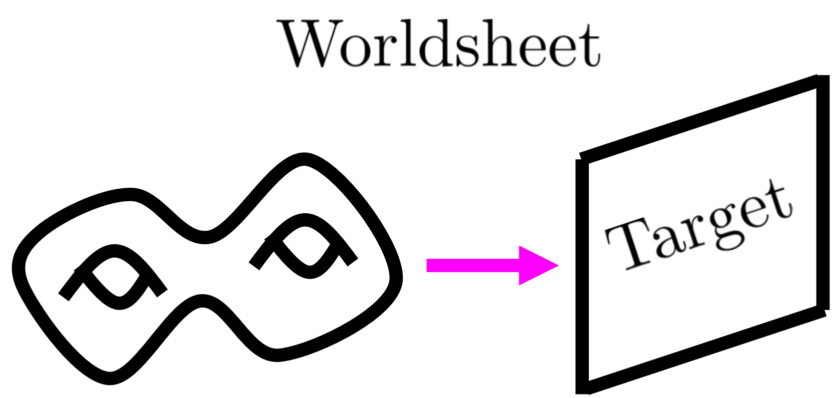

UV Motivated Approaches

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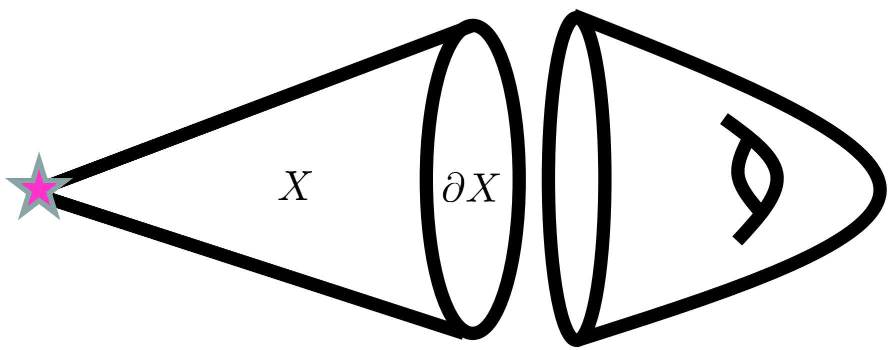
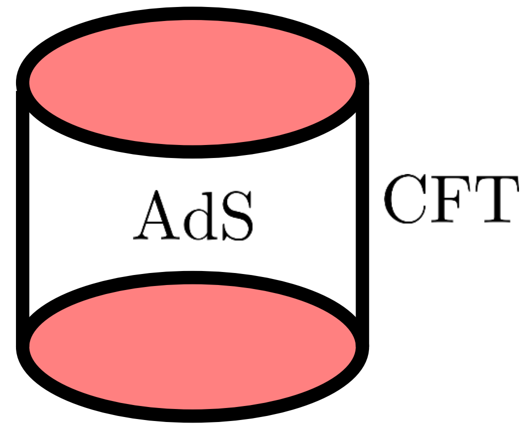


Holography



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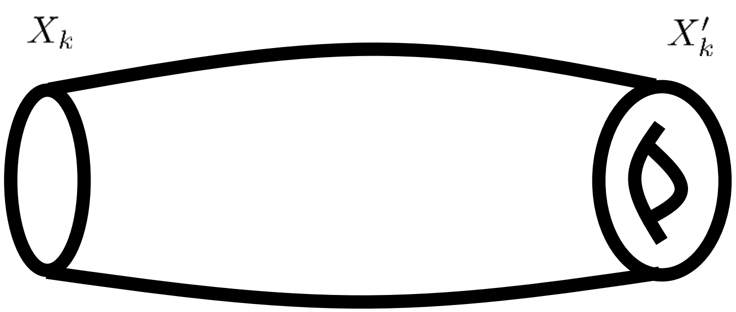
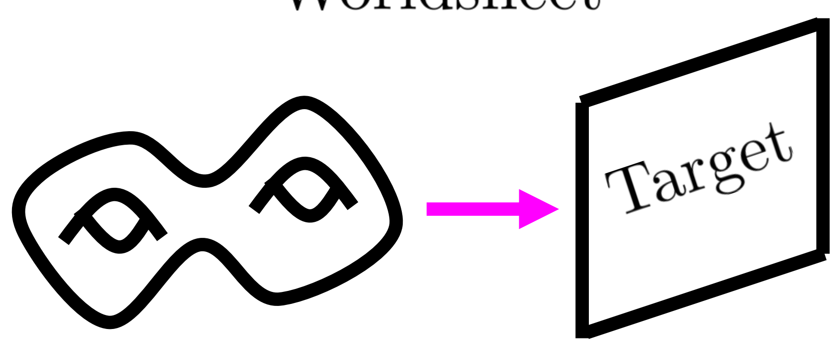
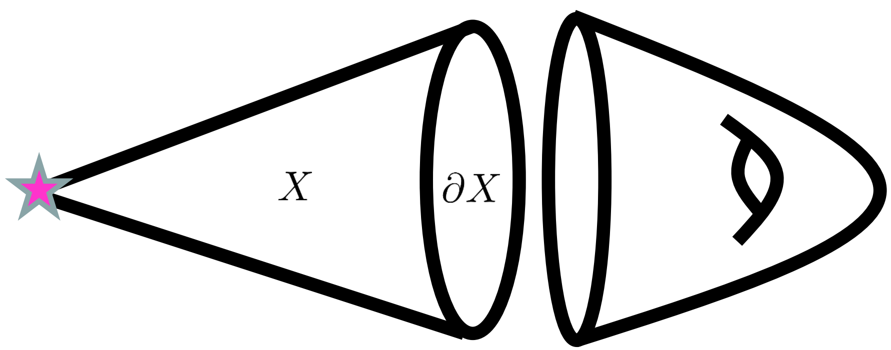


Worldsheet



Focus for Today

String Compactification



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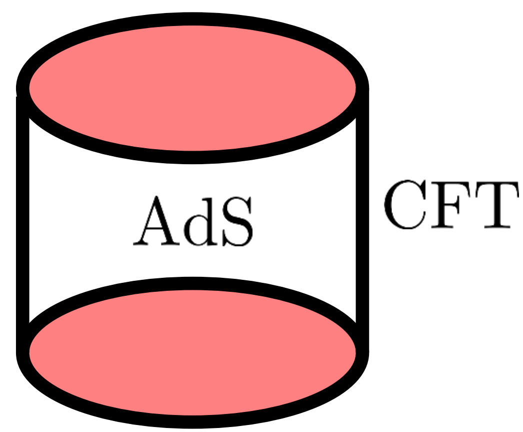
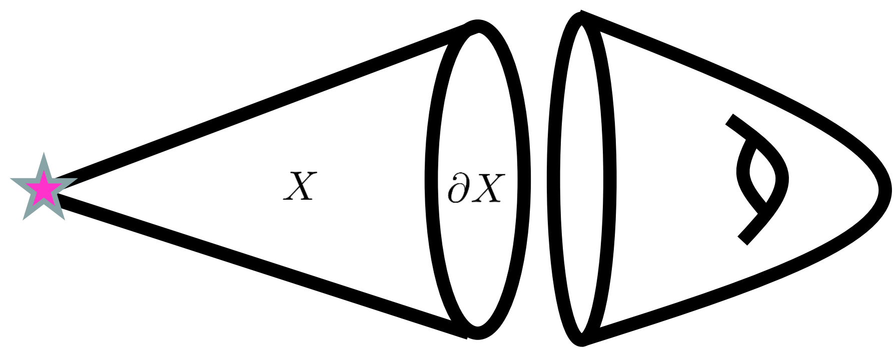
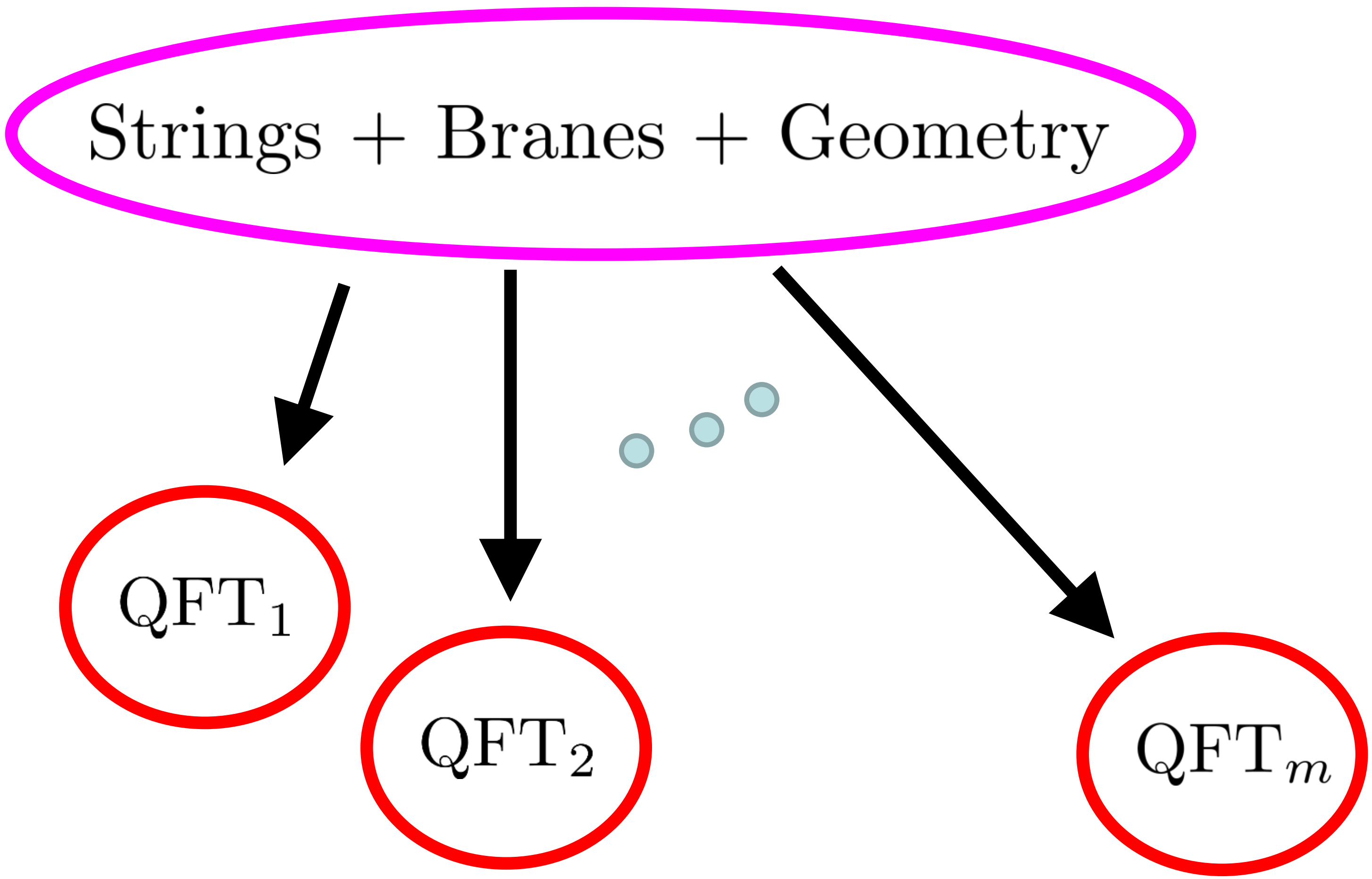


Approach 1:
String Compactification

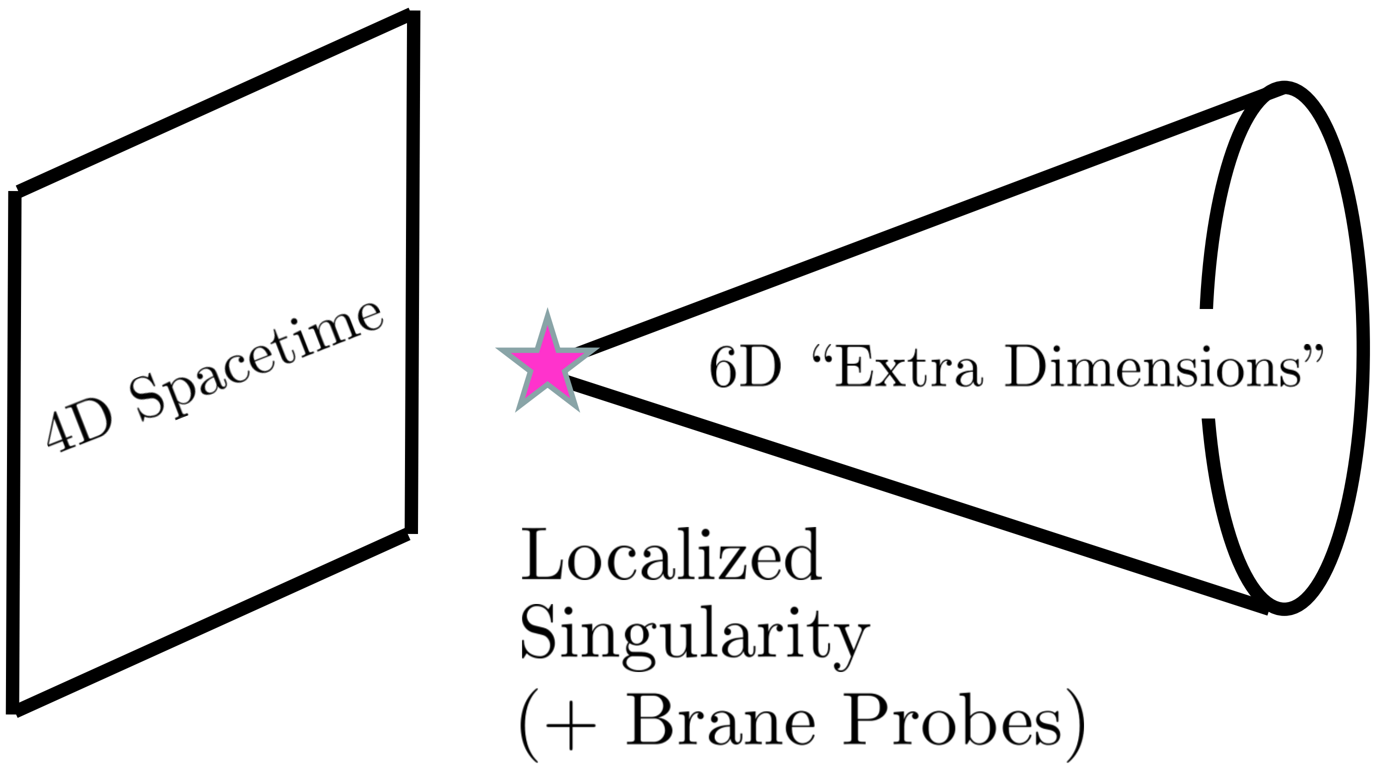


Long Distance Limits

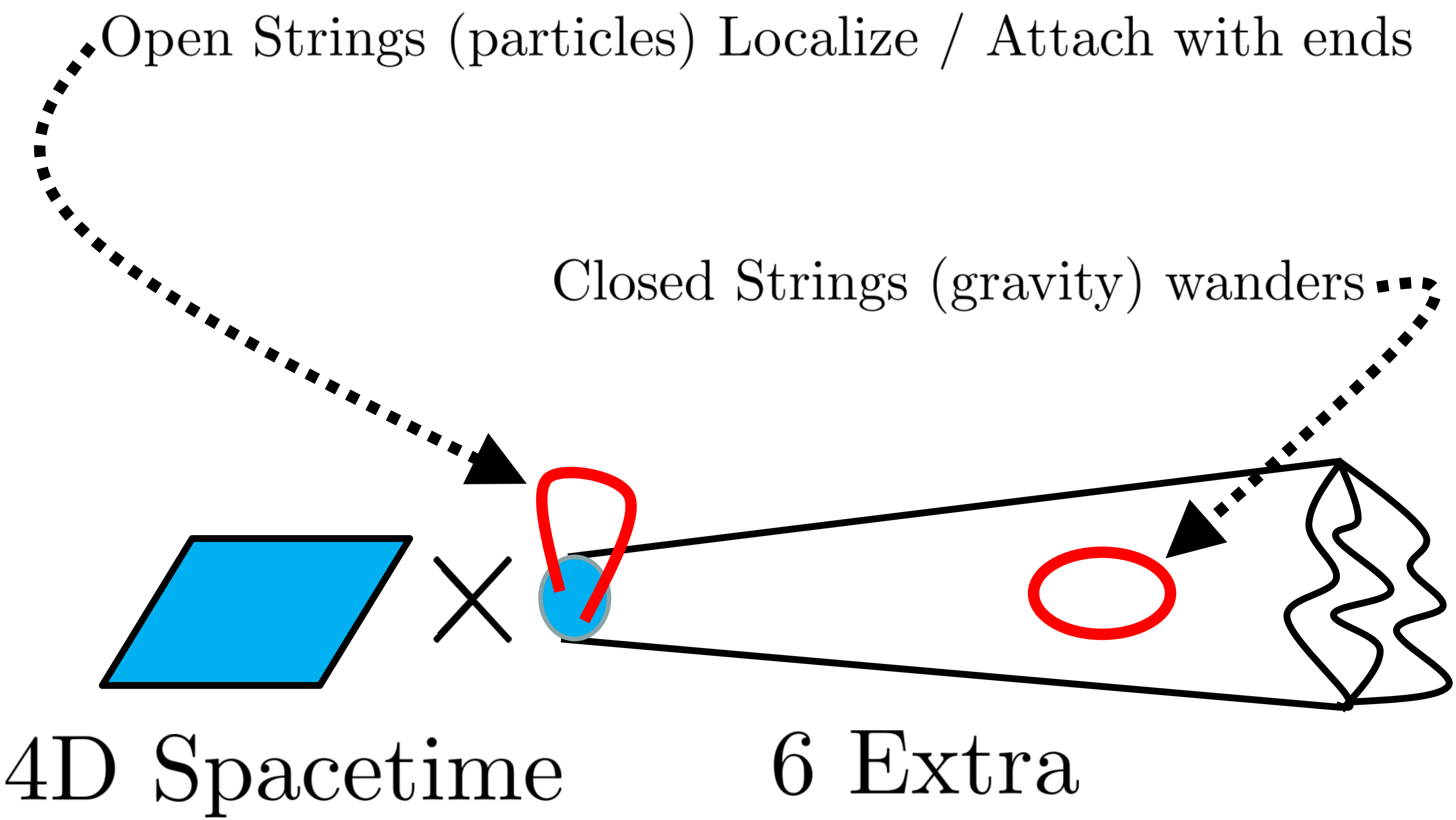


$G_N \to 0$ in Quantum Gravity

10D string backgrounds of the form:

