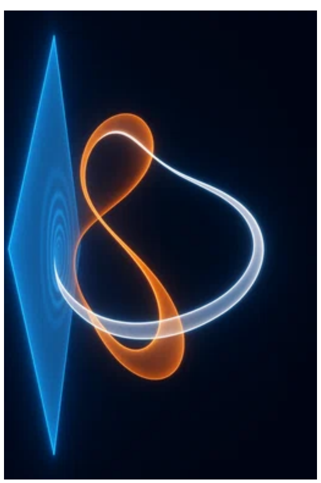
Constructing fully extended functorial 2d chiral CFTs from Conformal Nets

Nivedita

University of Oxford

(from Sept 2026) **Perimeter Institute + IAS**



Global Categorical Symmetries 2026

Institut Henri Poincaré

Paris

2d Chiral Conformal Field Theory

Vertex Operator Algebra

Conformal Nets

Functorial Chiral CFTs

(Segal’s modular functor)

2d Chiral Conformal Field Theory

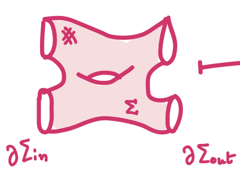
Vertex Operator Algebra

$V \rightsquigarrow \mathrm{Rep}(V)$

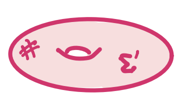
Functorial Chiral CFTs

(Segal's modular functor)

 $\longmapsto \mathcal{C}$



$\longmapsto F_\Sigma : \mathcal{C}_{\partial\Sigma_{in}} \to \mathcal{C}_{\partial\Sigma_{out}}$



$\longmapsto \mathcal{H}_{\Sigma'}$ the space of conformal blocks

Conformal Nets

2d Chiral Conformal Field Theory

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(Segal's modular functor)

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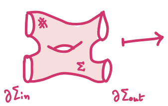

$\longmapsto F_\Sigma : \mathcal{C}_{\partial\Sigma_{in}} \to \mathcal{C}_{\partial\Sigma_{out}}$

$\longmapsto \mathcal{H}_{\Sigma'}$ the space of conformal blocks

* a Hilbert space $\mathcal{H}_0$, a unit vector $\Omega \in \mathcal{H}_0$
* a strongly cont. representation $\text{Möb} \to \mathcal{U}(\mathcal{H}_0)$

extending to a strongly cont. projective rep $U : \text{Diff}(S^1) \to P\mathcal{U}(\mathcal{H}_0)$

* An assignment $I \subset S^1 \mapsto \mathcal{A}(I) \subset \mathcal{B}(\mathcal{H}_0)$

Isotony + Locality
- $I \subset J \Rightarrow \mathcal{A}(I) \subset \mathcal{A}(J)$, $\mathring{I} \cap \mathring{J} = \phi \Rightarrow [\mathcal{A}(I), \mathcal{A}(J)]_{\mathcal{B}(\mathcal{H}_0)} = 0$

Covariance
- $U(\varphi)\,\mathcal{A}(I)\,U(\varphi)^* = \mathcal{A}(\varphi I)$ for $\varphi \in \text{Diff}(S^1)$

  if $\varphi|_I = id_I$ then $Ad(U(\varphi))|_{\mathcal{A}(I)} = id_{\mathcal{A}(I)}$

Positivity of energy
- $U(Rot_t) = e^{itL_0}$, L_0 is required to have positive spectrum

Vacuum vector
- $U|_{\text{Möb}}$ has 1-dim fixed point space spanned by Ω

  Ω is cyclic for $\bigvee_{I \subset S^1} \mathcal{A}(I)$

2d Chiral Conformal Field Theory

unitary

Vertex Operator Algebra

$$A_V(I) := W^*\big(\{\underbrace{Y(a,f)}_{\text{smeared vertex operator}} : a \in V,\ f \in C^\infty S^1,\ \operatorname{supp} f \subset I\}\big)$$

Carpi-Kawahigashi-Longo-Weiner 2018

Conformal Nets

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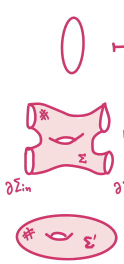
Functorial Chiral CFTs

(Segal's modular functor)



$\bigcirc \longmapsto \mathcal{C}$

$\Sigma \longmapsto F_\Sigma : \mathcal{C}_{\partial\Sigma_{in}} \to \mathcal{C}_{\partial\Sigma_{out}}$

$\Sigma' \longmapsto \mathcal{H}_{\Sigma'}$ the space of conformal blocks

2d Chiral Conformal Field Theory

Vertex Operator Algebra

Henriques-Tener 2025

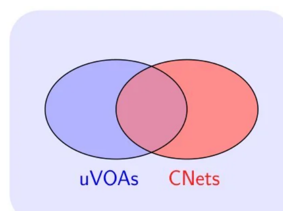
Conformal Nets

Th (Henriques-Tener 25)

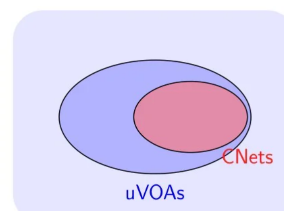
Every conformal net has an associated unitary vertex operator algebra.

Functorial Chiral CFTs

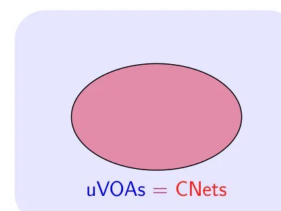
(Segal's modular functor)



Carpi-Kawahigashi-Longo-Weiner 2018



Henriques-Tener 2025



Conjecturally.

