Intrinsic DQCP Transitions from Twin Phases

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Based on [2605.31601] and [2605.31602]
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Introduction and Summary

Are all phase transitions dual to SSB transitions?

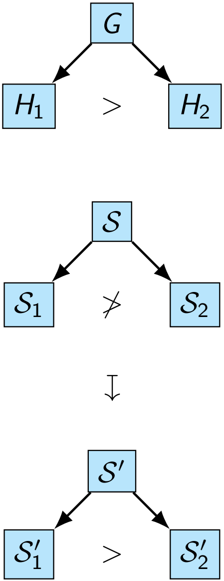
- Landau theory: continuous phase transitions are spontaneous symmetry breaking (SSB) transitions (reviews of generalized Landau [McGreevy],[Chen])
- Many 1+1d transitions appear beyond Landau but become Landau for a dual symmetry obtained by (partial) gauging
- Examples: $\mathcal{S} = \mathbb{Z}_2^a \times \mathbb{Z}_2^b$

  1) Triv $\not>$ SPT $\longmapsto$ Triv $>$ $\mathbb{Z}_2^2$ SSB

     [Kennedy, Tasaki] "Hidden Symmetry Breaking"

  2) With mixed anomaly $\mathbb{Z}_2^a \not> \mathbb{Z}_2^b \longmapsto$ Triv $> \mathbb{Z}_4$ SSB

     [Zhang, Levin]
- Are all phase transitions dual to symmetry-breaking transitions?



Phase Transitions Without Hidden Symmetry Breaking

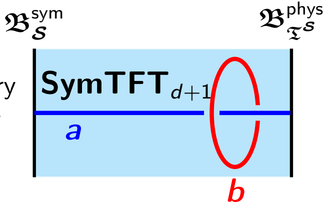
- Are all phase transitions dual to symmetry-breaking transitions?
- No [AW, Gai, Schäfer-Nameki]
- Why? Because we can have **twin phases**: a pair of inequivalent phases with the same number of ground states for all (partial) gaugings $\Rightarrow$ no relative SSB
- Mathematically, they come from **twin algebras**: condensable algebras with same object but different algebra structures
- Direct and stable **phase transitions between twin phases are intrinsically beyond Landau**: not dual to any SSB transition

$$\begin{tikzcd}
 & \boxed{\mathcal{S}} \arrow[dl] \arrow[dr] & \\
\boxed{\mathcal{S}_1} & \not\simeq & \boxed{\mathcal{S}_2} \\
 & {} \arrow[d, maps to] & \\
 & {} & \\
 & \boxed{\mathcal{S}'} \arrow[dl] \arrow[dr] & \\
\boxed{\mathcal{S}'_1} & \not\simeq & \boxed{\mathcal{S}'_2}
\end{tikzcd}$$

- Deconfined Quantum Critical Point (DQCP) is a direct stable transition between incompatible subgroup SSB. **Twins** $\Rightarrow$ **Intrinsic DQCP Transitions**

Quantum Phases with Generalized Symmetries from the SymTFT

Symmetry boundary fixes symmetry $\mathcal{S}$



Physical boundary dynamics of $\mathfrak{T}$

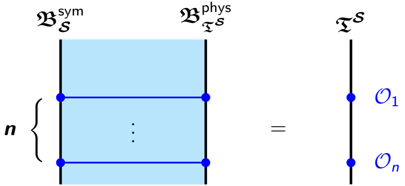
- Generalized charges of operators: from linking in the SymTFT bulk
- Example: $\mathbb{Z}_N$ SymTFT from $U(1)$ BF-theory:

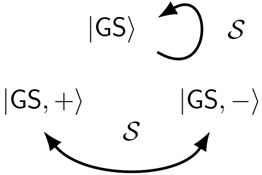
$$S_{BF} = \frac{N}{2\pi} \int_{d+1} b \wedge da$$