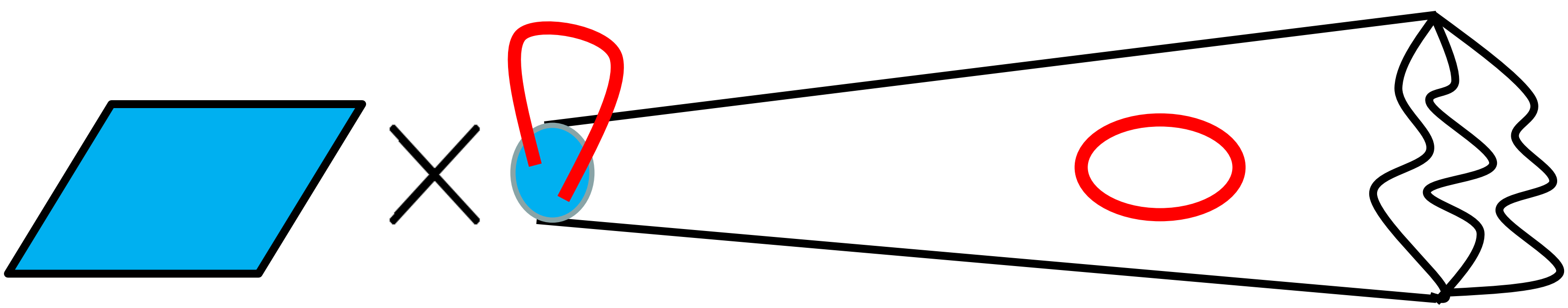


Turning Off Gravity



Turning Off Gravity

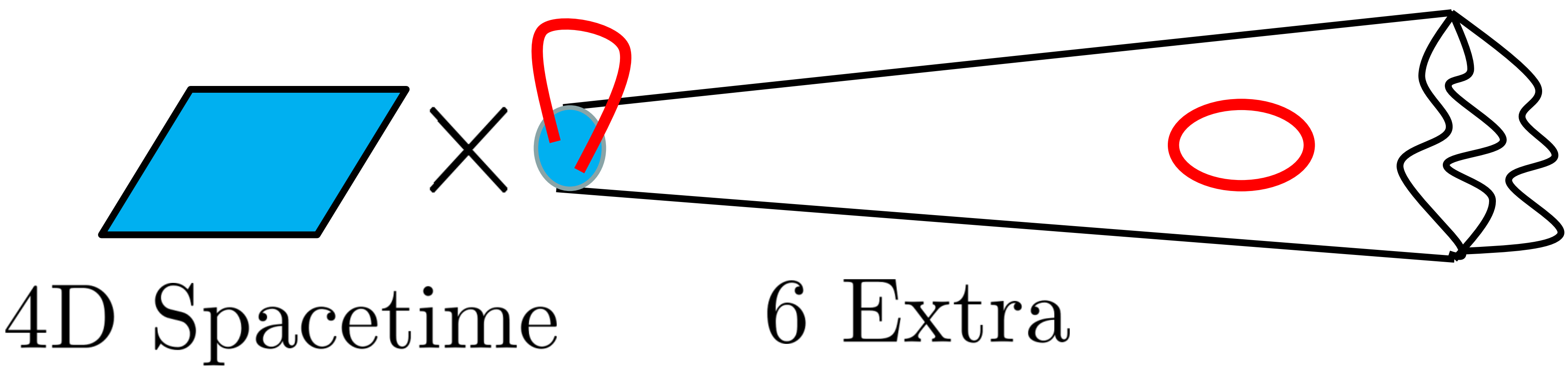
$$G_{\text{Newton}} \sim \frac{1}{\text{Vol(Extra Dimensions)}}$$



4D Spacetime 6 Extra

Turning Off G_{Newton}

$$G_{\text{Newton}} \sim \frac{1}{\text{Vol(Extra Dimensions)}} \to 0$$



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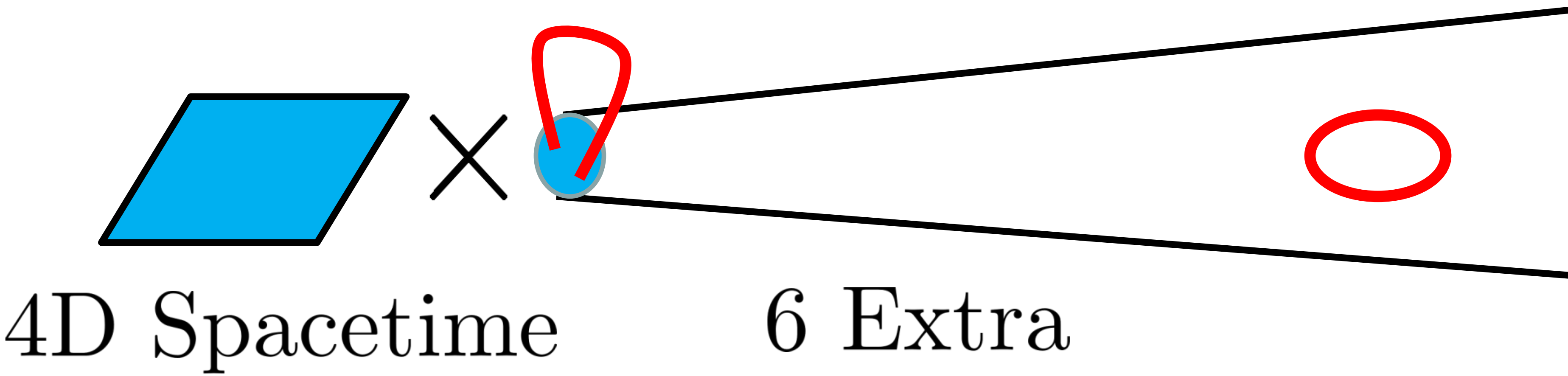

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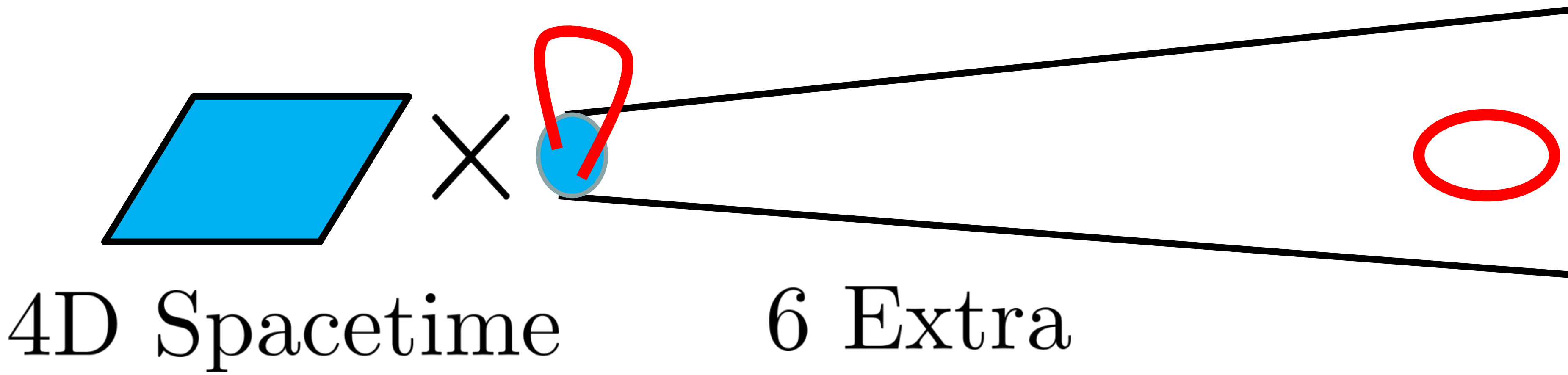

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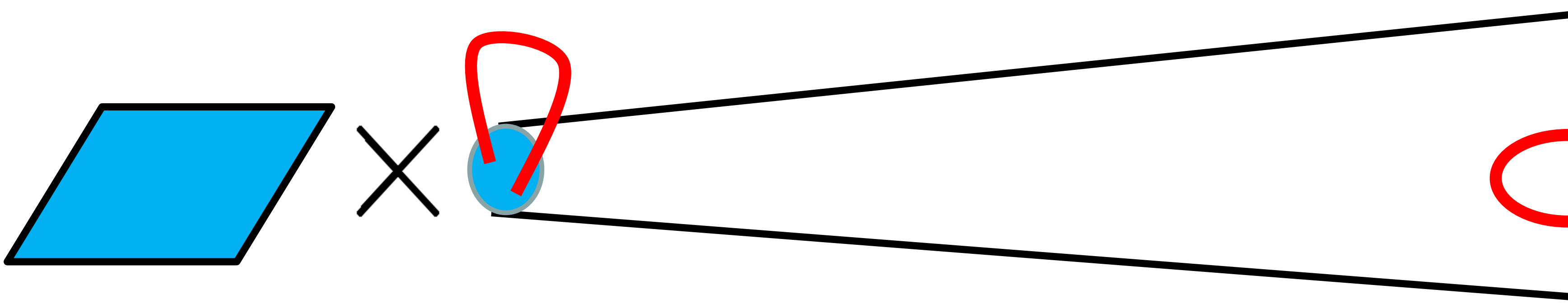

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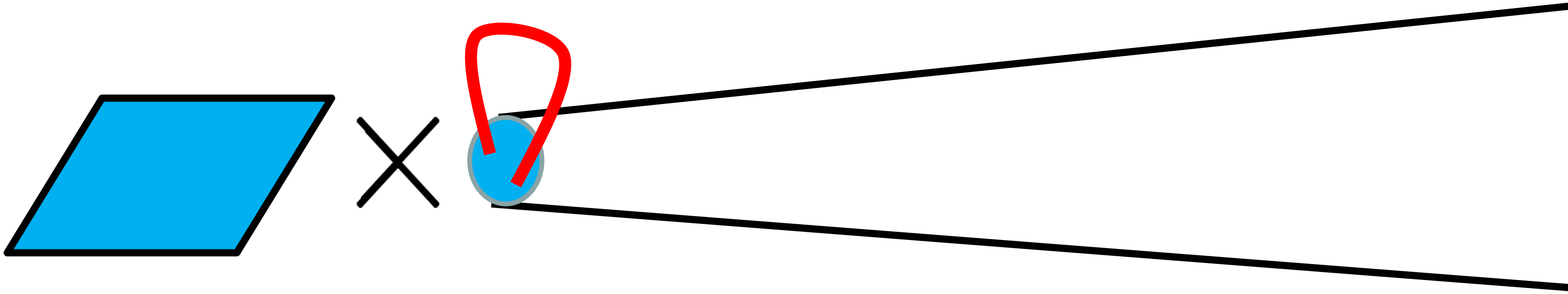


4D Spacetime

6 Extra

Stringy Generalized Symms

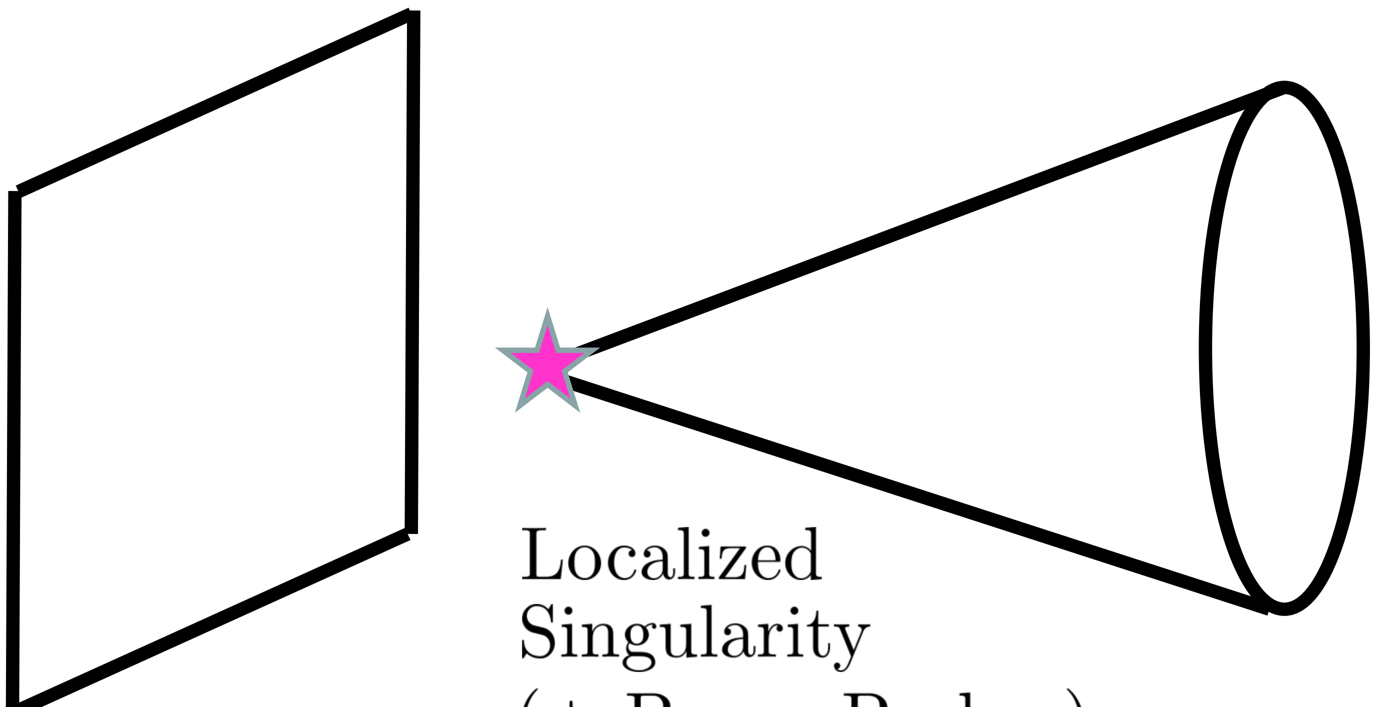
- Engineer a QFT (via Geometry)

- Defects (via Branes / Geometry / ...)

- Symmetry Ops (via Branes / Geometry / ...)