2d Chiral Conformal Field Theory

Vertex Operator Algebra

Conformal Nets

Functorial Chiral CFTs

$$
\begin{tikzcd}
\widetilde{B\mathrm{Ann}}(S^1) \arrow[r] \arrow[rrr, bend left, "\mathrm{Const}\, Z_\chi^{top}(S^1)"] \arrow[rrr, phantom, shift left=6ex, "\simeq \Downarrow T"] & B\mathrm{Ann}(S^1) \arrow[r] & \mathrm{Bord}^{conf,or}_{2+\varepsilon,2,1} \arrow[r, "Z_\chi^{top}"] \arrow[r, bend right=60, "\mathrm{Const}\,\mathbb{1}"'] \arrow[r, phantom, shift right=3ex, "\Downarrow Z_\chi^{hol}"] & 2\text{-}\mathcal{V}
\end{tikzcd}
$$

Relative theory.

closed 1-manifold $S \longmapsto$ an algebra A_S

a complex cobordism Σ. $\longmapsto$ $A_{\partial in \Sigma}$ - $A_{\partial out \Sigma}$ bimodule H_Σ, $\Omega_\Sigma \in H_\Sigma$.

+ axiom that implies that H_{Σ_1} depends - topologically on Σ and $\Omega_\Sigma \in H_\Sigma$ depends holomorphically on Σ.

if $\Sigma_1 \underset{\text{diffeo}}{\sim} \Sigma_2$ then $H_{\Sigma_1} \underset{\text{iso}}{\cong} H_{\Sigma_2}$ as bimodules.

2d Chiral Conformal Field Theory

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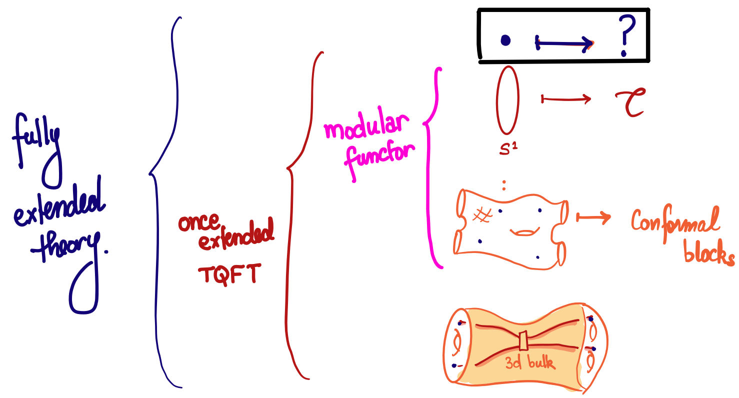
$$\begin{tikzcd}
\widetilde{B\mathrm{Ann}}(S^1) \arrow[r] \arrow[rrr, bend left, "\mathrm{Const}\, Z_X^{top}(S^1)"] & B\,\mathrm{Ann}(S^1) \arrow[r] \arrow[r, phantom, shift left=6, "\sim\Downarrow T"] & \mathrm{Bord}^{conf,\,or}_{2+\varepsilon,2,1,0} \arrow[r, "Z_X^{top}"] \arrow[rr, bend right, "\mathrm{Const}\ \mathbb{1}"'] \arrow[r, phantom, shift right=6, "\Downarrow Z_X^{hol}"] & \cancel{2\text{-}\mathcal{V}}\ 3\text{-}\mathcal{V}
\end{tikzcd}$$



fully-extended

Target categories $\underbrace{\Omega_{\mathbb{1}}\ n\text{-}\mathcal{V}}_{\mathrm{End}_{n\text{-}\mathcal{V}}(\mathbb{1})} \simeq (n-1)\text{-}\mathcal{V}$

$$\Omega_{\mathbb{1}}\, n\text{-}\mathcal{V} \simeq (n-1)\text{-}\mathcal{V}$$

Morita theory $n \geq 2$, $n = 1$ $\mathcal{V} = \mathrm{Vect}_{\mathbb{C}}$

$$\mathrm{Mor}\left(\mathbb{E}_1\left((n-1)\text{-}\mathcal{V}\right)\right) \xrightarrow{\boxed{}\text{-}\mathrm{Mod}} \mathrm{Lin}\,(n-1)\mathrm{Cat}$$

associative algebra objects.

$$\Omega_{\mathbb{1}}\, n\text{-}\mathcal{V} \simeq (n-1)\text{-}\mathcal{V}$$

Morita theory $n \geq 2$, $n = 1$ $\mathcal{V} = \mathrm{Vect}_{\mathbb{C}}$

$$\mathrm{Mor}(\mathbb{E}_1((n-1)\text{-}\mathcal{V})) \xrightarrow{\square\text{-}\mathrm{Mod}} \mathrm{Lin}\,(n-1)\mathrm{Cat}$$

$\mathrm{Vect}_{\mathbb{C}}$

$$\mathrm{Mor}(\mathbb{E}_1(\mathrm{Vect}_{\mathbb{C}})) \qquad \mathrm{Mor}(\mathbb{E}_1(\mathrm{Lin\,Cat}))$$

$\mathrm{Alg}_{\mathbb{C}}^{\mathrm{bimod}}$ LinCat $\mathrm{TensCat}^{\mathrm{bimod}}$ $\mathrm{Lin\ 2\text{-}Cat}$