Bulk topological line operators with commutation relation:

$$e^{i\int_\Sigma b}\, e^{i\int_M a} = e^{\frac{2\pi i\, \mathrm{Link}(\Sigma, M)}{N}}\, e^{i\int_M a}\, e^{i\int_\Sigma b}$$

[Freed, Moore, Teleman], [Kaidi, Ohmori, Zheng],[Chatterjee, Wen], [Bhardwaj, Bottini, Pajer, Schäfer-Nameki], [Wen, Potter], [Antinucci, Benini, Copetti, Galati, Rizi], [Bhardwaj, Pajer, Schäfer-Nameki, AW], [Bonetti, Del Zotto, Minasian] [Cordova, García-Sepúlveda, Harvey],[Gannon, Rayhaun]

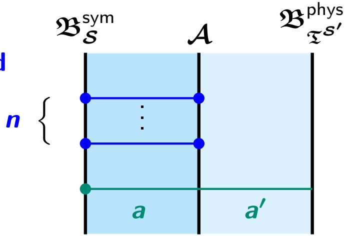




- **SPT**: symmetry protected topological phase
  $\boldsymbol{n} = \mathbf{1}$ symmetric ground state
- **SSB**: spontaneous symmetry breaking
  phase $\boldsymbol{n} > \mathbf{1}$ ground states related by $\mathcal{S}$
- **Topological order:** SSB for higher-form symmetry
- Systematic classification for any $\mathcal{S} \Rightarrow$ **Categorical Landau paradigm**
- This talk will focus on 1+1d: TQFTs are characterized by
  $\boldsymbol{n}$ = number of ground states $= \sum_a n^a_{\text{sym}} n^a_{\text{phys}}$ and symmetry action on them

- **Gapless** phases from **non-maximal** SymTFT condensable algebra $\mathcal{A}$
  SymTFT$(\mathcal{S})/\mathcal{A} =$ SymTFT$(\mathcal{S}')$ for a reduced symmetry $\mathcal{S}'$

anyons
- $\to$ end on $\mathfrak{B}^{\text{sym}}_{\mathcal{S}}$ and $\mathcal{I}_{\mathcal{A}} \Rightarrow$ **condensed**
- $\to$ non-local with $\mathcal{A} \Rightarrow$ **confined**
- $\to$ local with $\mathcal{A} \Rightarrow$ **de-confined**



- New phase transitions from old:

$$\mathfrak{T}^{\mathcal{S}'}_B \xleftarrow{\;-\mathcal{O}'\;} \mathfrak{T}^{\mathcal{S}'}_{\text{CFT}} \xrightarrow{\;+\mathcal{O}'\;} \mathfrak{T}^{\mathcal{S}'}_A$$

map of anyons

$$\Downarrow$$

$$\mathfrak{T}^{\mathcal{S}}_B \xleftarrow{\;-\mathcal{O}\;} \mathfrak{T}^{\mathcal{S}}_{\text{CFT}} \xrightarrow{\;+\mathcal{O}\;} \mathfrak{T}^{\mathcal{S}}_A$$

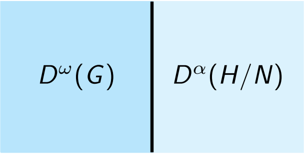
Twin Algebras and Twin Phases

Mathematical Definitions

- The topological defects of the SymTFT for a theory with fusion category symmetry $\mathcal{S}$ form the Drinfeld center $\mathcal{Z}(\mathcal{S})$.
- A *condensable algebra* $\mathcal{A}$ in $\mathcal{Z}(\mathcal{S})$ is a connected commutative symmetric normalized-special Frobenius algebra in $\mathcal{Z}(\mathcal{S})$.
  Physically, it encodes an algebra of anyons that can consistently be condensed (bosons with trivial mutual braiding) and determines a topological interface.
- A *Lagrangian algebra* $\mathcal{A}$ is a condensable algebra of maximal dimension: $\dim(\mathcal{A}) = \dim(\mathcal{S})$. Physically it encodes topological boundary conditions.
- We will focus on 1+1d theories with $\mathcal{S}$ related by gauging to a finite group G with possible anomaly $\omega \in H^3(G, U(1))$.
  The 2+1d SymTFT is the twisted quantum double $D^\omega(G)$ (Dijkgraaf-Witten theory) whose topological defects form $\mathcal{Z}(\mathrm{Vec}_G^\omega)$.