Typical Setup:

String backgrounds of the form:



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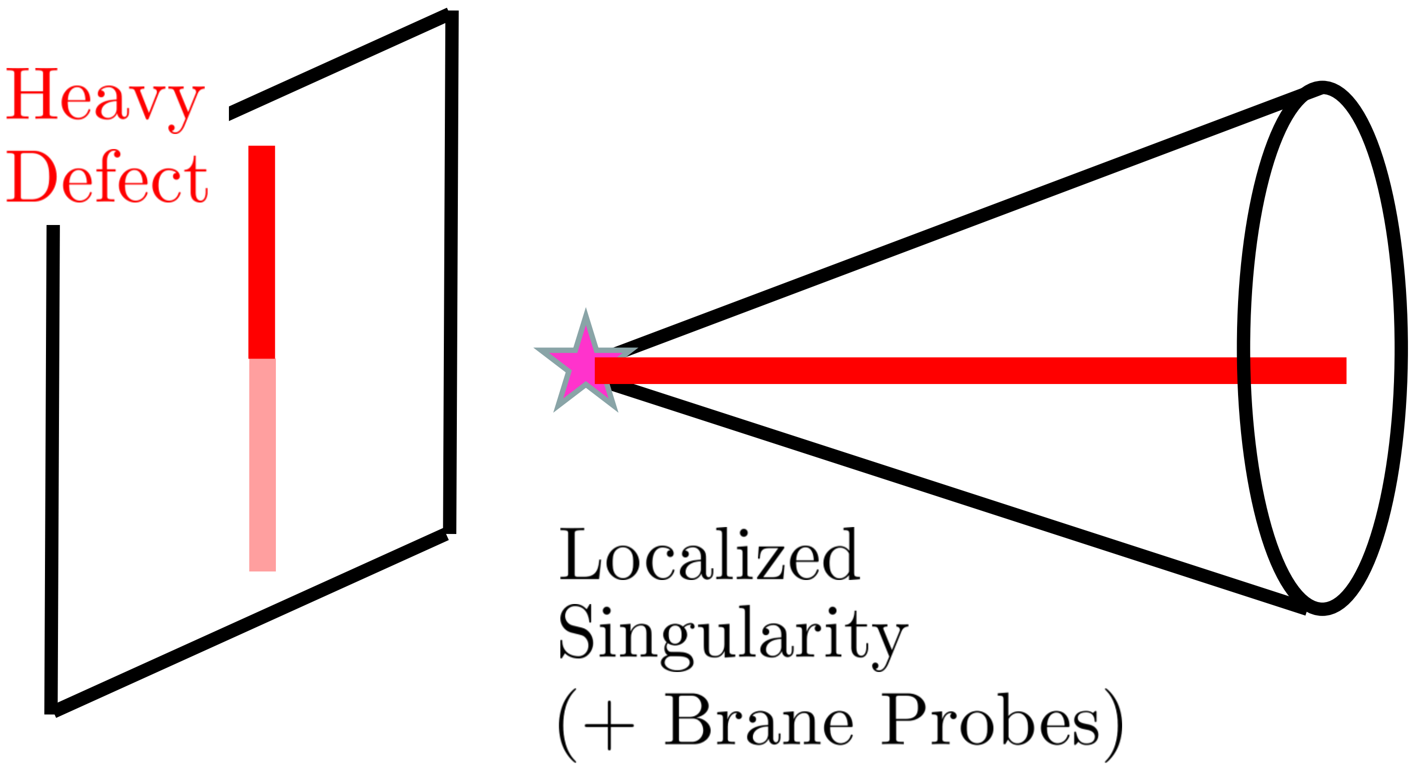

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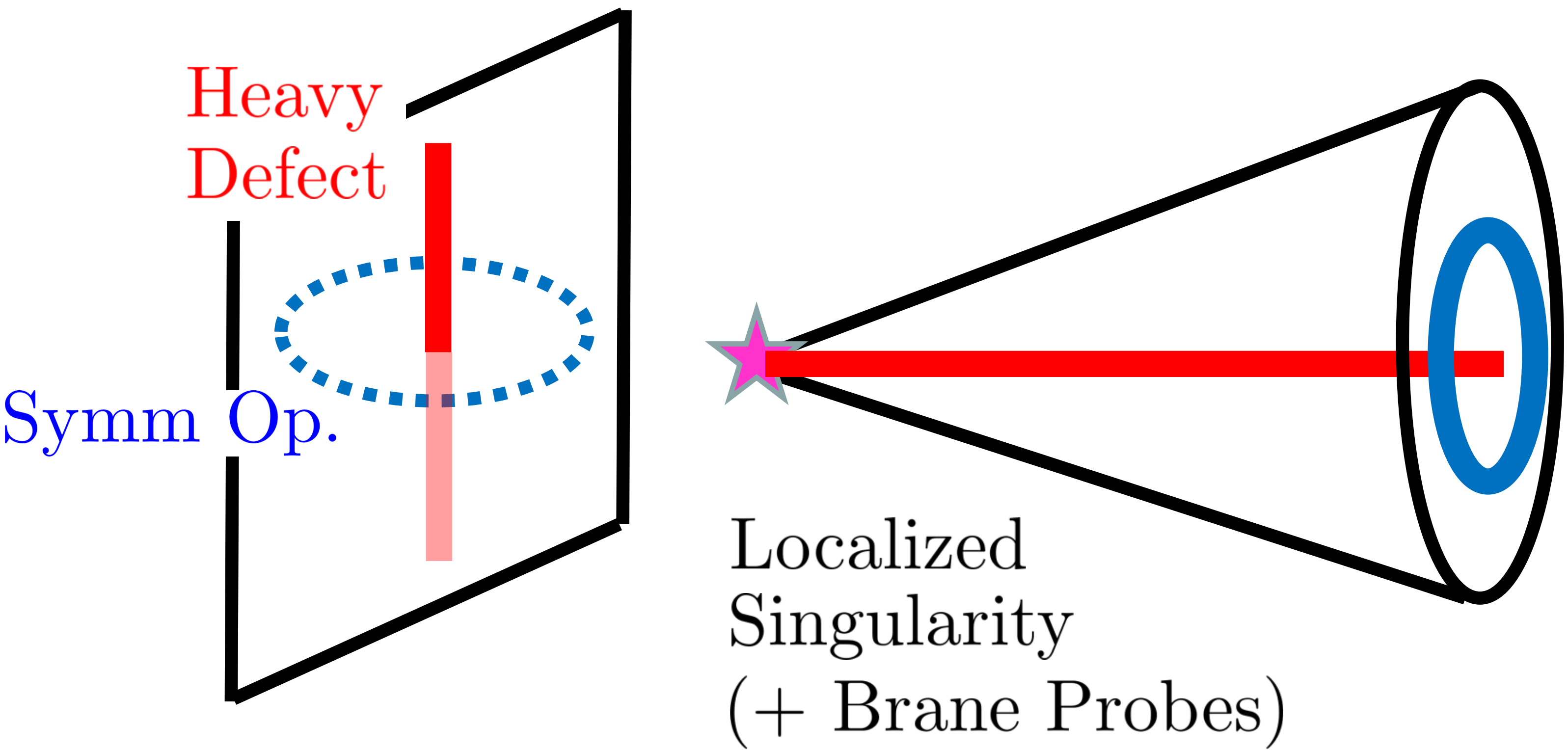

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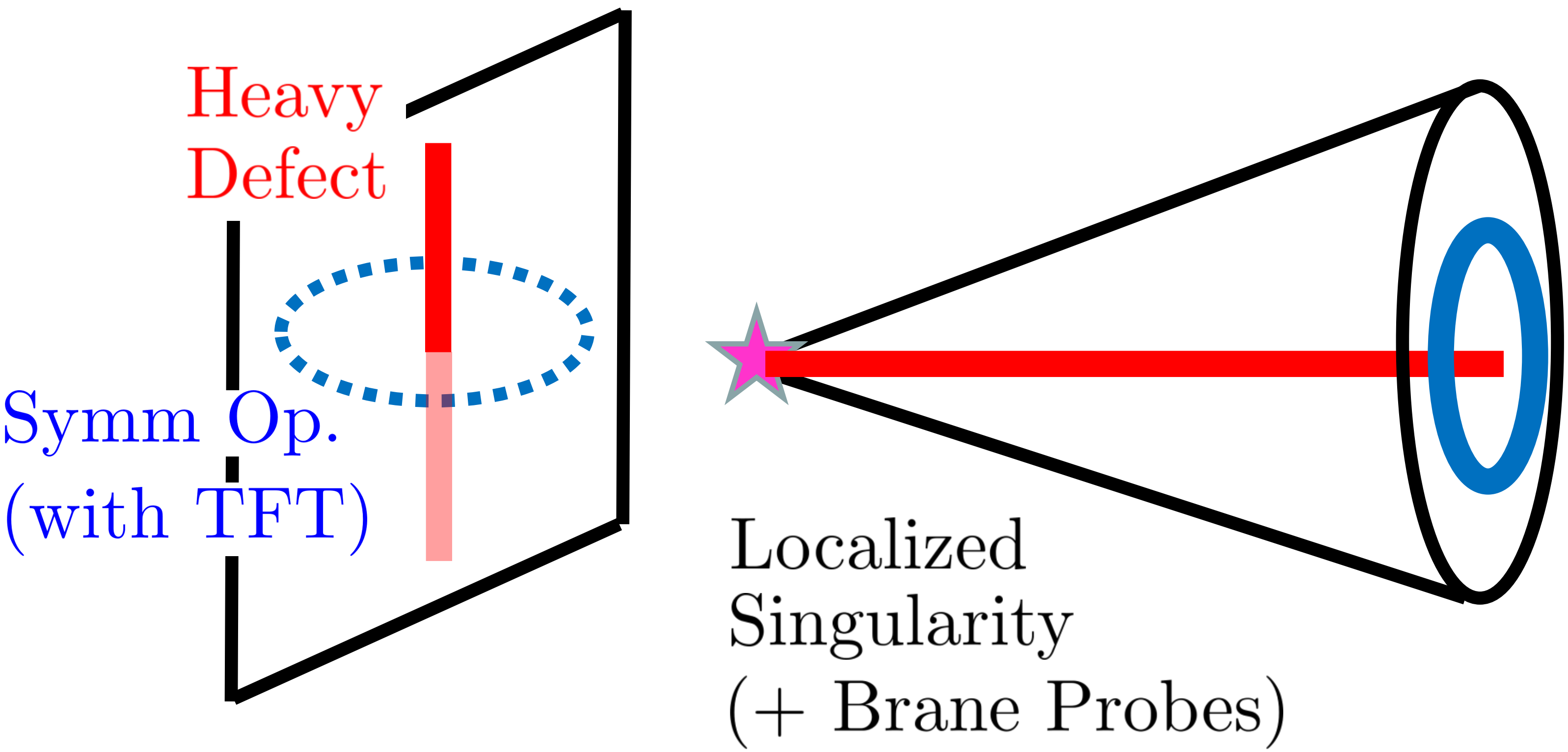
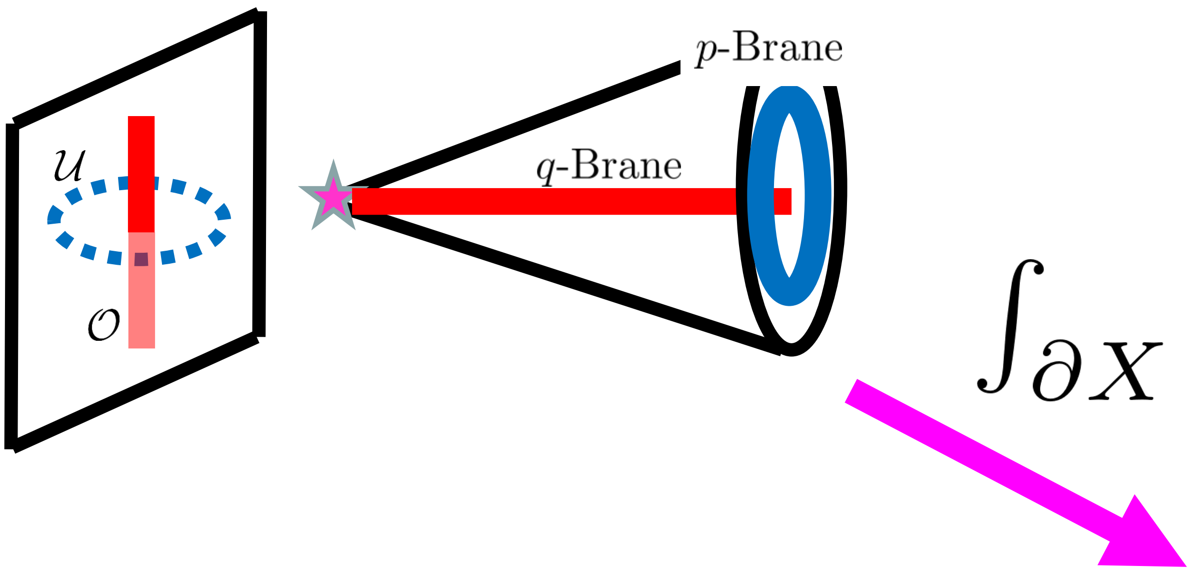
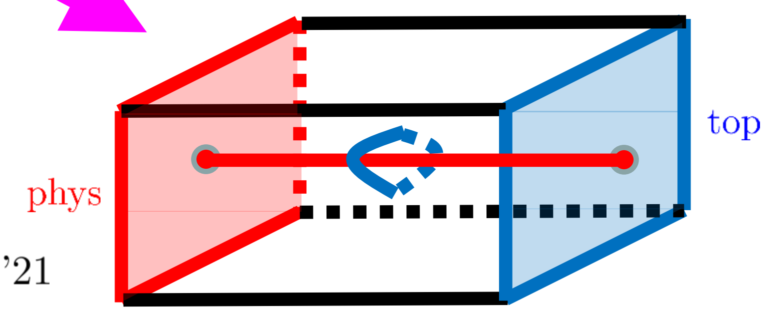


Symmetry Theories

Reshetikhin, Tuarev, Viro; Freed Moore Teleman ’12 + many many many

Matches well with SymTh point of view:





Apruzzi Bonetti Garcia Etxebarria Hosseini Schafer Nameki ’21

JJH Hübner et al. ’22 - ’26

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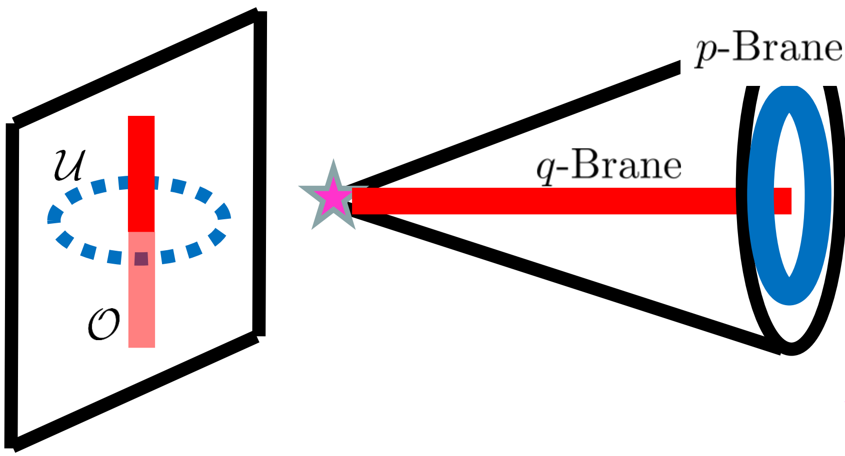
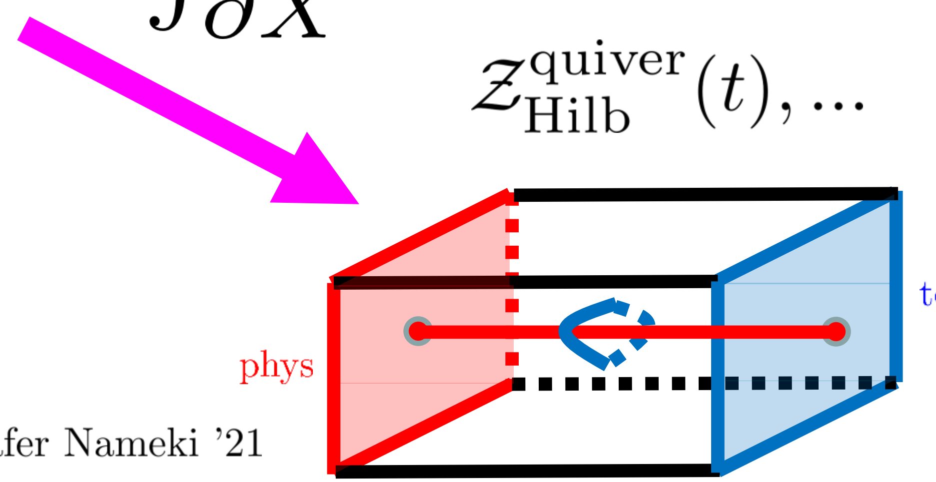


$\int_{\partial X}$

Encoded in

$\eta(\partial X)$

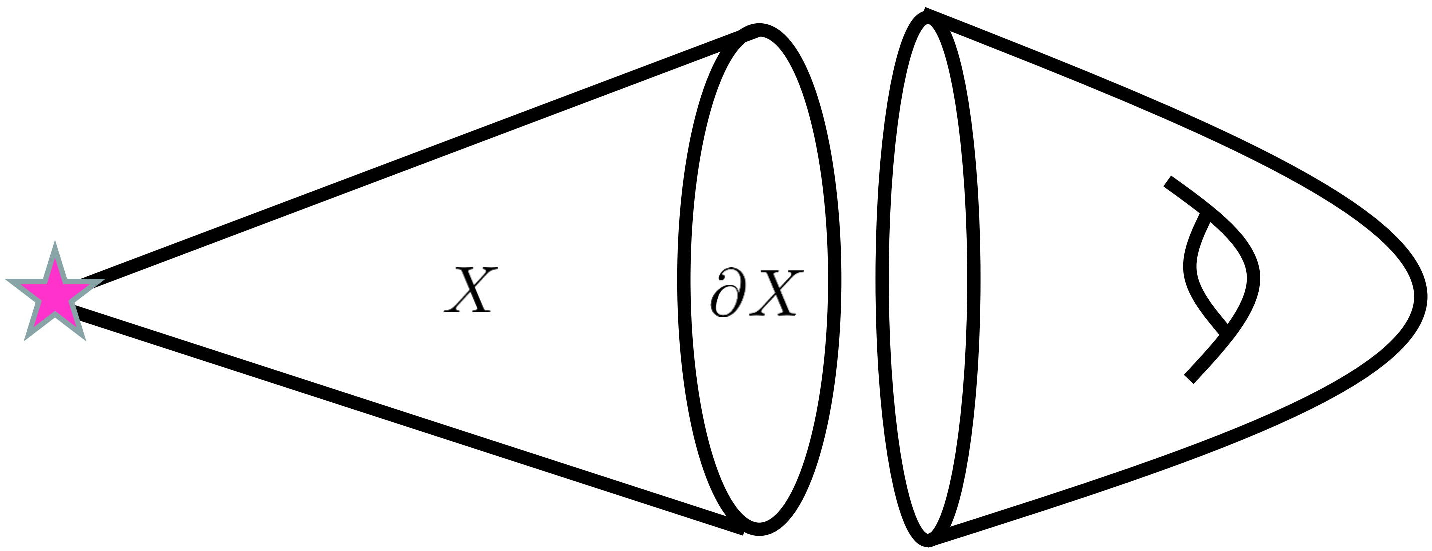
$\mathcal{Z}^{\text{quiver}}_{\text{Hilb}}(t), \ldots$



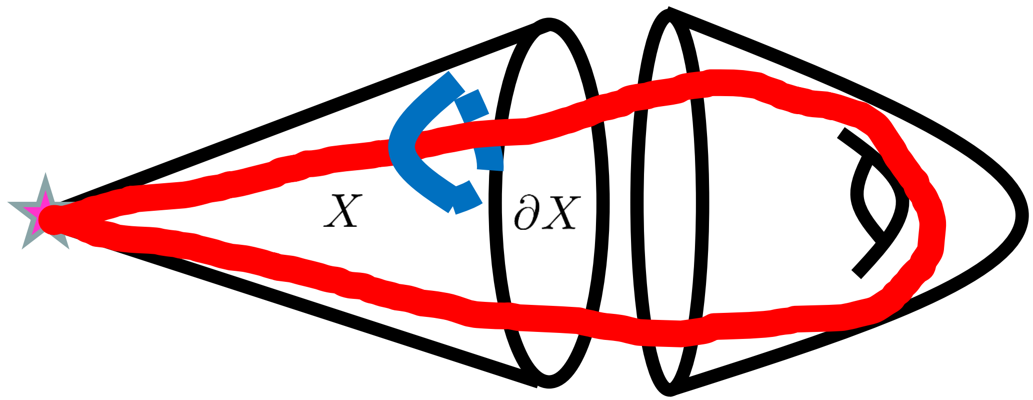
Apruzzi Bonetti Garcia Etxebarria Hosseini Schafer Nameki ’21

(Chakrabhavi) JJH Hübner et al. ’22 - ’26

Engineering QFTs + Gravity



¿Fate Of Symmetry Operators?