Composition of 1-morphism

$${}_A M \otimes_B N_C$$

identity 1-morphism ${}_A A_A$

Composition of 1-morphisms

$${}_{\mathcal{C}} M \boxtimes_{\mathcal{D}} N_{\mathcal{E}}$$

identity 1-morphism ${}_{\mathcal{C}}\mathcal{C}_{\mathcal{C}}$

Some constructions

vector space

algebra

linear category

tensor category

Some constructions

$$V \longmapsto \mathrm{End}(V)$$

vector space $\quad A \xleftarrow[\text{forget}]{} A \quad$ algebra

linear category $\qquad\qquad$ tensor category

$$\mathcal{C} \longmapsto \mathrm{End}(\mathcal{C})$$

$$\mathcal{T} \xleftarrow[\text{forget}]{} \mathcal{T}$$

Some constructions

$$V \longmapsto \mathrm{End}(V)$$

vector space $\qquad$ $A \xleftarrow{\text{forget}} A$ $\qquad$ algebra

$$\mathrm{Hom}_{\mathcal{C}}(x,y) \longleftarrow x, y \in \mathcal{C}$$

Vect is a linear category

$$A \longmapsto \mathrm{Bimod}(A)$$

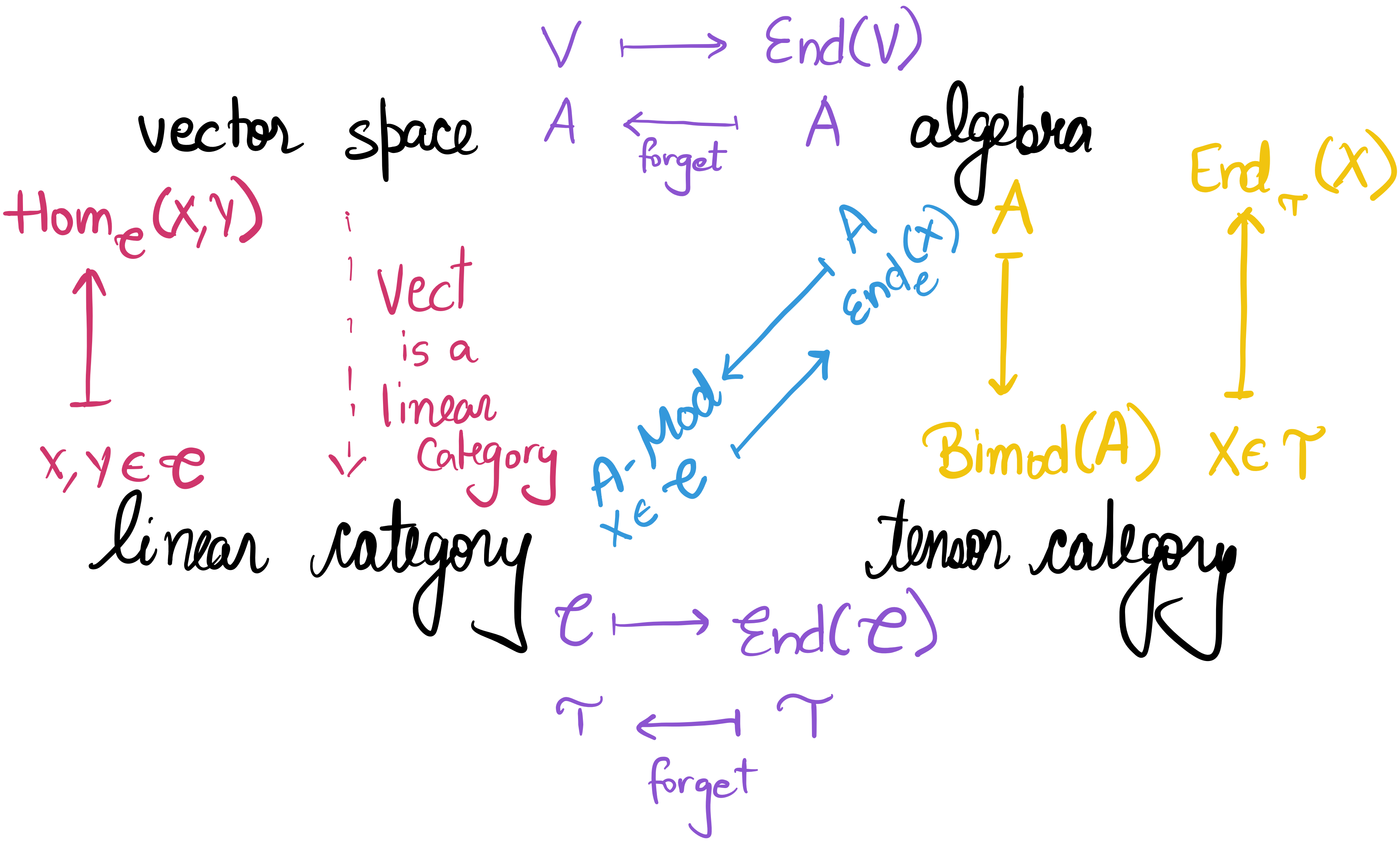
$$\mathrm{End}_{\mathcal{T}}(X) \longleftarrow X \in \mathcal{T}$$