- Condensable Algebras in $D^\omega(G)$ are classified by
  1. $H \subseteq G$ a subgroup (preserved symmetry)
  2. $N \triangleleft H$ a normal subgroup (symmetry acting trivially)
  3. $\gamma : N \times N \to U(1)$ such that $d\gamma = \omega|_N$
     $\mathcal{O}_{n_1}\mathcal{O}_{n_2} = \gamma(n_1, n_2)\mathcal{O}_{n_1 n_2}$ (N-projective representation)
  4. $\epsilon : H \times N \to U(1)$ (H-action on N) $h\,\mathcal{O}_n h^{-1} = \epsilon(h, n)\mathcal{O}_{hnh^{-1}}$

$\mathcal{I}_{\mathcal{A}(H,N,\gamma,\epsilon)}$



[Davydov, Simmons], [Ostrik], [Natale], [Gai, Schafer-Nameki, AW]

- The interface $\mathcal{I}_{\mathcal{A}(H,N,\gamma,\epsilon)}$ encodes the equivalence of modular tensor categories

$$\mathcal{Z}(\mathrm{Vec}_G^\omega)^{\mathrm{loc}}_{\mathcal{A}(H,N,\gamma,\epsilon)} \cong \mathcal{Z}(\mathrm{Vec}^\alpha_{H/N})$$

Isomorphism relation on condensable algebras

- Two condensable algebras $\mathcal{A}(H, N, \gamma, \epsilon)$ and $\mathcal{A}(H', N', \gamma', \epsilon')$ in $\mathcal{Z}(\mathrm{Vec}_G^\omega)$ are isomorphic, if and only if there exists $g \in G$ such that

$$\boxed{\begin{aligned} H' &= {}^gH, \quad \gamma'({}^g n_1, {}^g n_2) = \frac{\gamma(n_1, n_2)}{\Omega_g(n_1, n_2)}, \\ N' &= {}^gN, \qquad \epsilon'({}^g h, {}^g n) = \frac{\Omega_g({}^h n, h)}{\Omega_g(h, n)}\epsilon(h, n), \end{aligned}}$$

where for $h \in H$ and $n_1, n_2, n \in N$ we denote ${}^g x = gxg^{-1}$, $\quad {}^gH = gHg^{-1}$ and

$$\Omega_f(g, h) = \frac{\omega(f, g, h)\omega({}^fg, {}^fh, f)}{\omega({}^fg, f, h)}$$

- Example: for $G = \mathbb{Z}_2^{(a)} \times \mathbb{Z}_2^{(b)} \times \mathbb{Z}_2^{(c)}$ and anomaly $\omega_{\mathrm{III}} = \hat{a} \cup \hat{b} \cup \hat{c}$, the algebras with $H = N = \mathbb{Z}_2^{(a)} \times \mathbb{Z}_2^{(b)}$ and trivial or non-trivial $\gamma \in H^2(\mathbb{Z}_2^2, U(1))$ (which would be inequivalent without anomaly) become equivalent!