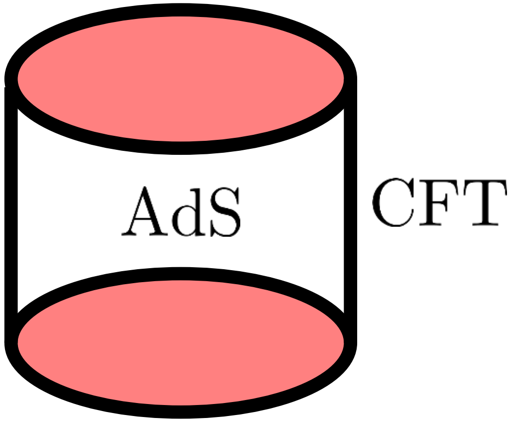


QFT $\longrightarrow$ QFT + Gravity

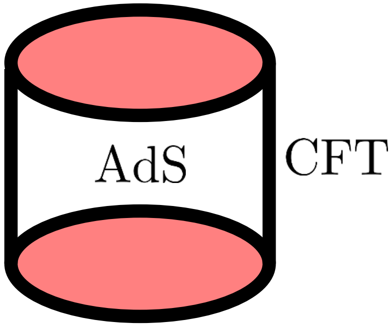
Top. Symm Op. $\longrightarrow$ Dynamical Brane

Approach 2: Holography

Maldacena '97



Dictionary Entries



AdS_{D+1}		CFT_D
Gauge Symm.	$\longleftrightarrow$	Global Symm.
$\vdots$		$\vdots$

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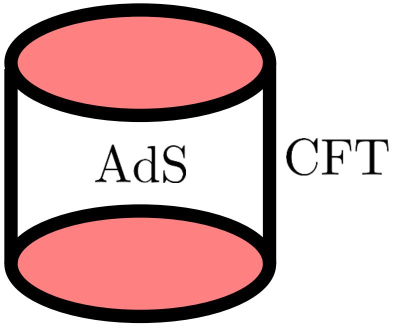
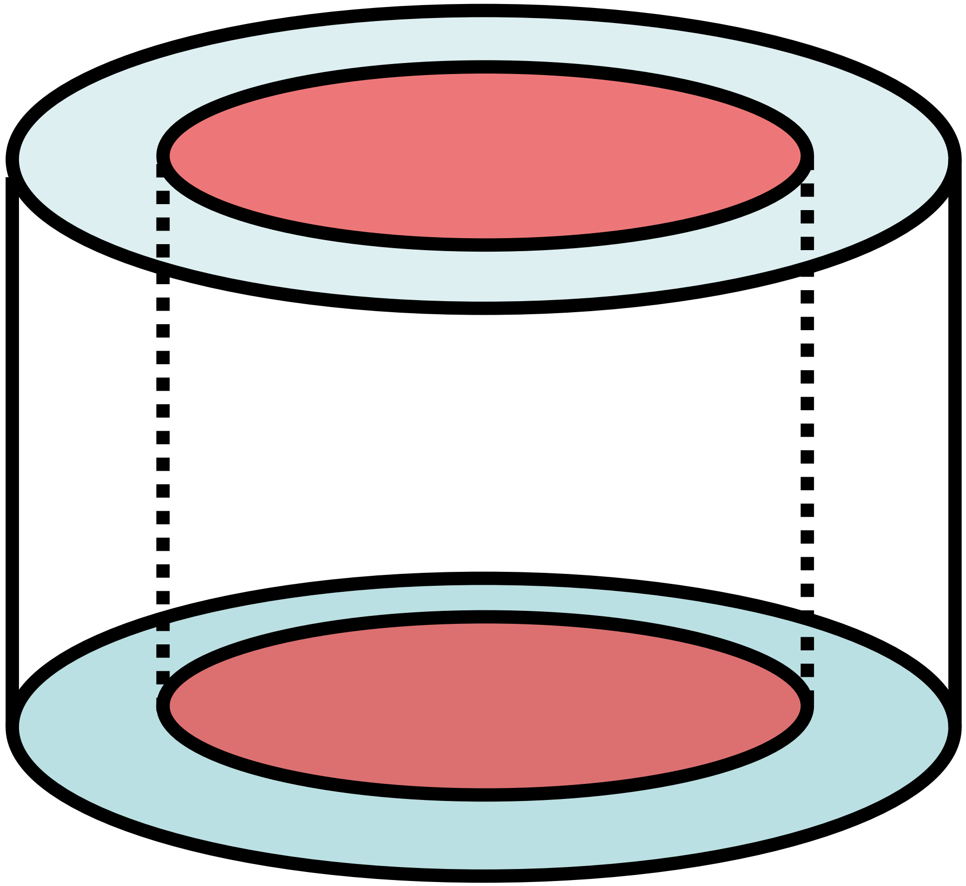


AdS_{D+1}		CFT_D
		Hamilton Kabat Lifschytz Lowe '05 '06
Radial Evol^n	$\longleftrightarrow$	Smearing
Gauge Symm.	$\longleftrightarrow$	Global Symm.
$\vdots$		$\vdots$

Brane Tracking



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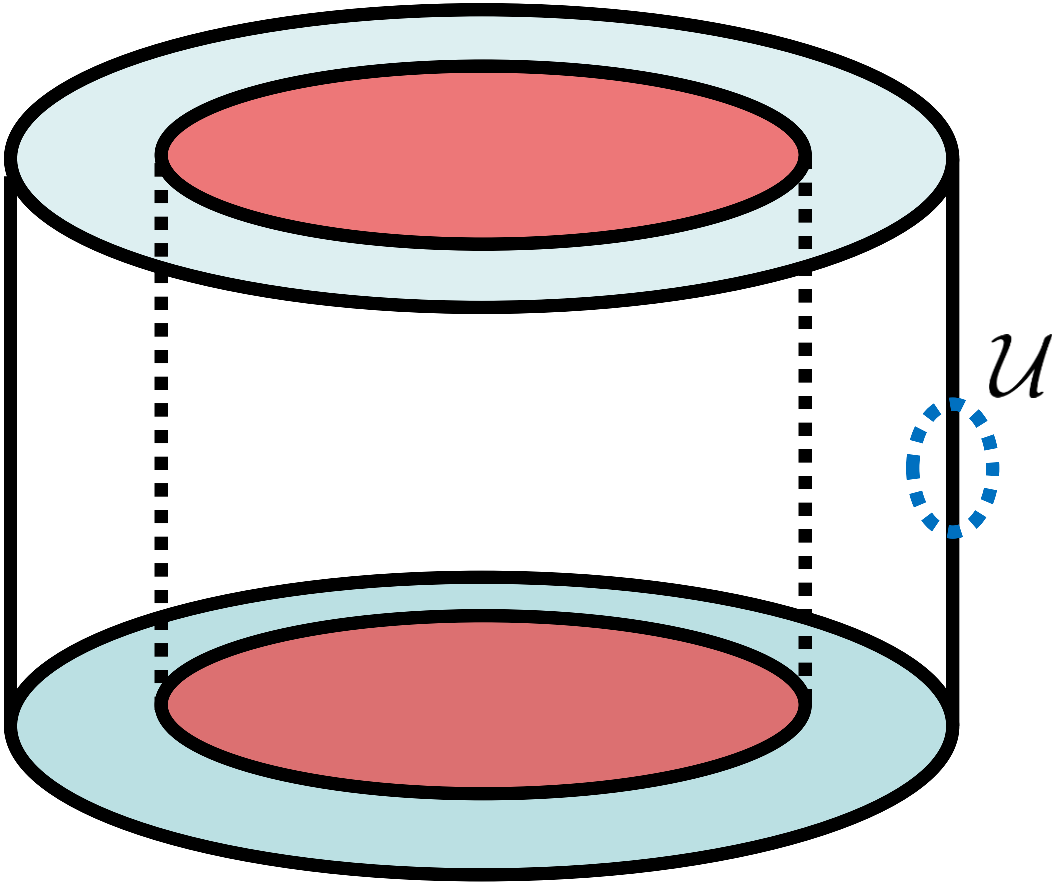

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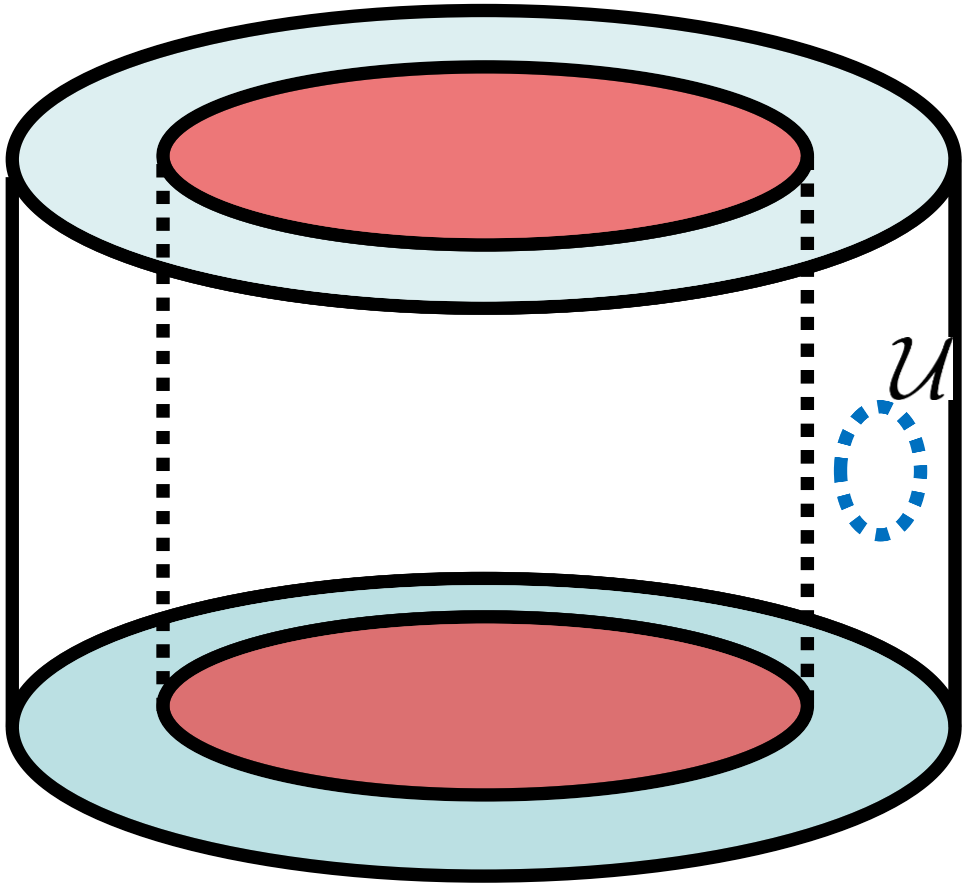

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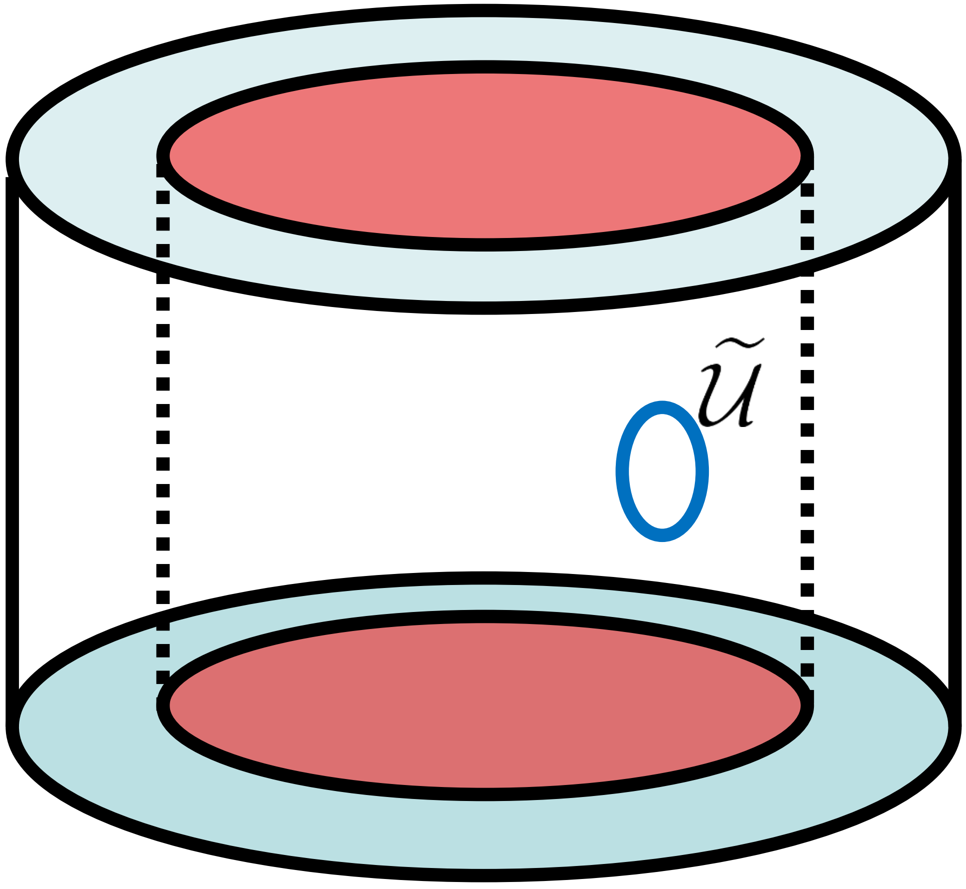

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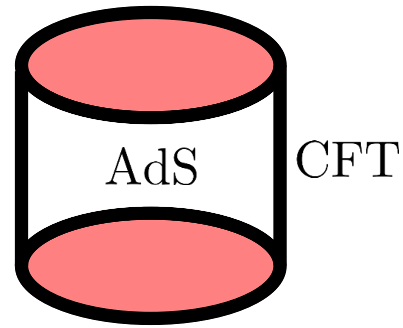


AdS_{D+1} $\qquad\qquad$ CFT_D

Radial Evol^n $\longleftrightarrow$ Smearing

Gauge Symm. $\longleftrightarrow$ Global Symm.

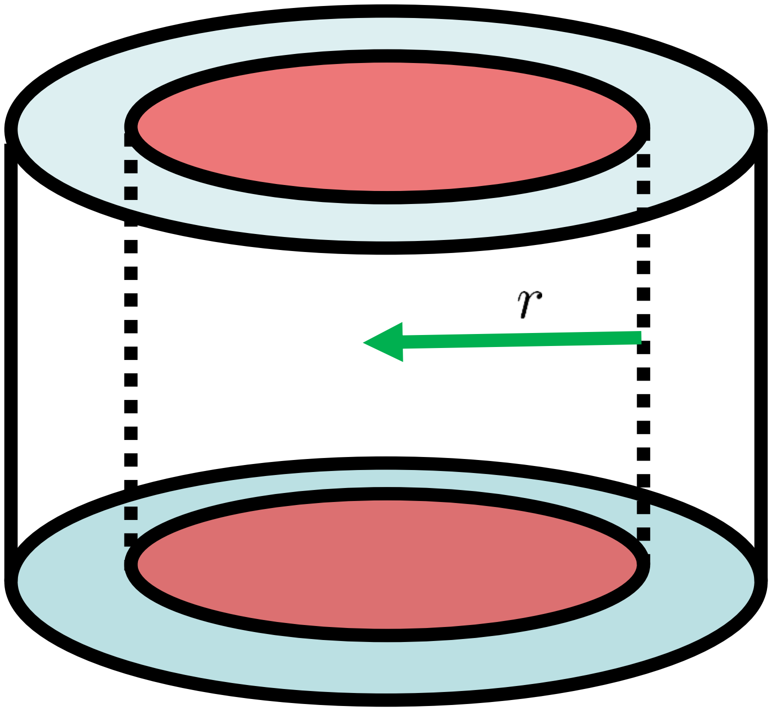
$\widetilde{\mathcal{U}}_{\text{Brane}} \longrightarrow \mathcal{U}_{\text{Symmetry Operator}}$

Top Down Approach: Apruzzi Bah Bonetti Schafer-Nameki '22;
Garcia Etxebarria '22;
JJH Hubner Torres Zhang '22 + ...