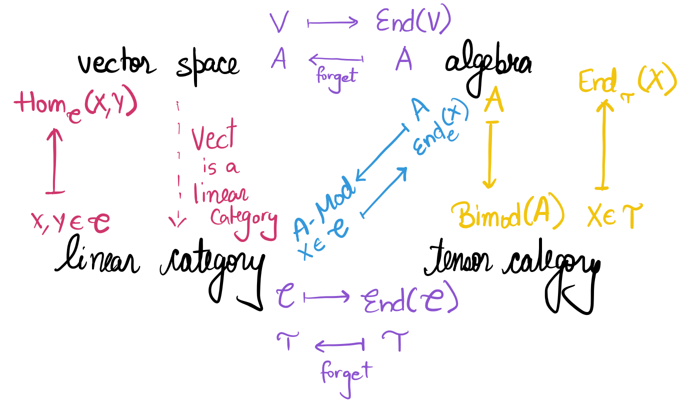
linear category $\qquad$ tensor category

$$\mathcal{C} \longmapsto \mathrm{End}(\mathcal{C})$$

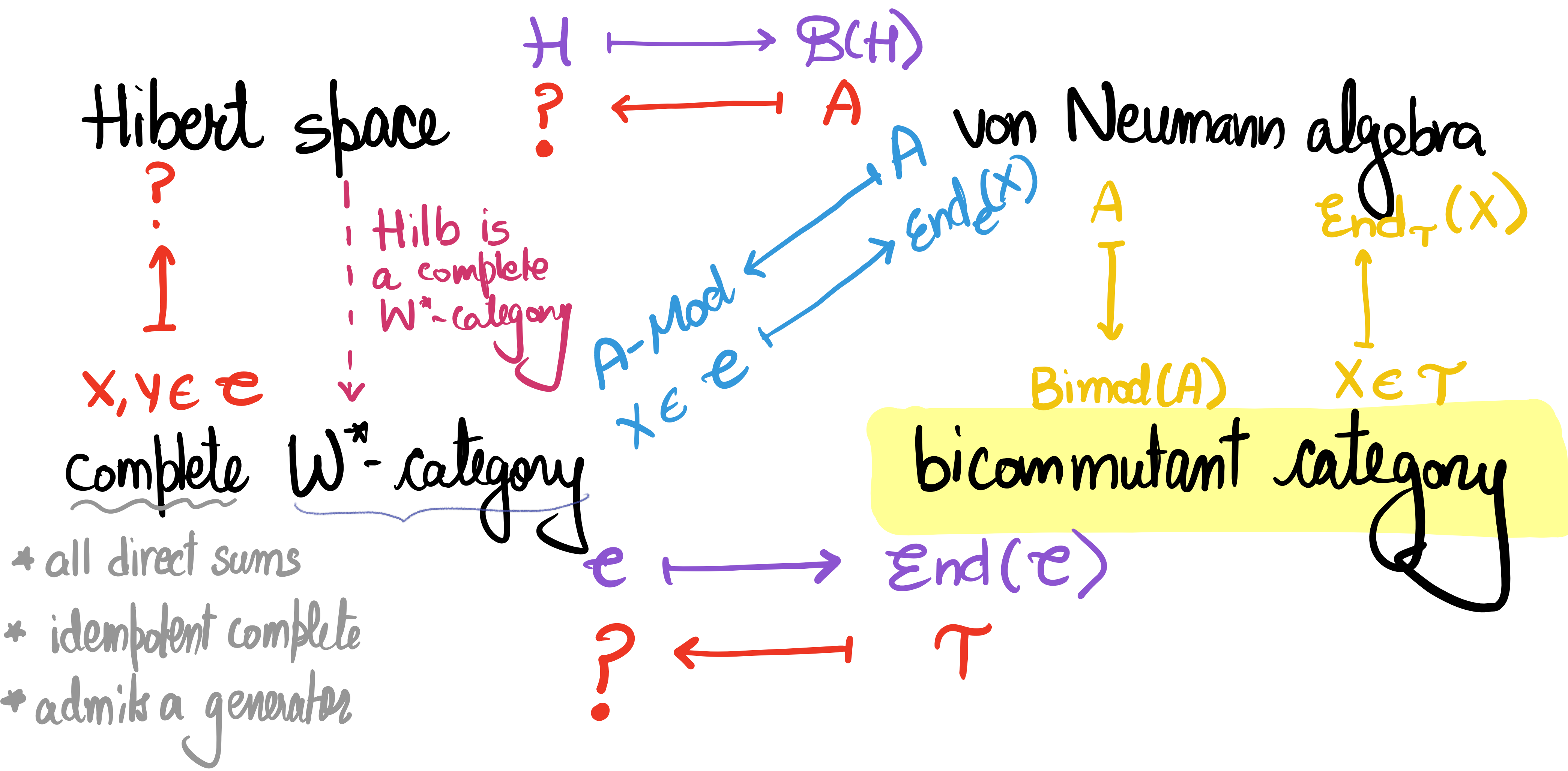
$$\mathcal{T} \xleftarrow{\text{forget}} \mathcal{T}$$

Some constructions





V ⟼ End(V)
A ⟵ A
forget
vector space
algebra
Hom_C(X,Y)
X, Y ∈ C
Vect is a linear category
A-Mod
X ∈ C
A
End_C(X)
A
Bimod(A)
End_T(X)
X ∈ T
linear category
tensor category
C ⟼ End(C)
T ⟵ T
forget



H ⟼ B(H)
? ⟵ A
Hibert space
von Neumann algebra
?
X, Y ∈ C
Hilb is a complete W*-category
A-Mod
X ∈ C
A
End_C(X)
A
Bimod(A)
End_T(X)
X ∈ T
complete W*-category
bicommutant category
C ⟼ End(C)
? ⟵ T
* all direct sums
* idempotent complete
* admits a generator