Anyon decomposition of condensable algebras

- The condensable algebra $\mathcal{A}(H, N, \gamma, \epsilon)$ in $\mathcal{Z}(\mathrm{Vec}_G^\omega)$ has decomposition:

$$\boxed{\mathcal{A}(H, N, \gamma, \epsilon) \cong \bigoplus_{([x],\rho)} n_{\mathcal{A}}^{([x],\rho)}([x], \rho)}$$

with multiplicity of simple object $([x], \rho)$ given by scalar product of characters

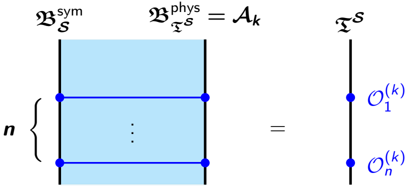
$$n_{\mathcal{A}}^{([x],\rho)} = \sum_{\substack{f\in[x],\\ g\in C_G(f)}} \left\{ \frac{\overline{\mathrm{Tr}(\rho_f(g))}}{|G|} \sum_{\substack{y\in R,\\ {}^{\overline{y}}f\in N, g^y\in H}} \frac{\omega({}^{\overline{y}}g, y^{-1}|f)}{\omega(y^{-1}, g|f)} \, \epsilon({}^{\overline{y}}g, {}^{\overline{y}}f) \right\}$$

with $\overline{\mathrm{Tr}(\rho_f(g))}$ the complex conjugation of $\mathrm{Tr}(\rho_f(g))$ and the transgression

$$\omega(f, g|h) = \frac{\omega(f, {}^g h, g)}{\omega(f, g, h)\omega({}^{fg}h, f, g)}$$

Twin Algebras and Twin Phases

- The simple objects in condensable algebras are anyons (topological lines) that end at the interface/boundary with $n^a_{\mathcal{A}} \in \mathbb{N}$, $\mathcal{A} \cong_{\text{obj}} \sum_a n^a_{\mathcal{A}} a$
- Condensable algebras are defined with an algebra structure on the anyons
- **Twin algebras**: two algebras $\mathcal{A}_1$ and $\mathcal{A}_2$ with the same anyon decomposition but inequivalent algebra structures [Gai, Schäfer-Nameki, AW]
- **Twin phases**: the theories obtained from the SymTFT with the twin algebras $\mathcal{A}_k$ $(k = 1, 2)$ on $\mathfrak{B}^{\text{phys}}$ [AW, Gai, Schäfer-Nameki]



Types of twin algebras

Twin algebras: two algebras $\mathcal{A}(H_i, N_i, \gamma_i, \epsilon_i)$ with the same anyon decomposition (objects) but inequivalent algebra structures.

$$\boxed{\text{Four distinct mechanisms for realizing twin phases}}$$

- H-**type twins:** non-conjugate subgroups $H_i \subset G$
  constructed from Gassmann triples
  $$|H_1 \cap [g]| = |H_2 \cap [g]|\,, \quad \forall\ [g] \in \mathrm{ConjClasses}(G)$$
  Twin algebras can be maximal (Lagrangian) or non-maximal.
- N-**type twins:** same H but non-conjugate normal subgroups N_i
  The subgroups N_i form a Gassmann triple.
- γ-**type twins:** same H and N, but inequivalent SPT data γ.
  Includes both known Lagrangian examples and new non-maximal examples for anomalous symmetries.
- ϵ-**type twins:** same H, N, and γ, but inequivalent H-action phases ϵ.
  These new twins are non-maximal (because for maximal algebras $\gamma \Rightarrow \epsilon$).

Physical properties of twin phases

- Twin phases are very similar:
  1. they share the same anyon decomposition $\Rightarrow$ same number of ground states $n = \sum_a n^a_{\text{sym}} n^a_{\text{phys}}$ and order parameters are part of the same generalized charge
  2. gauging i.e. change $\mathfrak{B}^{\text{sym}}$ doesn't modify the twin algebras on $\mathfrak{B}^{\text{phys}}$ $\Rightarrow$ no relative SSB for any symmetry $\mathcal{S}$
- But they are distinct phases:
  1. G twin SPT phases can be distinguished by generalized string order parameters [Pollmann, Turner], [Chavda, Delcamp, Turzillo, You]
  2. Twin SSB phases can be distinguished by different symmetry action or by different OPE [Kobayashi, Watanabe], [Gai, Schäfer-Nameki, AW]
  3. Twin gapless phases give rise to different maps of anyons across the SymTFT interface [Gai, Schäfer-Nameki, AW]
- Direct and stable **phase transitions between twin phases are intrinsically beyond Landau**: not dual to any SSB transition [AW, Gai, Schäfer-Nameki], [Gai, Schäfer-Nameki, AW]