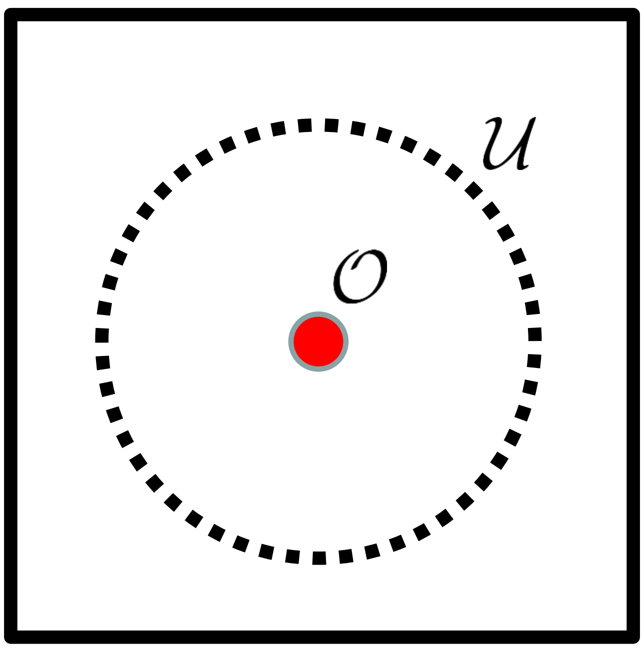
Smearing $\mathcal{U} \oplus \mathcal{O}$

Look at how $\mathcal{U}$ acts on charged $\mathcal{O}$:

CFT + $r_{\mathrm{UV-cutoff}}$



$r = r_{\mathrm{UV}}$



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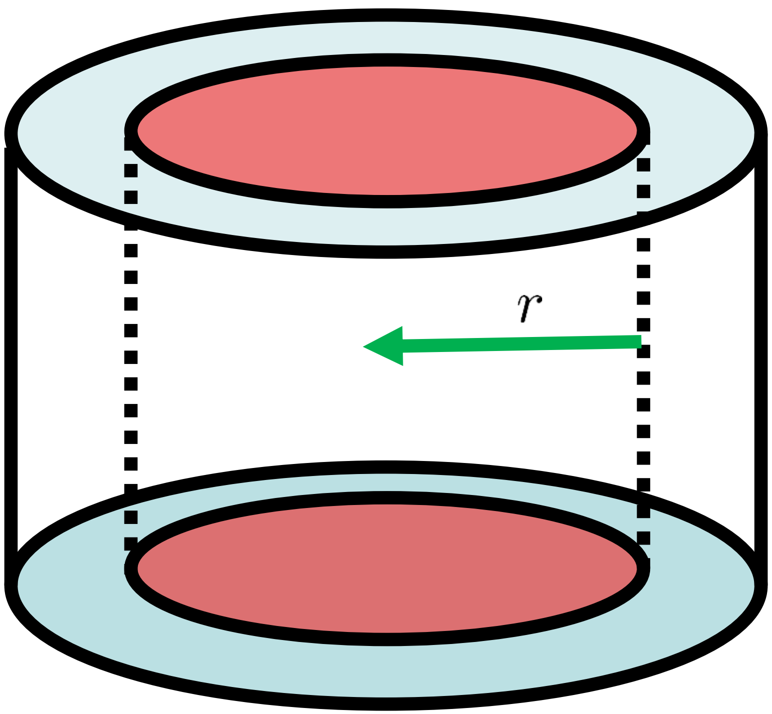
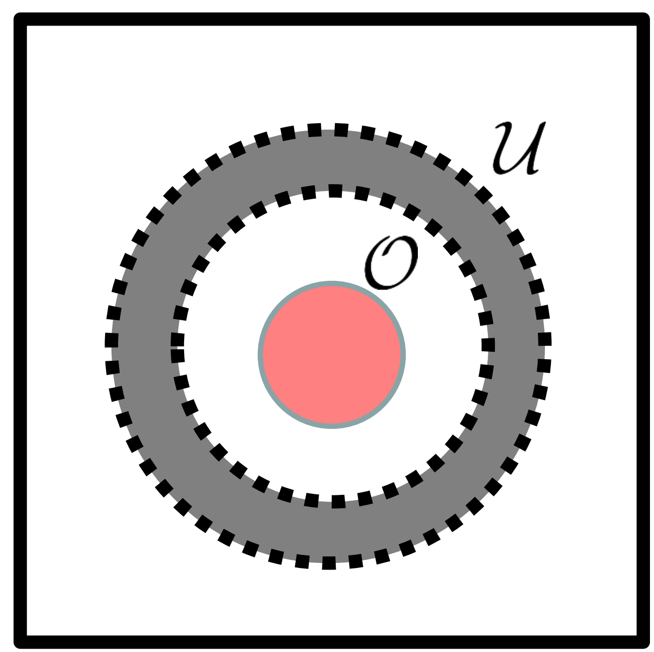


$r = r_{\text{mid}}$



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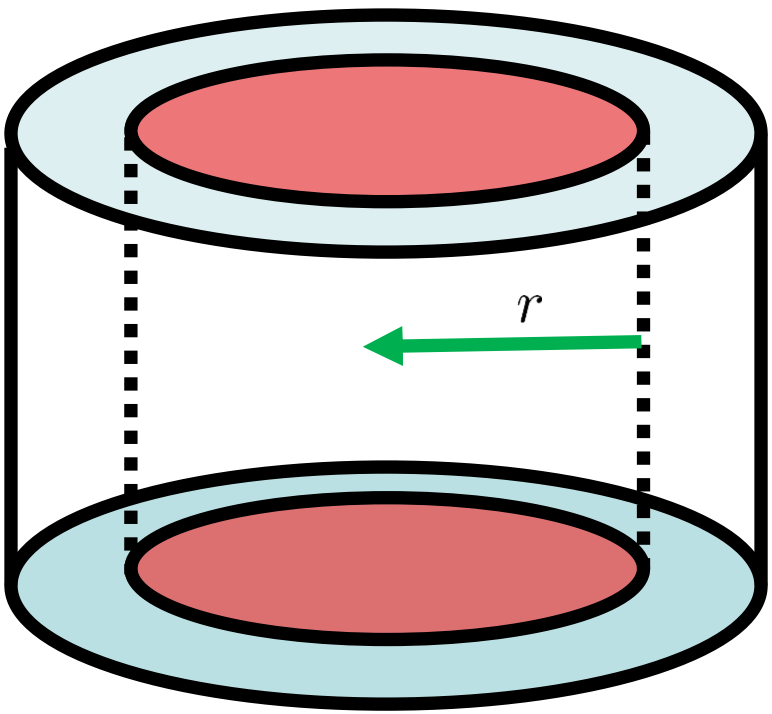
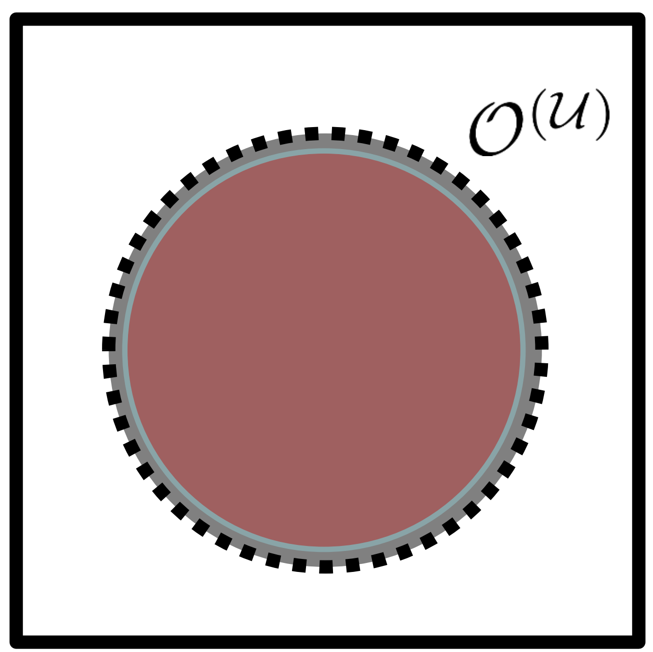


$r < r_*$



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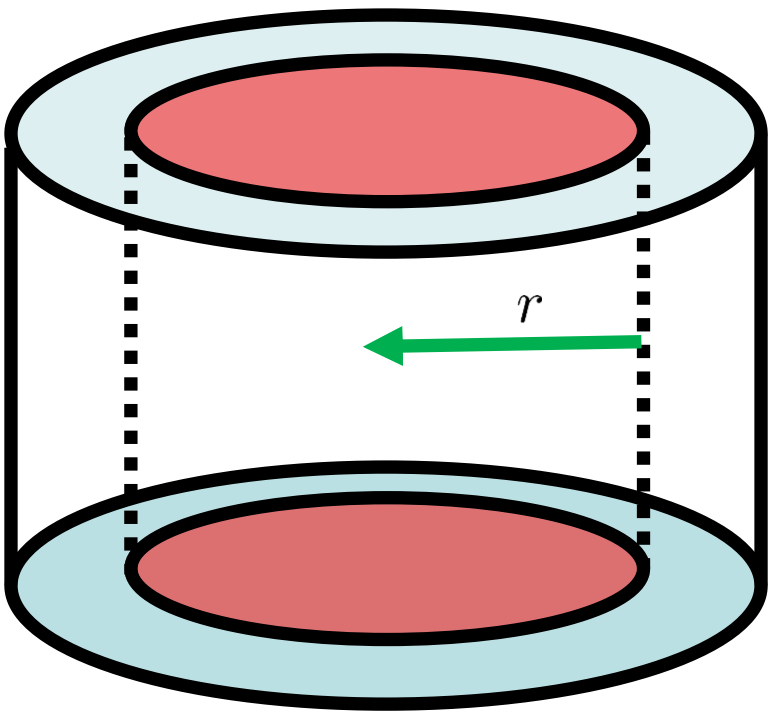

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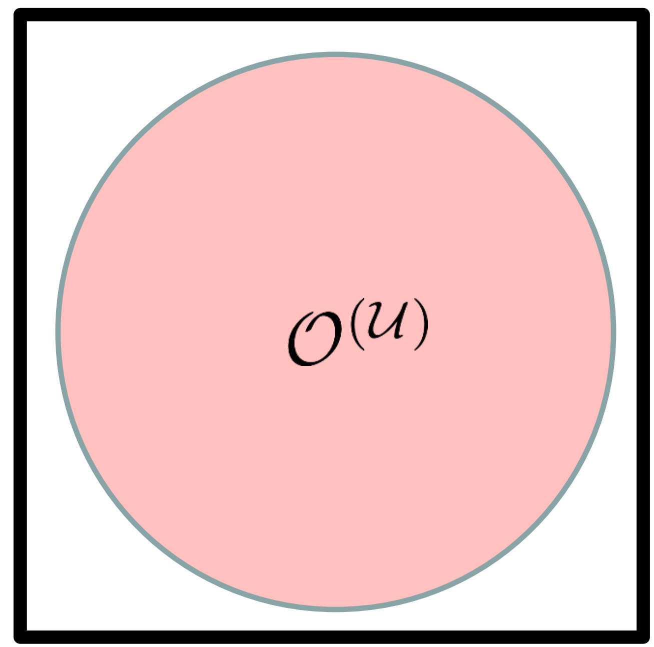
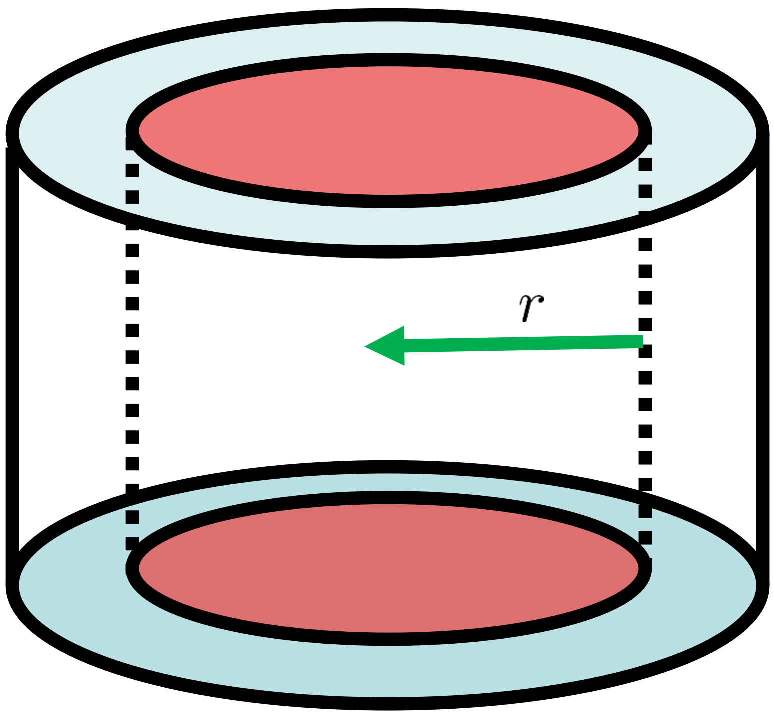


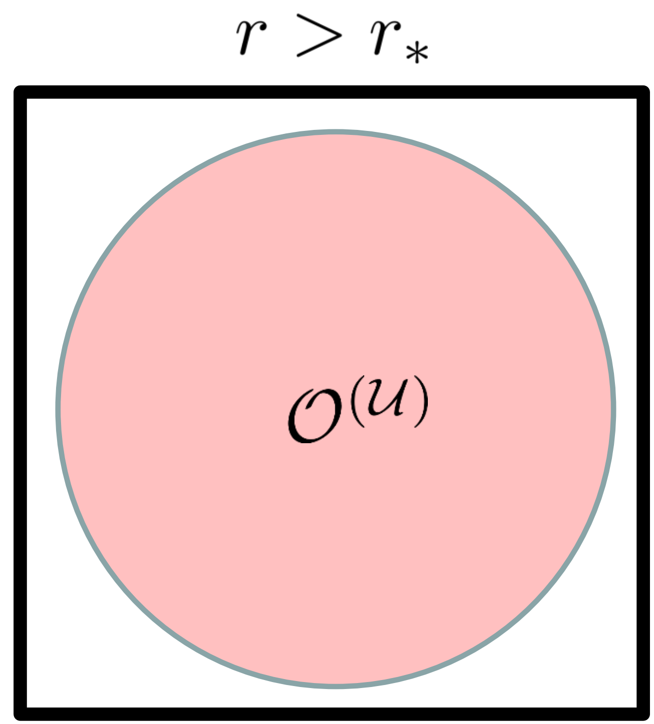
Smearing $\mathcal{U} \oplus \mathcal{O}$

Look at how $\mathcal{U}$ acts on charged $\mathcal{O}$:

CFT + $r_{\mathrm{UV-cutoff}}$

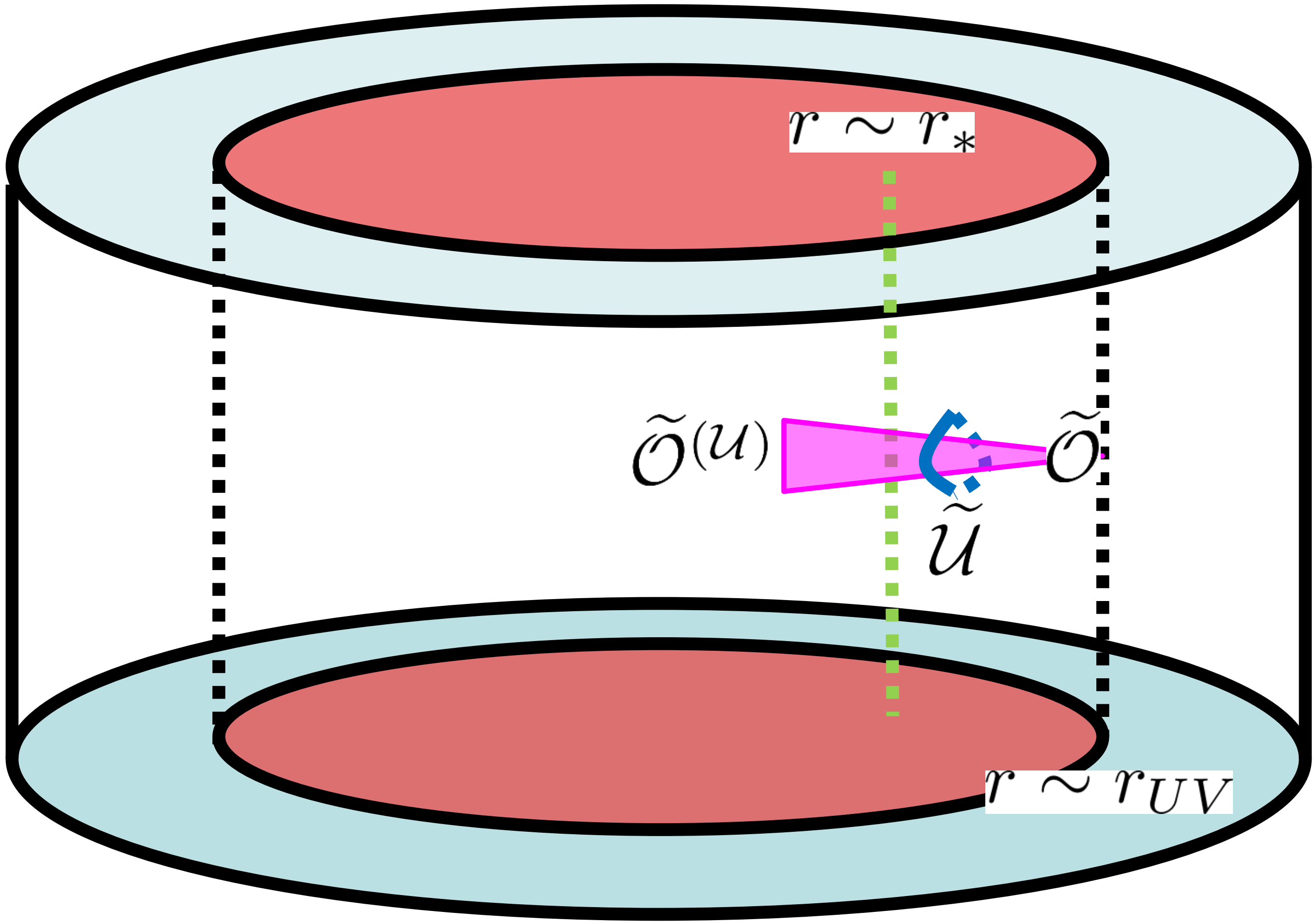
r_* set by extremal surface
Ryu Takayanagi '06;
Hubeny Rangamani Takayanagi '07





$\widetilde{\mathcal{U}}$ Sources $T_{\mu\nu}$: It's a Brane!