Hilbert spaces	Complete W^*-categories
Inner-product $\langle\ ,\ \rangle : \overline{H} \times H \to \mathbb{C}$	Cannonical Hilb-valued $\langle\ ,\ \rangle : \overline{\mathcal{C}} \times \mathcal{C} \to \mathrm{Hilb}$
$H \to \{ f : \overline{H} \to \mathbb{C} \mid f \text{ is bounded} \}$ $\xi \mapsto \langle\ , \xi \rangle$ is an isomorphism.	$\mathcal{C} \to \mathrm{Func}(\overline{\mathcal{C}}, \mathrm{Hilb})$ $X \mapsto \langle\ , X \rangle$ is an equivalence
An anti-linear isometry 'adjoint' $\dagger : \mathrm{Hom}(H_1, H_2) \xrightarrow{\cong} \mathrm{Hom}(H_2, H_1)$ characterized by $\langle \xi, f^\dagger \eta \rangle_{H_1} = \langle f\xi, \eta \rangle_{H_2}$	An anti-linear equivalence 'adjoint' $\dagger : \mathrm{Func}(\mathcal{C}_1, \mathcal{C}_2) \xrightarrow{\cong} \mathrm{Func}(\mathcal{C}_2, \mathcal{C}_1)$ characterised by natural unitary isos $\langle X, F^\dagger Y \rangle_{\mathcal{C}_1} \cong \langle FX, Y \rangle_{\mathcal{C}_2}$

$H \longmapsto B(H)$

Hilbert space $L^2 A \longleftarrow\!\shortmid A$ von Neumann algebra

standard form

cannonical, but **NOT** universal

?

$x, y \in \mathcal{C}$

Hilb is a complete W^*-category

A-Mod $\longleftarrow\!\shortmid A$

$x \in \mathcal{C} \longmapsto \mathrm{End}_{\mathcal{C}}(x)$

$A \longmapsto \mathrm{Bimod}(A)$

$\mathrm{End}_{\mathcal{T}}(X) \longleftarrow\!\shortmid X \in \mathcal{T}$

complete $\underbrace{W^*\text{-category}}$ bicommutant category

* all direct sums
* idempotent complete
* admits a generator

$\mathcal{C} \longmapsto \mathrm{End}(\mathcal{C})$

$? \longleftarrow\!\shortmid \mathcal{T}$

Hilbert space $\qquad H \longmapsto B(H) \qquad$ von Neumann algebra

$$L^2A \longleftarrow\!\!\!\shortmid \; A$$

standard form

cannonical, but NOT universal

$$\langle X, Y\rangle_{\mathcal{C}} := p_Y \, L^2 \, \underbrace{\mathrm{End}_{\mathcal{C}}(X \oplus Y)}_{\left(\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}\right) \; 2\times 2} \, p_X$$

$$\uparrow$$

$X, Y \in \mathcal{C}$

Hilb is a complete W^*-category

$$A\text{-Mod} \longleftarrow\!\!\!\shortmid \; A$$

$$X \in \mathcal{C} \longmapsto \mathrm{End}_{\mathcal{C}}(X)$$

$$A \mapsto \mathrm{Bimod}(A) \qquad \mathrm{End}_{T}(X) \leftarrow\!\!\!\shortmid \; X \in T$$

complete W^*-category $\qquad$ bicommutant category

$$\mathcal{C} \longmapsto \mathrm{End}(\mathcal{C})$$

$$? \longleftarrow\!\!\!\shortmid \; T$$

* all direct sums
* idempotent complete
* admits a generator

Hibert space $\mathcal{H} \longmapsto B(\mathcal{H})$ von Neumann algebra

$L^2 A \longleftarrow A$

standard form

cannonical, but **NOT** universal

$\langle X, Y \rangle_{\mathcal{C}} := p_Y L^2 \underbrace{\mathrm{End}_{\mathcal{C}}(X \oplus Y)}_{2\times 2} p_X$

$\uparrow$

$X, Y \in \mathcal{C}$

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complete W^*-category

* all direct sums
* idempotent complete
* admits a generator

bicommutant category

$\mathcal{C} \longmapsto \mathrm{End}(\mathcal{C})$

$\mathcal{T}^{abs} \longleftarrow \mathcal{T}$

2-sided ideal of absorbing objects

$\Omega \otimes X \cong \Omega \cong X \otimes \Omega \quad \forall X \in \mathcal{T}$

$\mathcal{L}^2(H) = L^2(B(H))$ Hilbert-Schmidt operators $\longleftarrow B(H)$

2-sided $*$-ideal

$\mathrm{Vect}_{\mathbb{C}}$

$\mathrm{Mor}(\mathbb{E}_1(\mathrm{Vect}_{\mathbb{C}}))$ — $\mathrm{Alg}_{\mathbb{C}}^{\mathrm{bimod}}$, LinCat

Composition of 1-morphism ${}_AM \otimes_B N_C$

identity 1-morphism ${}_AA_A$

$\mathrm{Mor}(\mathbb{E}_1(\mathrm{Lin\,Cat}))$ — $\mathrm{TensCat}^{\mathrm{bimod}}$, $\mathrm{Lin\,2\text{-}Cat}$

Composition of 1-morphisms ${}_{\mathcal{C}}M \boxtimes_{\mathcal{D}} N_{\mathcal{E}}$

identity 1-morphism ${}_{\mathcal{C}}\mathcal{C}_{\mathcal{C}}$

Some constructions

Hilb

$\mathrm{vNAlg}^{\mathrm{bimod}} \underset{\text{Thm}}{\simeq} \mathrm{W^*Cat}$ $\qquad \mathrm{BicommCat}^{\mathrm{bimod}} \qquad \mathrm{W^*2\text{-}Cat}$

Thm [Henriques, N, Penneys 2024]

composition of 1-mor

Connes fusion ${}_AH \boxtimes_B K_C$

actions $\begin{cases} \lambda : B \to \mathcal{B}(L^2B) \\ \rho : B^{op} \to \mathcal{B}(L^2B) \end{cases}$

completion of the vector space

$$\mathrm{Hom}_B(L^2B_*, {}_AH_\bullet) \otimes_B L^2B \otimes_B \mathrm{Hom}_B(L^2B_*, {}_\bullet K_C)$$

wrt $\langle \varphi_1 \otimes \xi_1 \otimes \psi_1, \varphi_2 \otimes \xi_2 \otimes \psi_2 \rangle := \langle \xi_1, \lambda^{-1}(\varphi_1^* \circ \varphi_2)\, \xi_2\, \rho^{-1}(\psi_1^* \circ \psi_2) \rangle_{L^2B}$