Examples of twin phases
and intrinsic DQCP transitions

Twin algebra examples

- Performing a systematic search with GAP, we find the following twin algebras:

Group G with twin algebras in $\mathcal{Z}(\mathrm{Vec}_G)$	Type of twins
$G_{32,43} = \mathbb{Z}_8 \rtimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$	Non-Lag $H' \neq {}^gH$ Non-Lag $\epsilon'^g \neq \epsilon$
$G_{32,44} = (\mathbb{Z}_2 \times Q_8) \rtimes \mathbb{Z}_2$	Non-Lag $\epsilon'^g \neq \epsilon$
$G_{48,29} = GL(2,3)$	Non-Lag $H' \neq {}^gH$ Non-Lag $N' \neq {}^gN$ Lag $H' \neq {}^gH$
$G_{48,31} = \mathbb{Z}_4 \times A_4$	Non-Lag $\epsilon'^g \neq \epsilon$
$G_{48,32} = \mathbb{Z}_2 \times SL(2,3)$	Non-Lag $\epsilon'^g \neq \epsilon$
$G_{48,33} = ((\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2) \rtimes \mathbb{Z}_3$	Non-Lag $\epsilon'^g \neq \epsilon$
$G_{48,49} = \mathbb{Z}_2 \times \mathbb{Z}_2 \times A_4$	Non-Lag $H' \neq {}^gH$ Non-Lag $N' \neq {}^gN$ Non-Lag $\epsilon'^g \neq \epsilon$

- We will focus on $G_{48,29} = GL(2,3)$ with twin Lagrangian algebras [AW, Gai, Schäfer-Nameki]. Note that with ω, the smallest group admitting twin Lagranian algebras is $G_{32,43} = \mathbb{Z}_8 \rtimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$ [Gai, Schäfer-Nameki, AW].

Twin gapped phases for $GL(2,3)$

- The quantum double of $GL(2,3) \cong Q_8 \rtimes S_3 \subset$ Clifford group on one qubit admits two Lagrangian twin algebras: $\mathcal{A}_{S_3^{(1)}}$ and $\mathcal{A}_{S_3^{(2)}}$ with $S_3^{(1)} \not\sim S_3^{(2)}$
- They have the same anyon decomposition (objects):

$$\mathcal{A}_{S_3^{(k)}} \cong_{\text{obj}} 1 \oplus \rho_3 \oplus \rho_4 \oplus [a]_{++} \oplus [a]_{+-} \oplus [h]_{++} \oplus [h]_{+-}$$

  where ρ_n are n-dimensional irreps of G and $S_3^{(k)} = \mathbb{Z}_3^{(a)} \rtimes \mathbb{Z}_2^{(c^{k-1}h)}$
- For G symmetry, the twin phases are have eight ground states labeled by the cosets $S_3^{(k)}q$ with $q \in Q_8$ but inequivalent preserved symmetry $S_3^{(k)}$
- $\rho_4|_{S_3^{(k)}}$ decomposes into S_3 irreps as:

$$\rho_4|_{S_3^{(1)}} = 1 \oplus \rho_{1-} \oplus \rho_2 \quad \text{and} \quad \rho_4|_{S_3^{(2)}} = \rho_{1-} \oplus 1 \oplus \rho_2$$

  the component that has a vev is the trivial irrep 1, which appears in entry $(k,k) \Rightarrow$ twin order parameters are different component of the same irrep ρ_4