JJH Hübner Murdia '24



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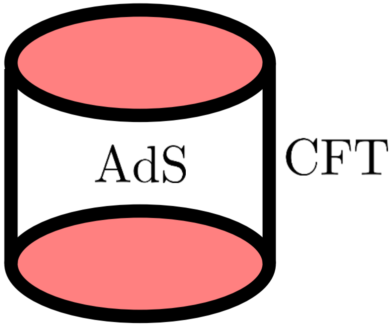
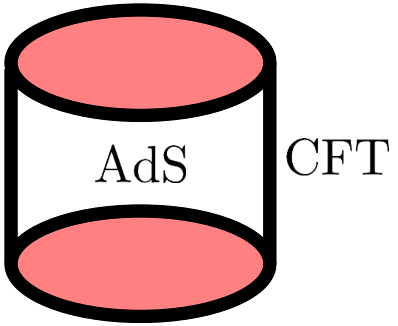


AdS_{D+1}		CFT_D
Radial Evol^n	$\longleftrightarrow$	Smearing
Gauge Symm.	$\longleftrightarrow$	Global Symm.
$\widetilde{\mathcal{U}}_{\mathrm{Brane}}$	$\longleftrightarrow$	$\mathcal{U}_{\mathrm{Symmetry\ Operator}}$

Dictionary Entries



AdS_{D+1}		CFT_D
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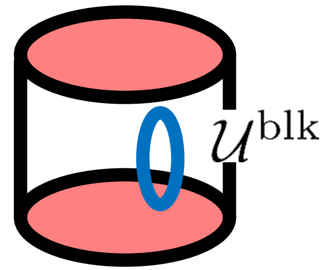
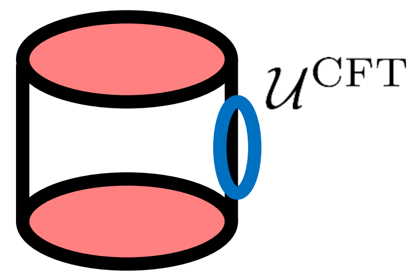
JJH Hübner Murdia '24; JJH Hübner Murdia '25; Cummings JJH '26; complementary to Harlow Ooguri '18

No Global Symmetries!

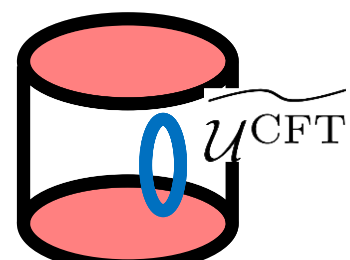
Suppose $\mathcal{U}^{\text{blk}}$ topological



$\Rightarrow \mathcal{U}^{\text{CFT}}$ symmetry operator in CFT



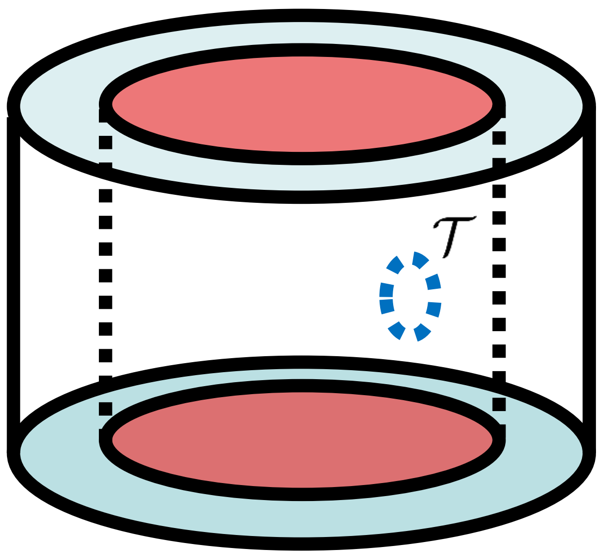
$\Rightarrow \widetilde{\mathcal{U}^{\text{CFT}}}$ Non-Topological in Bulk!



¿A Loophole?

What if $I_{\text{bulk}} = I_{\text{grav}} + I_{\text{TQFT}}$?

$$\mathcal{Z} \sim \int [\mathcal{D}\Phi_{\text{grav}}][\mathcal{D}\Phi_{\text{TQFT}}] e^{-I_{\text{grav}}[\Phi_{\text{grav}}] - I_{\text{TQFT}}[\Phi_{\text{TQFT}}]}$$



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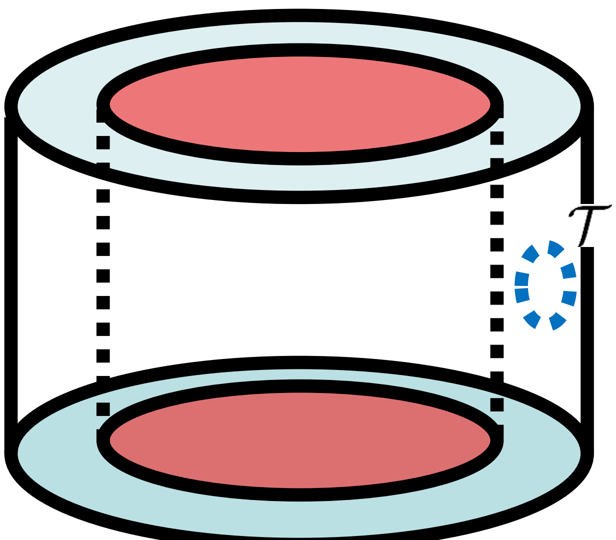

¿A Loophole?

What if $I_{\text{bulk}} = I_{\text{grav}} + I_{\text{TQFT}}$?

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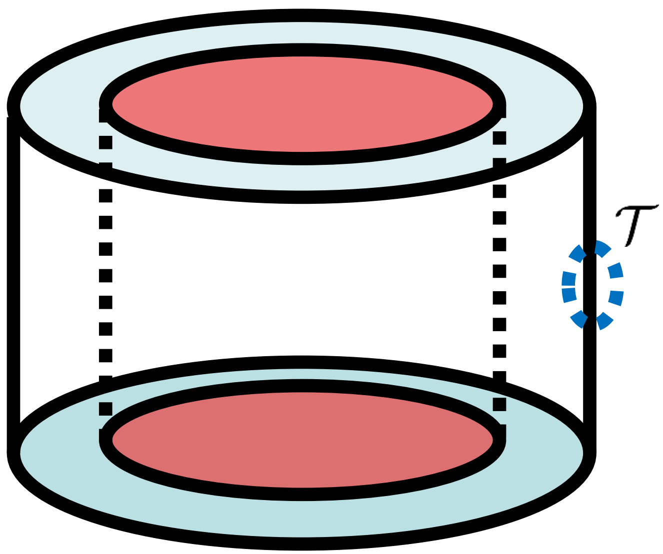


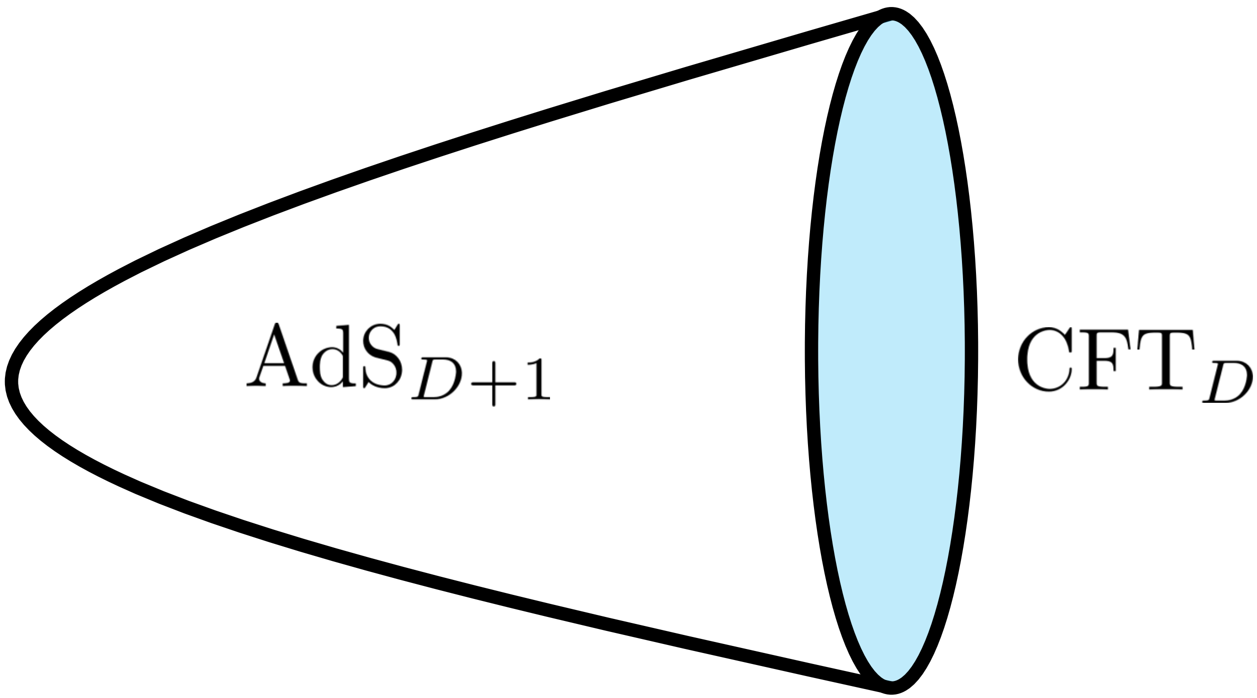
TQFTs vs Gravity

Cummings JJH '26

First: Recall “Just Gravity”

Euclidean AdS/CFT

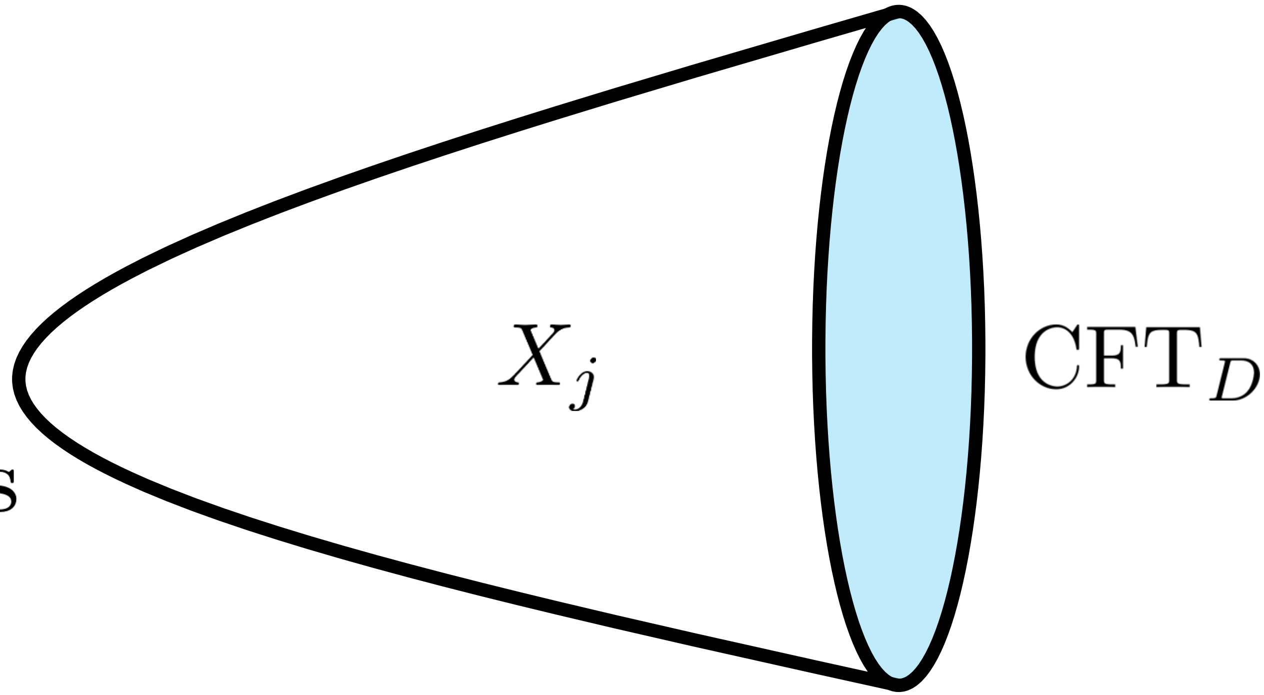
(Lorentzian Version of Argument also Works)



$$Z_{\text{bulk}}[J] = Z_{\text{bdry}}[J]$$

Better Approximation...

$$\sum_{j \in \text{saddles}}$$



$$\partial X_j = M_{\text{CFT}} \text{ for all } j$$

Main Idea: Multiple Saddles

JJH Hübner Murdia '25; Cummings JJH '26

Asymptotic AdS_{D+1}

$$\partial X_j^{(D+1)} = S^p \times S^q$$

Boundary CFT_D

$$S^p \times S^q$$

$$\boxed{\frac{\sum_j e^{-I_j[\widetilde{\mathcal{U}}]}}{\sum_j e^{-I_j}} = \langle \mathcal{U} \rangle_{\mathrm{CFT}} = \frac{Z[\mathcal{U}]}{Z}}$$

Main Idea: Multiple Saddles

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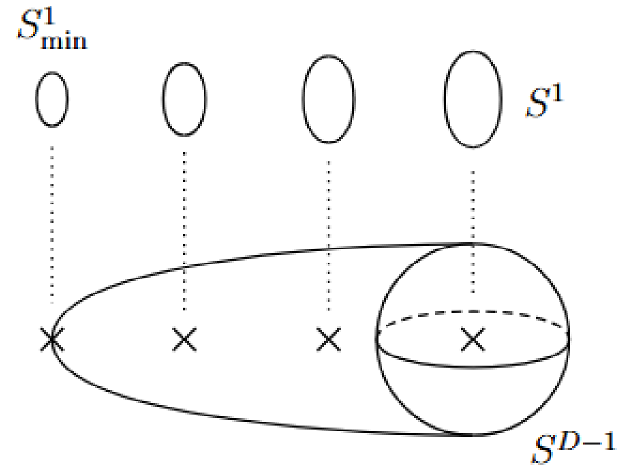
Focus For Today: $S^1 \times S^{D-1}$

Gravitational Saddles

Hawking Page '82

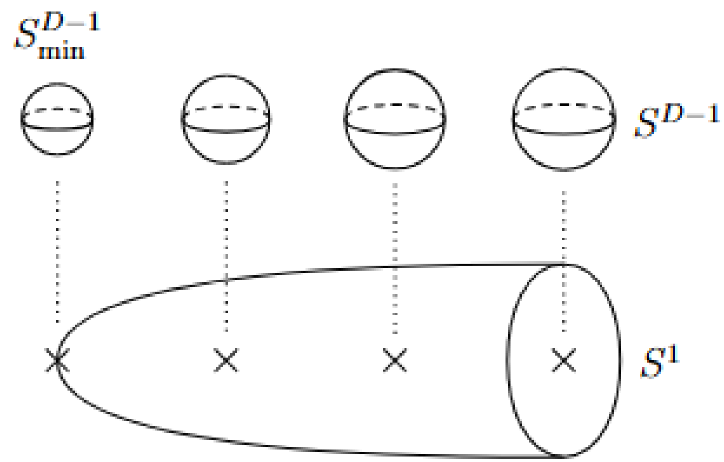
Witten '98

($S^p \times S^q$ case: Aharony Urbach Weiss '19)



Thermal AdS:

$S^1 \to \text{Cone}(S^{D-1})$



AdS-Schwarzschild:

$S^{D-1} \to \text{Cone}(S^1)$

Gravitational Saddles

Thermal AdS:
$S^1 \to \mathrm{Cone}(S^{D-1})$

AdS-Schwarzschild:
$S^{D-1} \to \mathrm{Cone}(S^1)$

$$\frac{e^{-I_{\mathrm{bh}}}}{e^{-I_{\mathrm{th}}}} = \exp\left(\mathcal{S}_{\mathrm{BH}} \times \frac{r_+^2 - \ell_{\mathrm{AdS}}^2}{D r_+^2 + (D-2)\ell_{\mathrm{AdS}}^2}\right)$$

Gravitational Saddles

Thermal AdS:
$S^1 \to \mathrm{Cone}(S^{D-1})$

AdS-Schwarzschild:
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Non-Perturbative: $\mathcal{S}_{\mathrm{BH}} = \frac{A}{4G_N} = \left(\frac{M_{\mathrm{pl}}}{E_{\mathrm{hor}}}\right)^{D-2}$