L^2A is the unit

$\mathrm{vNAlg}^{\mathrm{bimod}} \underset{\text{Thm}}{\cong} \mathrm{W}^*\text{-}\mathrm{Cat}$ $\qquad$ $\mathrm{BicommCat}^{\mathrm{bimod}}$ $\qquad$ $\mathrm{W}^*\,2\text{-}\mathrm{Cat}$

[Henriques, N, Penneys 2024]

A bicommutant cate is a categorified analogue of von Neumann algebra.

	A	$\mathcal{T}$
•	algebra$_{\mathbb{C}}$	a $\mathbb{C}$-linear monoidal category
•	$*$-algebra	bi-involutive structure $\quad \dagger_0$ anti-linear anti-tensor involution $(*)\quad \dagger_1$ anti-linear
•	commutant A' in B $\{b \in B : ab = ba \ \forall a \in A\}$ commutant of a $*$-alg is a $*$-algebra	given $\iota : \mathcal{T} \to \mathcal{E}$ $\mathcal{Z}(\iota : \mathcal{T} \to \mathcal{E})$ is relative center obj: $(X, \{e_{X,Y} : X \otimes \iota(Y) \xrightarrow[\cong]{\text{data}} \iota(Y) \otimes X\}_{Y \in \mathcal{T}})$ mor: $f : X_1 \to X_2$ in $\mathcal{E}$ such that bi-involutive monoidal
•	${}_A L^2\boxed{A}_A$	${}_{\mathcal{T}}\boxed{\mathcal{T}}^{\mathrm{abs}}_{\mathcal{T}}$ (generally) non-unital monoidal complete W^*-category

$\{$ Hilb, Bim(A), End(W)

Bicommutant property. $A'' = A$!

Defn. Let $\mathcal{T}$ be a complete bi-involutive W^*-tensor category that admits absorbing objects.

We have commuting W^*-tensor functors

$$L : \mathcal{T} \to \mathrm{End}(\mathcal{T}^{abs}) \qquad R : \mathcal{T}^{mop} \to \mathrm{End}(\mathcal{T}^{abs})$$
$$X \mapsto X \otimes \_ \qquad\qquad X \mapsto \_ \otimes X$$

data : equip $\mathcal{T}$ with structure maps that make L, R bi-involutive functors.

$$\gamma^L : L(\overline{\ }) \Rightarrow L(\ )^\dagger$$
$$\gamma^R : R(\overline{\ }) \Rightarrow R(\ )^\dagger$$

Property :

$$\mathcal{T} \to Z(R : \mathcal{T}^{mop} \longrightarrow \mathrm{End}(\mathcal{T}^{abs}))$$
$$\mathcal{T}^{mop} \to Z(L : \mathcal{T} \longrightarrow \mathrm{End}(\mathcal{T}^{abs}))$$

are equivalence of categories

+ lots of coherences ...

Categorified Connes fusion of modules.

Connes fusion

completion of a vector space w.r.t a cannonical inner product.

L^2A is the unit

Admits an alternative description

$$H \underset{A}{\boxtimes} K := \mathrm{proj}_{2,1}\, L^2\, \mathrm{End}_{A\text{-Mod}}\left({}_A\overline{H} \oplus {}_A K\right)$$

where

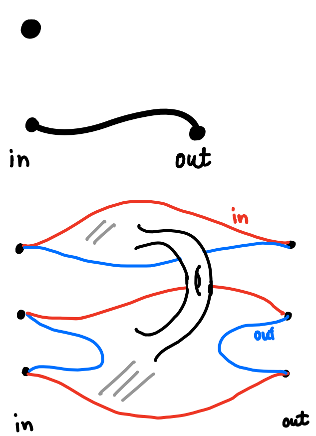
$$\mathrm{proj}_{2,1}(\xi) := \begin{pmatrix} 0 & 0 \\ 0 & 1_K \end{pmatrix} \xi \begin{pmatrix} 1_H & 0 \\ 0 & 0 \end{pmatrix}$$

$$M \underset{T}{\boxtimes} N := \mathrm{proj}_{2,1}\left(\mathrm{End}_{T\text{-Mod}}(\overline{M} \oplus N)^{abs} \right)$$

$$T^{abs} \underset{T}{\boxtimes} T^{abs} \simeq T^{abs}$$

functorial X-CFT from Conformal net

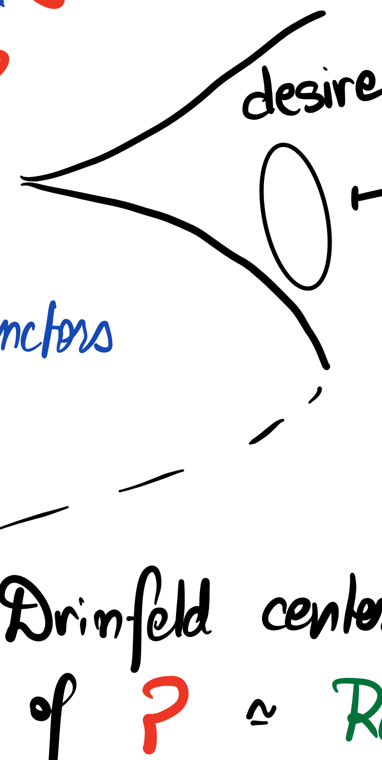
* fully local / fully extended *



$\mapsto$ bicommutant category ?