The non-maximal algebra for the phase transition

- The transition is obtained from a non-maximal condensable algebra $\mathcal{A}(H,N)$ $H = \langle S_3^{(1)} \cup S_3^{(2)} \rangle \cong S_3 \times \mathbb{Z}_2$, $N = \langle S_3^{(1)} \cap S_3^{(2)} \rangle \cong \mathbb{Z}_3$, $\mathcal{A}_{(H,N)} \cong_{\text{obj}} 1 \oplus \rho_3 \oplus [a]_{++}$
- Without anomaly the gapped phase preserving H would allow the two consecutive Landau transitions $S_3^{(1)} < H > S_3^{(2)}$
- With mixed anomaly the H-preserving gapped phase is absent $\Rightarrow$ direct and stable transition between twin gapped phases
- $\mathcal{A}(H,N) \Rightarrow$ interface to reduced TO $D^{\omega_{\text{II}}}(\mathbb{Z}_2^2)$ with map of anyons

$\mathcal{A}(H,N)$

$D^{\omega}(G)$	$D^{\omega_{\text{II}}}(\mathbb{Z}_2^2)$
ρ_4	$e_1,\ e_2$
$[h]_{++}$	$m_1,\ m_2$

- Input $\mathrm{CFT}^{\mathbb{Z}_2^2,\omega_{\mathrm{II}}}$ (the $c=1$ compact boson with ω_{II} between shift and winding) on $\mathfrak{B}^{\mathrm{phys}}$ of the Reduced TO $D^{\omega_{\mathrm{II}}}(\mathbb{Z}_2^2)$ and map via $\mathcal{A}(H,N)$

$$\mathcal{P}_{\langle m_1\rangle} \xleftarrow{\mathcal{O}_{e_2}} \mathrm{CFT}^{\mathbb{Z}_2^2,\omega^{\mathrm{II}}} \xrightarrow{\mathcal{O}_{e_1}} \mathcal{P}_{\langle m_2\rangle}$$

$\Downarrow$ Map via $\mathcal{A}(H,N)$

$$\mathcal{P}_{S_3^{(1)}} \xleftarrow{\mathcal{O}_{\rho_4}^{(1,1)}} \mathrm{CFT}^{G,\omega} \xrightarrow{\mathcal{O}_{\rho_4}^{(2,2)}} \mathcal{P}_{S_3^{(2)}}$$

- It remains a DQCP transition even after changing $\mathfrak{B}^{\mathrm{sym}}$ by gauging

$$\mathbb{Z}_4 \text{ SSB} \longleftarrow \mathrm{CFT}^{\mathbb{Z}_4} \longrightarrow \mathbb{Z}_4 \text{ Trivial}$$

$\Downarrow$ Map via $\mathcal{A}(H,N)$

$$\text{Full } \mathcal{S}\text{-SSB 1} \longleftarrow \mathrm{CFT}^{\mathcal{S}} \longrightarrow \text{Full } \mathcal{S}\text{-SSB 2}$$

- 1+1d lattice model with local Hilbert space spanned by $\{|f\rangle : f \in G\}$ with:
  1. left and right multiplication operators $L_j^{g,\omega}$, $R^{g,\omega}$ (disordering)
  2. irrep operators $Z^{\rho}_{n,m}|g\rangle = \rho(g)_{n,m}|g\rangle$ (ordering)

- $H(\lambda)$ interpolating between the twin phases preserving $S_3^{(k)} = \mathbb{Z}_3^{(a)} \rtimes \mathbb{Z}_2^{(hc^{k-1})}$

$$
\begin{aligned}
H(\lambda) &= (1-\lambda)H_{S_3^{(1)}} + \lambda H_{S_3^{(2)}} \\
&\simeq -\sum_j \frac{1}{8}\left[\mathbb{I}_{j,j+1} + 3\left(Z_j^{\rho_3}\cdot Z_{j+1}^{\rho_3\dagger}\right)_{1,1}\right]_j - \frac{1}{3}\sum_j\left(\mathbb{I}_j + L_j^{a,\omega} + L_j^{a^2,\omega}\right) - \frac{1}{2}\sum_j \mathbb{I}_j \\
&\quad - \frac{1}{4}\sum_j (1-\lambda)\left[L_j^{h,\omega} + \left(Z_j^{\rho_4}\cdot Z_{j+1}^{\rho_4\dagger}\right)_{1,1}\right] - \frac{1}{4}\sum_j \lambda\left[L_j^{hc,\omega} + \left(Z_j^{\rho_4}\cdot Z_{j+1}^{\rho_4\dagger}\right)_{2,2}\right]
\end{aligned}
$$