On-Shell Action + $\widetilde{\mathcal{U}}$

$$\boxed{\frac{\sum_j e^{-I_j[\widetilde{\mathcal{U}}]}}{\sum_j e^{-I_j}} = \langle \mathcal{U} \rangle_\beta = \frac{\mathrm{Tr}(e^{-\beta H}\mathcal{U})}{\mathrm{Tr}(e^{-\beta H})}}$$

Wrap Brane on minimal cycle of X_j;

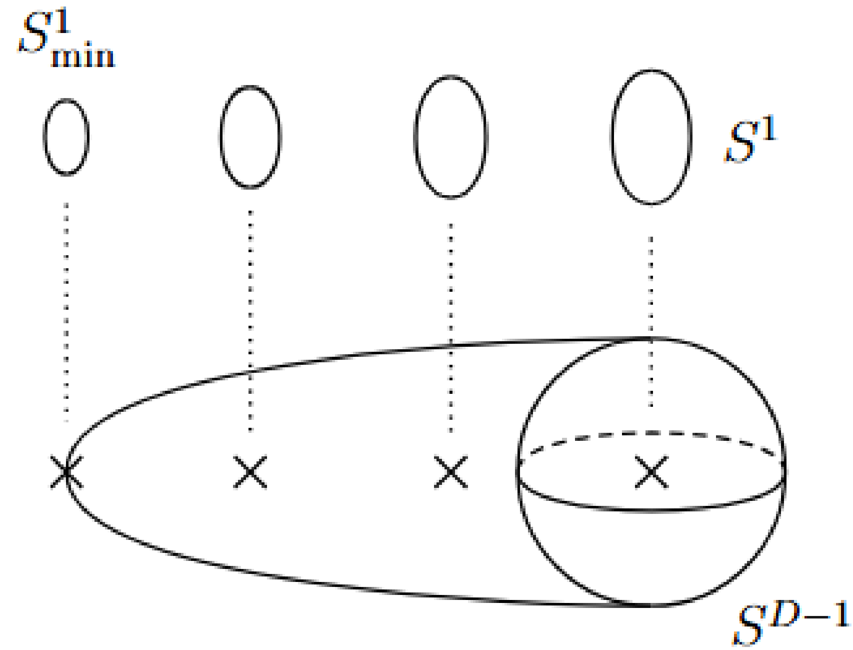
$\partial X_j = S^1_\beta \times S^{D-1}$

Thermal AdS

$$S^1 \to \mathrm{Cone}(S^{D-1})$$

$$\mathrm{Vol}(\gamma_{\min}) \to 0$$

$$\Rightarrow e^{-I_{\rm th}[\widetilde{\mathcal{U}}]} \simeq e^{-I_{\rm th}}$$

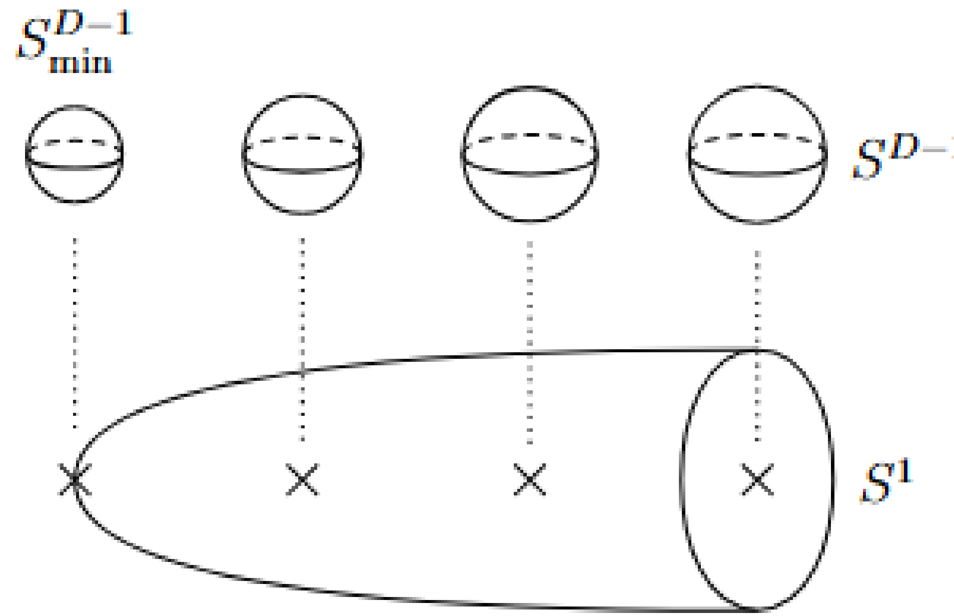


AdS-Schwarzschild + $\widetilde{\mathcal{U}}$

$S^{D-1} \to \mathrm{Cone}(S^1)$

$\Rightarrow \widetilde{\mathcal{U}}$ on outer horizon

$\mathrm{Vol}(\gamma_{\min}) = \mathrm{Area}(\mathrm{Hor})$



On-Shell Action + $\widetilde{\mathcal{U}}$

$$\boxed{\frac{\sum_j e^{-I_j[\widetilde{\mathcal{U}}]}}{\sum_j e^{-I_j}} = \langle \mathcal{U} \rangle_\beta = \frac{\mathrm{Tr}(e^{-\beta H}\mathcal{U})}{\mathrm{Tr}(e^{-\beta H})}}$$

$$\boxed{\frac{e^{-I_{\mathrm{th}}} + e^{-I_{\mathrm{bh}}[\widetilde{\mathcal{U}}]}}{e^{-I_{\mathrm{th}}} + e^{-I_{\mathrm{bh}}}} = \langle \mathcal{U} \rangle_\beta = \frac{\mathrm{Tr}(e^{-\beta H}\mathcal{U})}{\mathrm{Tr}(e^{-\beta H})}}$$

JJH Hübner Murdia ’25

Examples

Symmetries of $\mathcal{N} = 4$ SYM

- Duality / Triality Defects

  Constant τ_{IIB} $[p, q]$ 7-branes @ infinity

  JJH Hübner Torres Zhang Yu '22

- R-symmetries

  KK monopoles from metric isometries

  Cvetič JJH Hübner Murdia '25

  (see also Bah Bonetti Chitoto Leung '25; Mignosa Rodriguez-Gomez '26)

Example:
Duality / Triality Defects
4D $\mathcal{N} = 4$ $\mathfrak{su}(N)$ SYM

Duality Defect: CFT$_4$ Picture

Global Form: $G = (SU(N)/\mathbb{Z}_K)_m$

$$\tau_{\mathrm{IIB}} = \tau_{\mathrm{YM}} = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$$

Kaidi Ohmori Zheng '21; Choi Cordova Hsin Lam Shao '21;
JJH Hübner Torres Yu Zhang '22 + Many...

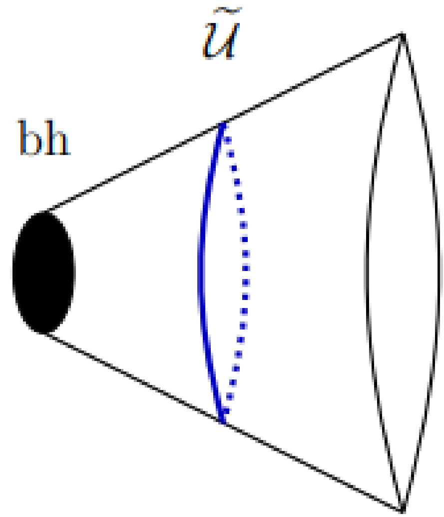
G

$\tau = i$

$G' = G$

$\tau' = -1/\tau = i$

Gravity Picture



Const. τ_{IIB} 7-brane on $S^3 \times S^5 \subset \mathrm{AdS}_5 \times S^5$

$$\exp\left(-\frac{\chi}{2\pi} \times \mathcal{S}_{\mathrm{BH}}\right) \simeq \langle \mathcal{U} \rangle_\beta$$

JJH Hübner Torres Yu Zhang '22
JJH Hübner Murdia '25

Gravity Picture

τ_{IIB}	Lie Algebra	Kodaira Fiber	7-branes	χ
$\exp(2\pi i/6)$	$\mathfrak{e}_8$	II^*	A^7BC^2	$10 \times \frac{2\pi}{12}$
$\exp(2\pi i/4)$	$\mathfrak{e}_7$	III^*	A^6BC^2	$9 \times \frac{2\pi}{12}$
$\exp(2\pi i/6)$	$\mathfrak{e}_6$	IV^*	A^5BC^2	$8 \times \frac{2\pi}{12}$
$\exp(2\pi i/6)$	$\mathfrak{su}_3$	IV	A^3C	$4 \times \frac{2\pi}{12}$
$\exp(2\pi i/4)$	$\mathfrak{su}_2$	III	A^2C	$3 \times \frac{2\pi}{12}$
$\exp(2\pi i/6)$	$\mathfrak{su}_1$	II	AC	$2 \times \frac{2\pi}{12}$

Const. τ_{IIB} 7-brane on $S^3 \times S^5 \subset \mathrm{AdS}_5 \times S^5$

$$\exp\left(-\frac{\chi}{2\pi} \times \mathcal{S}_{\mathrm{BH}}\right) \simeq \langle \mathcal{U} \rangle_\beta$$

JJH Hübner Torres Yu Zhang '22
JJH Hübner Murdia '25

Example: R-Symmetries 4D $\mathcal{N} = 4$ $\mathfrak{su}(N)$ SYM

JJH Hübner Murdia ’25

JJH Hübner Murdia Cvetič ’25

Isom(S^5) & R-Symmetries

10D Picture: $\mathrm{AdS}_5 \times S^5$

Focus on Cartan: $U(1)^3 \subset SO(6)$

CFT_4: $\mathcal{U}_\alpha = \exp(2\pi i(\alpha_1 Q_1 + \alpha_2 Q_2 + \alpha_3 Q_3))$

AdS_5: $\widetilde{\mathcal{U}}_\alpha =$ KK monopole for $U(1)^3$

JJH Hübner Murdia Cvetič '25

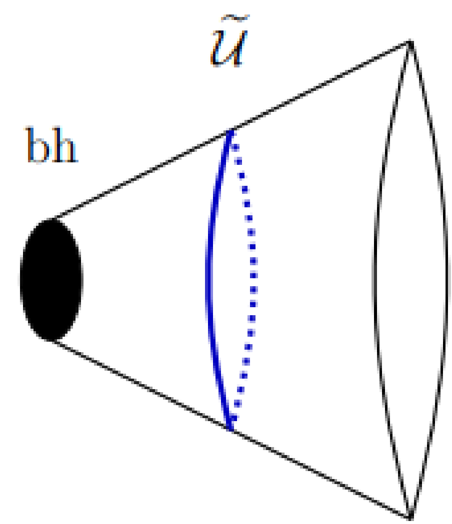
Tension of $\widetilde{\mathcal{U}}_\alpha$

Tension: $T = \frac{\chi}{8\pi G_N} = \frac{\alpha_1+\alpha_2+\alpha_3}{4G_N}$

Arav Gauntlett Jiao Roberts Rosen '24; Bomans Tranchedone '24

Cvetič JJH Hübner Murdia '25

$$e^{-(\alpha_1+\alpha_2+\alpha_3)\mathcal{S}_{\rm BH}} \simeq \langle \mathcal{U}_\alpha \rangle_\beta$$



Cross-Check:
5D BPS Black Holes

JJH Hübner Murdia ’25

Black Holes in AdS_5

R-charges: Q_1, Q_2, Q_3
Ang. Momenta: J_1, J_2

Chong Cvetič Lu Pope '04;
Kunduri Lucietti Reall '05;
Hosseini Hristov Zaffaroni '17;
Cabo-Bizet Cassani Martelli Murthy '18;
Choi Kim Kim Nahmgoong '18;
Benini Milan '18;
....

Entropy Function: $\Gamma_{\text{crit}} = \mathcal{S}_{\text{BH}}$

Sen '05
Hosseini Hristov Zaffaroni '17

$$\Gamma = I - 2\pi i(\sum_a Q_a \varphi_a + \sum_j J_j \omega_j)$$
$$-2\pi i \Lambda(\sum_a \varphi_a - \sum_j \omega_j + 1)$$

On-Shell Action: $I = -\pi i N^2 \frac{\varphi_1 \varphi_2 \varphi_3}{\omega_1 \omega_2}$

CFT_4 Index

R-charges: Q_1, Q_2, Q_3
Ang. Momenta: J_1, J_2

Cabo-Bizet Cassani Martelli Murthy '18;
Choi Kim Kim Nahmgoong '18;
Benini Milan '18;
....

$$\langle \mathcal{U}_\alpha \rangle_{\text{Index}} = \exp\left(2\pi i \sum_a \alpha_a Q_a - (\alpha_1 + \alpha_2 + \alpha_3)\, \mathcal{S}_{\text{BH}} \right)$$

JJH Hübner Murdia '25

$$\langle \mathcal{U}_\alpha \rangle_{\text{Index}} = \frac{\mathrm{Tr}\Big((-1)^F e^{-\beta\{\mathcal{Q},\overline{\mathcal{Q}}\}} p^{J_1 + \frac{1}{2}R} q^{J_2 + \frac{1}{2}R} y_1^{q_1} y_2^{q_2} \mathcal{U}_\alpha \Big)}{\mathrm{Tr}\Big((-1)^F e^{-\beta\{\mathcal{Q},\overline{\mathcal{Q}}\}} p^{J_1 + \frac{1}{2}R} q^{J_2 + \frac{1}{2}R} y_1^{q_1} y_2^{q_2} \Big)}$$

CFT$_4$ Index

R-charges: Q_1, Q_2, Q_3

Ang. Momenta: J_1, J_2

$$\langle \mathcal{U}_\alpha \rangle_{\text{Index}} = \exp\left(2\pi i \sum_a \alpha_a Q_a - (\alpha_1 + \alpha_2 + \alpha_3)\, \mathcal{S}_{\text{BH}} \right)$$

Horizon Wrapping

$$\frac{\chi}{2\pi} \times \frac{\text{Area}}{4G_N}$$

CFT$_4$ Index

R-charges: Q_1, Q_2, Q_3

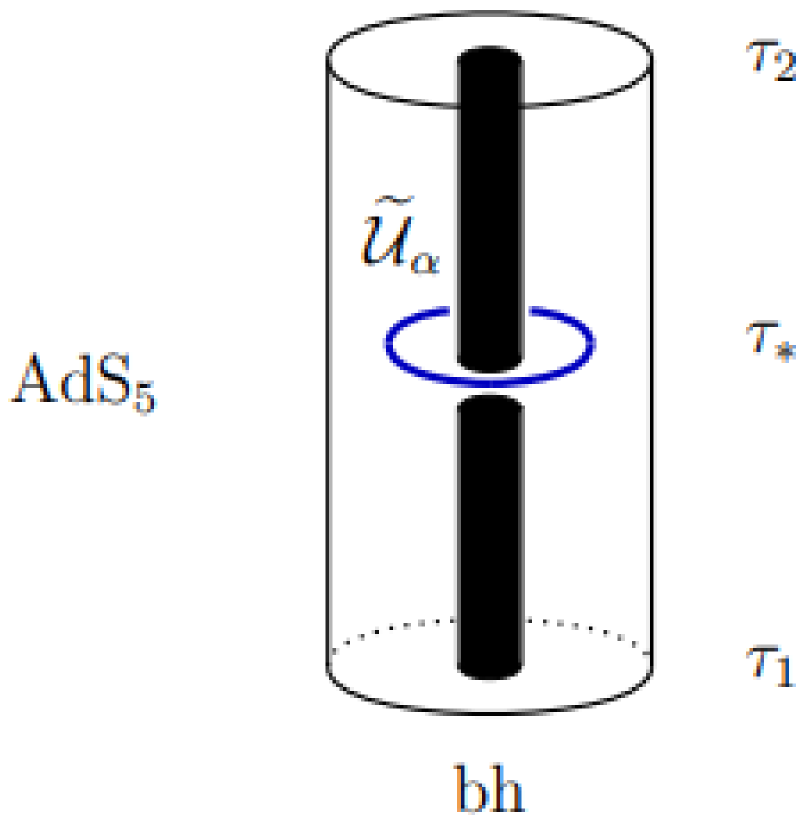
Ang. Momenta: J_1, J_2

$$\langle \mathcal{U}_\alpha \rangle_{\text{Index}} = \exp\left(2\pi i \underbrace{\sum_a \alpha_a Q_a}_{\text{Topological Linking}} - \underbrace{(\alpha_1 + \alpha_2 + \alpha_3)\, \mathcal{S}_{\text{BH}}}_{\text{Horizon Wrapping}} \right)$$

$$\frac{\chi}{2\pi} \times \frac{\text{Area}}{4G_N}$$

Linking / Wrapping

$$\langle \mathcal{U}_\alpha \rangle_{\text{Index}} = \exp\left(2\pi i \sum_a \alpha_a Q_a - (\alpha_1 + \alpha_2 + \alpha_3)\, \mathcal{S}_{\text{BH}}\right)$$



TQFTs vs Gravity

Cummings JJH '26

Keeping a TQFT “Decoupled”

$$I_{\text{full}} = I_{\text{grav}} + I_{\text{TQFT}}$$

Question: Does this produce a contradiction?

Keeping a TQFT "Decoupled"

$I_{\rm full} = I_{\rm grav} + I_{\rm TQFT}$

Question: Does this produce a contradiction?