$\mapsto$ bimodule categories

$\mapsto$ bi-linear / bi-equivariant functors



$\rightarrow$ something like $\mathrm{Rep}(\cancel{V})$ $\mathcal{A}$ for ~~VOA V~~. conformal net $\mathcal{A}$.

Drinfeld center of ? $\simeq$ $\mathrm{Rep}(\mathcal{A})$.

Quick fact.

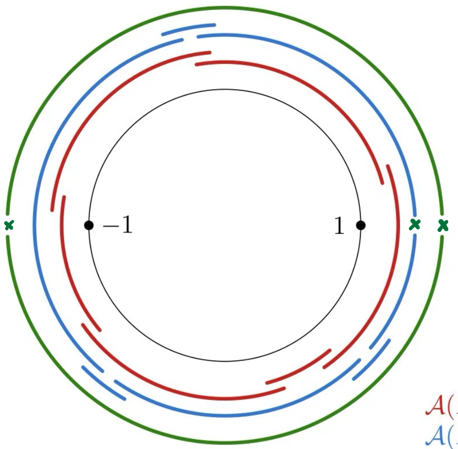
Conformal net $\mathcal{A}$ can be used to assign von Neumann algebra to arbitrary, abstract 1-manifolds.

Different flavours of representations of conformal nets.

Fact given a von Neumann algebra (type-III factor) A
there is an equivalence of monoidal categories

A)

$A \xrightarrow{\rho} A \xrightarrow{\lambda} B(L^2 A)$



$(H, \{\alpha_I : A(I) \to B(H)\}_{\dots})$

$\mathcal{A}(I)\ \forall I \subset S^1$ acts: Representation of conformal net $\mathcal{A}$ — **Rep($\mathcal{A}$)**

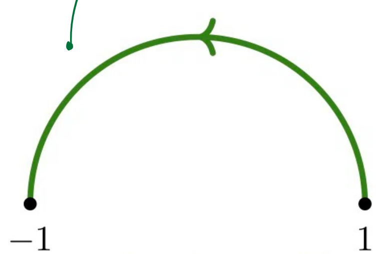
$\mathcal{A}(I)\ \forall I \subset S^1 \mid 1 \notin \mathring{I}$ acts: Solitonic Representation of $\mathcal{A}$ — **Sol($\mathcal{A}$)**

$\mathcal{A}(I)\ \forall I \subseteq S_+ \ \wedge\ \forall I \subseteq S_-$ acts: $\mathcal{A}(S_+)\bar{\otimes}\mathcal{A}(S_-)$-module

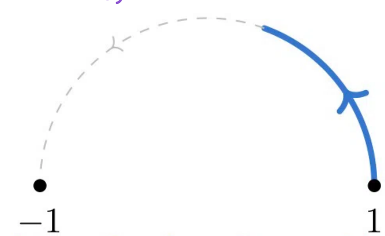
easy to establish fact.

well known equivalence

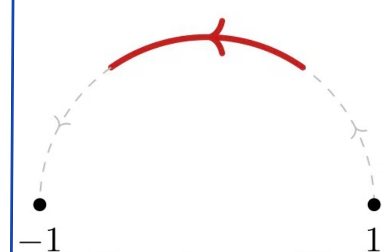
$\rho : \mathcal{A}(I) \to \mathcal{A}(I)$
s.t. $\exists J,\ \partial_+ I \in J$
$\rho|_{\mathcal{A}(J)} = \mathrm{id}_{\mathcal{A}(J)}$.



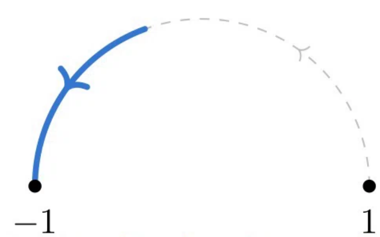
general endomorphism



left-localised endomorphisms



DHR endomorphism



right-localised endomorphism

- **Thm.** (Henriques, N)
  $\mathrm{Sol}(\mathcal{A}) \simeq$ left/right localised endomorphisms.

- **Thm** (Henriques, N) Drinfeld center of $\mathrm{Sol}(\mathcal{A}) \simeq \mathrm{Rep}(\mathcal{A})$.

Bord	Algebraic Assignment
Point P + 1-dim collar	A Bicommutant category $\mathcal{W}_P$
1-cobordism $I\colon P_{\text{in}} \to P_{\text{out}}$	$\mathcal{W}_{\text{out}}$-$\mathcal{W}_{\text{in}}$-bimodule W*-category ${}_{\mathcal{W}_{\text{in}}}\mathcal{M}_{\mathcal{W}_{\text{out}}}$ realised as A-Mod for a von Neumann algebra A
2-cobordism $\Sigma\colon M_{\text{in}} \to M_{\text{out}}$	A concrete equivariant W^*-functor F : A_{in}-Mod $\to$ A_{out}-Mod realised by a pointed A_{out}-A_{in}-bimodule