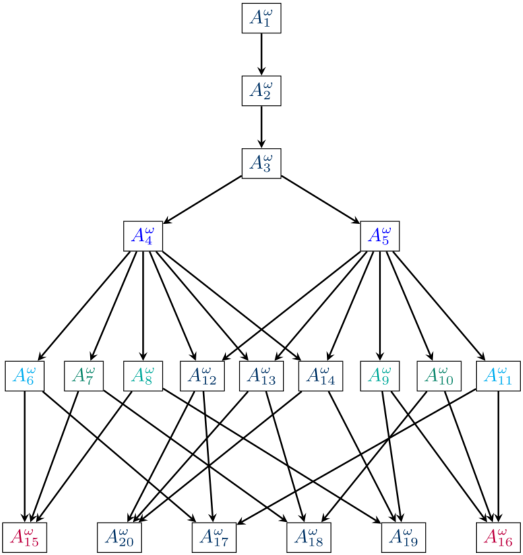
- The fixed-point twin gapped phases occur at $\lambda = 0$ and $\lambda = 1$ respectively
- The gapless phase transition is at $\lambda = 1/2$ and is an intrinsic DQCP

Another intrinsic DQCP example

- $G_{32} = (\mathbb{Z}_2^{(g_2)} \times \mathbb{Z}_2^{(g_3)}) \ltimes \mathbb{Z}_8^{(g_1)}$ with $\omega \in H^3(G, U(1))$ admits twin algebras

- The intrinsic DQCP transition between the twin gapped phases can be constructed from condensing A_3^ω, giving Reduced TO $D^{\omega_{\mathrm{III}}}(\mathbb{Z}_2^3) \cong D(D_8)$

  [Gai, Schäfer-Nameki, AW]



Twins: (A_4^ω, A_5^ω), $(A_6^\omega, A_{11}^\omega)$, (A_8^ω, A_9^ω), $(A_7^\omega, A_{10}^\omega)$, $(A_{15}^\omega, A_{16}^\omega)$

Summary

- Landau theory: continuous phase transitions are spontaneous symmetry breaking (SSB) transitions

$$\begin{tikzcd} & \boxed{G} \arrow[dl] \arrow[dr] & \\ \boxed{H_1} & > & \boxed{H_2} \end{tikzcd}$$

- **Twin phases** for a symmetry $\mathcal{S}$: inequivalent phases whose order parameters are part of the same generalized charges under $\mathcal{S}$

$$\begin{tikzcd} & \boxed{\mathcal{S}} \arrow[dl] \arrow[dr] & \\ \boxed{\mathcal{S}_1} & \not> & \boxed{\mathcal{S}_2} \end{tikzcd}$$

- Twin phases $\Rightarrow$ no relative SSB even after (partially) gauging $\mathcal{S}$

Stable and direct transitions between twin phases are **not** (dual to) symmetry-breaking transitions

[AW, Gai, Schäfer-Nameki], [Gai, Schäfer-Nameki, AW]

Outlook

- Extend beyond the group-theoretical 1+1d symmetries to:

  1) twin algebras in higher dimensions (difficulty in precise definitions, algebra equivalences and decomposition into simple objects)

  2) twin phases for continuous, higher-form and intrinsically non-invertible $\mathcal{S}$

- Explore the features of the critical theories of twin-phase transitions both from continuum CFT and numerical signatures from lattice models

- Understand the role of anomalies in stabilizing the twin-phase transitions

What is the complete general framework for phase transitions beyond symmetry breaking?

[Work in progress with Sakura Schäfer-Nameki and Rui Wen]

Thank you