Assume: No Coupling of TQFT fields to the metric,
"knows about" gravity via topology

Assume: $\exists$ TQFT operator $\mathcal{T}$ and some bkgnd X
such that $\langle \mathcal{T} \rangle_{X,{\rm TQFT}} \neq d_{\mathcal{T}}$

Main Idea: Multiple Saddles

Cummings JJH '26

Gravity + TQFT	$\mathrm{CFT}_D + \mathrm{QFT}_{\mathrm{extra}}$
$\partial X^{(j)}_{D+1} = S^p \times S^q$	$S^p \times S^q$
$Z_{\mathrm{bulk}}[\mathcal{T}] = \sum_j w_j Z^{(j)}_{\mathrm{TFT}}[\mathcal{T}]$	$Z_{\mathrm{bdry}}[\mathcal{T}] = Z_{\mathrm{CFT}} Z_{\mathrm{extra}}[\mathcal{T}]$

$$¿¿¿ Z_{\mathrm{bulk}}[\mathcal{T}] = Z_{\mathrm{bdry}}[\mathcal{T}]???$$

Mild Assumption

Cummings JJH '26

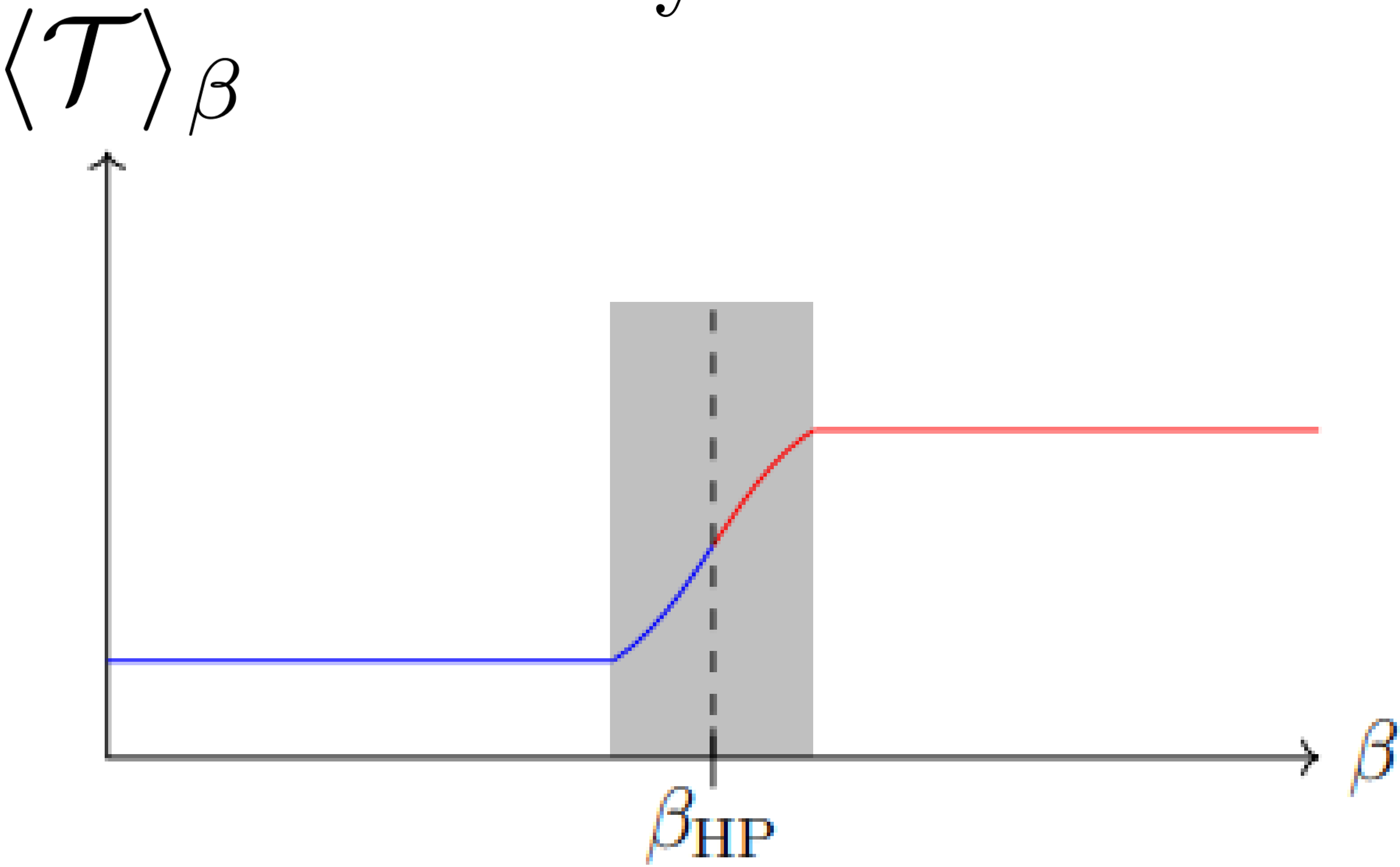
In the Bulk Geometry,

$\exists\ \mathcal{T}$ of TFT & $X \neq X'$ with $\partial X = \partial X' = S^p \times S^q$

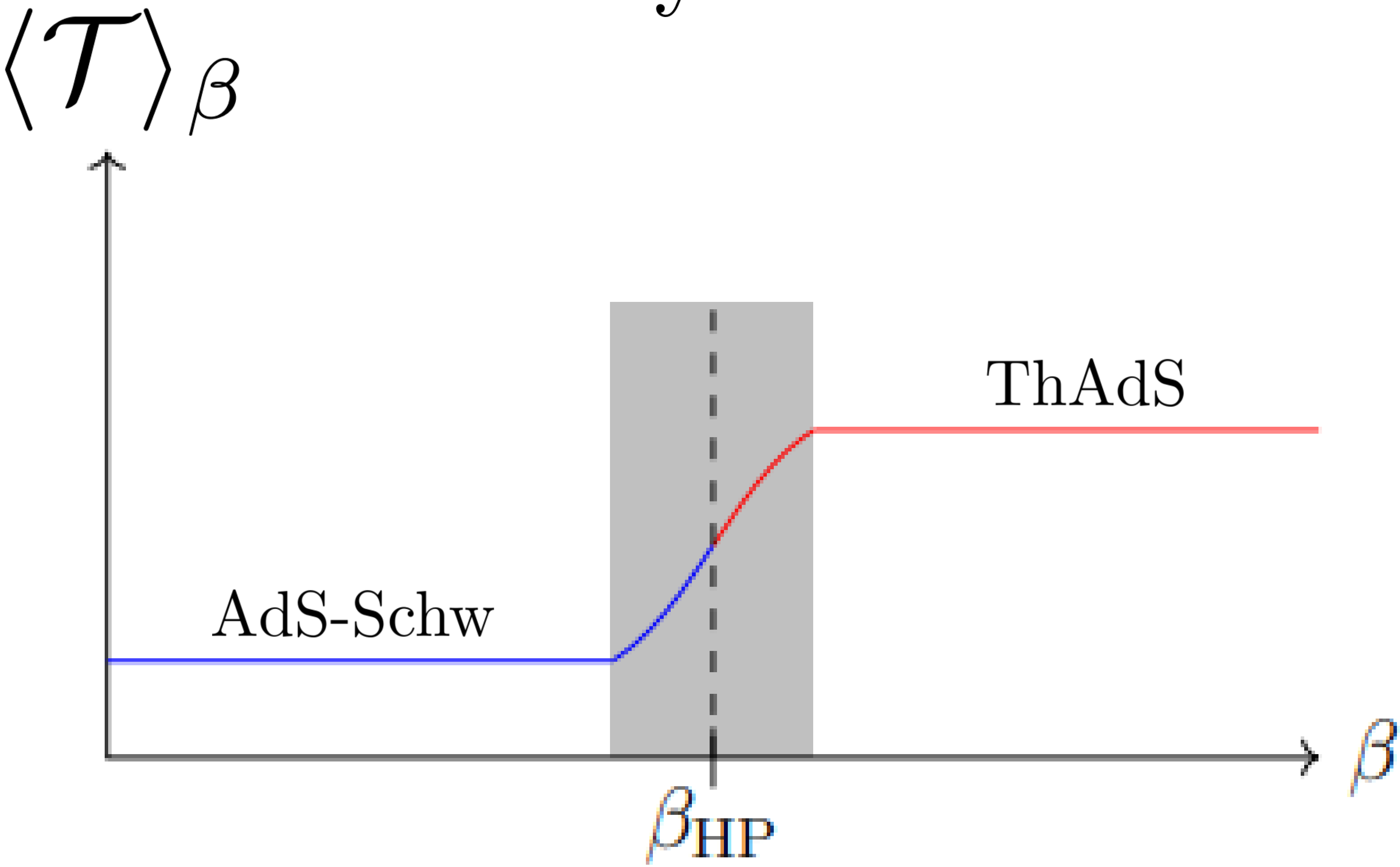
Such That: $\langle \mathcal{T} \rangle_{\mathrm{TFT},X} \neq \langle \mathcal{T} \rangle_{\mathrm{TFT},X'} \neq d_{\mathcal{T}}$

Example: $\mathcal{T} = e^{i \int_{\Sigma_p} \mathcal{O}}$ but only X has a p-cycle.

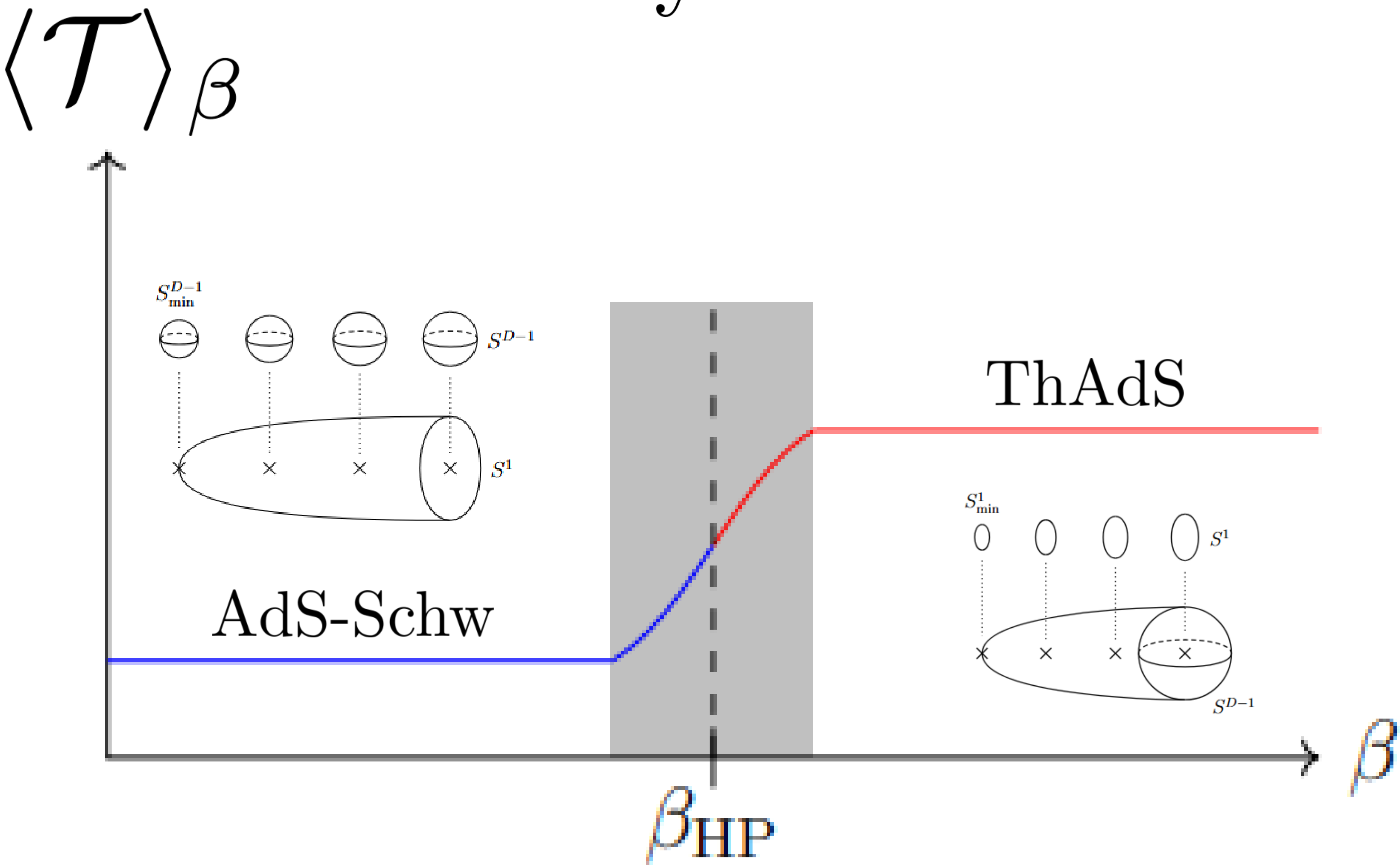
Gravity Answer



Gravity Answer



Gravity Answer



$$¿¿¿Z_{\mathrm{bulk}}[\mathcal{T}] = Z_{\mathrm{bdry}}[\mathcal{T}]???$$

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NO.

$$¿¿¿Z_{\text{bulk}}[\mathcal{T}] = Z_{\text{bdry}}[\mathcal{T}]???$$

NO.

$Z_{\text{bulk}}[\mathcal{T}]$ Depends on Dominant Saddle X_j

$Z_{\text{bdry}}[\mathcal{T}]$ Independent of all of this!

Contradiction.

$$¿¿¿Z_{\text{bulk}}[\mathcal{T}] = Z_{\text{bdry}}[\mathcal{T}]???$$

NO.

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$Z_{\text{bdry}}[\mathcal{T}]$ Independent of all of this!

Contradiction.

Contradiction $\Rightarrow$

Gravity + TQFT	$\mathrm{CFT}_D + \mathrm{QFT}_{\mathrm{extra}}$
TQFT NOT Decoupled	Doesn't Factorize

$$\widetilde{\mathcal{T}}_{\mathrm{brane}} \longleftrightarrow \mathcal{T}_{\mathrm{bdry}}$$

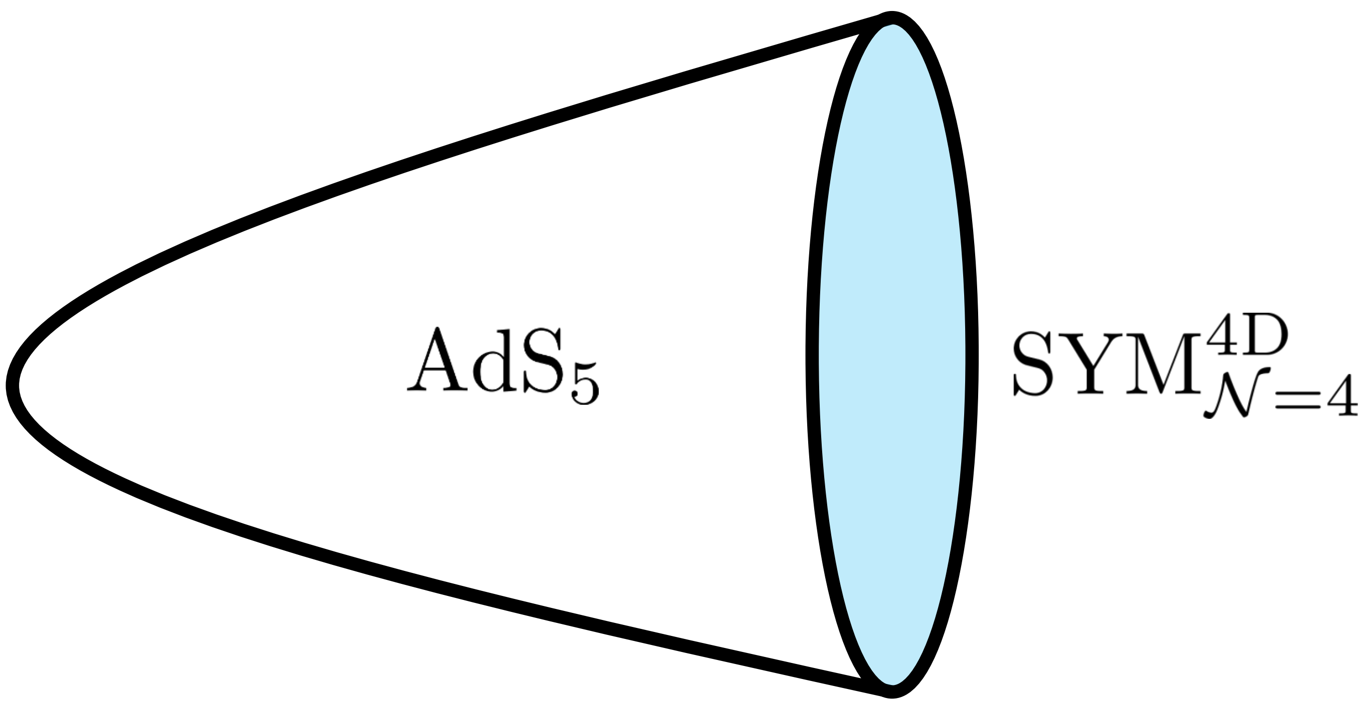
$\Rightarrow$ Local Source of $T_{\mu\nu}$ from TFT

$\Rightarrow$ *Perturbative* Coupling to $h_{\mu\nu}$

Example

Example: $\mathrm{AdS}_5 \times S^5$

$$S_{\mathrm{bulk}} \supset \int |dB|^2 + |dC|^2 + \frac{N}{2\pi} B \wedge dC$$



¿More Abstract TQFTs?

View TQFT as IR limit of a "Bigger QFT"

¿Concrete Examples?

Summary / Future

Summary / Future

- TQFTs are in the Swampland
- Bulk Saddles vs Bdry Factorization

- “Kinetic Terms” & more abstract TQFTs?
- Sharper Bounds for E_{gap}?