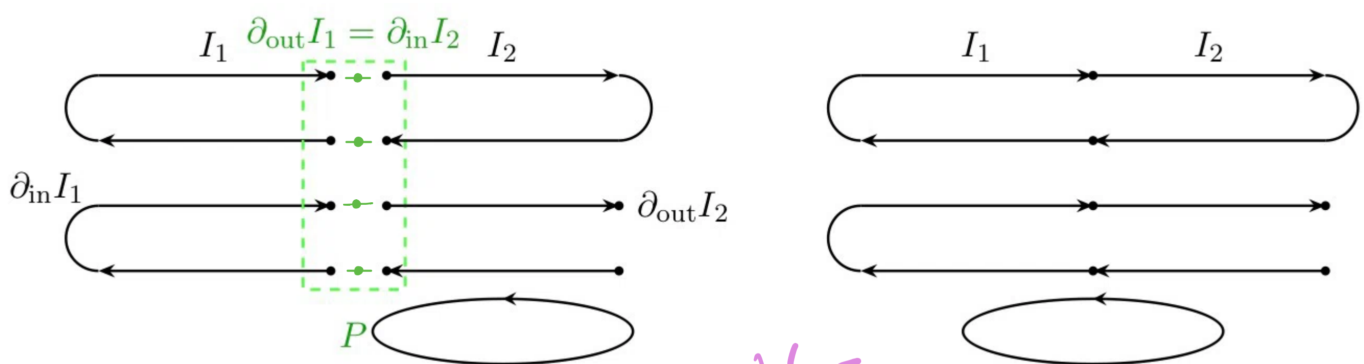
Assignment.

fact : the support of localised endomorphisms can be shrunk/extended as desired

Construction

W_P : solitonic representations

M_I : $A(I)$- Mod.



Thm. (N)

$$W_{\partial_{in} I_1} \circlearrowright M_{I_1} \boxtimes_{W_P} M_{I_2} \circlearrowleft W_{\partial_{out} I_2} \;\cong\; W_{\partial_{in} I_1} \circlearrowright M_{I_1 \cup_P I_2} \circlearrowleft W_{\partial_{out} I_2}$$

compatible with pointings

Bord	Algebraic Assignment
Point P + 1-dim collar	A Bicommutant category $\mathcal{W}_P$
1-cobordism $I \colon P_{\text{in}} \to P_{\text{out}}$	$\mathcal{W}_{\text{out}}$-$\mathcal{W}_{\text{in}}$-bimodule W*-category ${}_{\mathcal{W}_{\text{in}}}\mathcal{M}_{\mathcal{W}_{\text{out}}}$ realised as A-Mod for a von Neumann algebra A
2-cobordism $\Sigma \colon M_{\text{in}} \to M_{\text{out}}$	A concrete equivariant W^*-functor F : A_{in}-Mod $\to$ A_{out}-Mod realised by a pointed A_{out}-A_{in}-bimodule

Assignment.

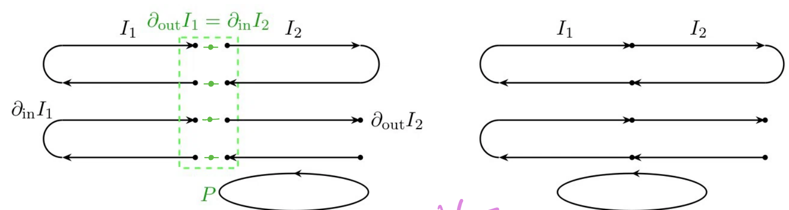
fact : the support of localised endomorphisms can be shrunk / extended as desired

Construction

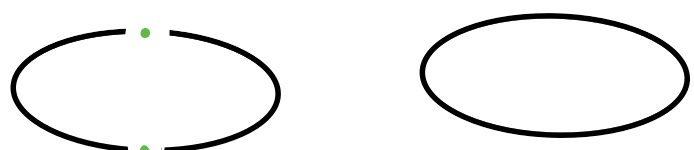
$\mathcal{W}_P$: solitonic representations

$\mathcal{M}_I$: $\mathcal{A}(I)$- Mod

] on 0- and 1-manifolds



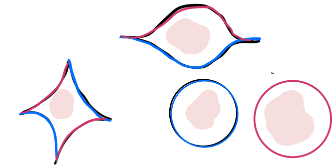
Thm. (N) $\quad {}_{W_{\partial_{in} I_1}}\mathcal{M}_{I_1} \boxtimes_{W_P} \mathcal{M}_{I_2}{}_{W_{\partial_{out} I_2}} \simeq {}_{W_{\partial_{in} I_1}}\mathcal{M}_{I_1 \cup_P I_2}{}_{W_{\partial_{out} I_2}}$



$\mathcal{A}(S)$ - Mod $\simeq$ Rep$(\mathcal{A})^{abs}$

for rational (finite index) conformal nets $\mathcal{A}(S)$ - Mod $\simeq$ Rep$(\mathcal{A})$

- on 2-manifolds (work in progress)

generators 

- Physical interpretation of $Sol(\mathcal{A})$

More generally $\mathcal{B} \subset \mathcal{A}$ we can consider $Sol_{\mathcal{B}}(\mathcal{A})$
solitonic representations of $\mathcal{A}$ that representations of $\mathcal{B}$.

topological lines in bulk

$Sol(\mathcal{A}) =: Sol_{\mathbb{C}}(\mathcal{A})$ $\qquad Sol_{Vir}(\mathcal{A})$ $\qquad Sol_{\mathcal{B}}(\mathcal{A})$ $\qquad Sol_{\mathcal{A}}(\mathcal{A}) := Rep(\mathcal{A})$

?

$$Z^{Drinf}(Sol_{\mathbb{C}}(\mathcal{A})) = Rep(\mathcal{A}) \cong Rep(\mathcal{A}) \boxtimes Rep(\mathbb{C})$$

$$Z^{Drinf}(Sol_{\mathcal{B}}(\mathcal{A}))$$

$$Z^{Drif}(Rep(\mathcal{A}))$$

in rational case

$$Rep(\mathcal{A}) \boxtimes Rep(\mathcal{A})^{op}$$

$\mathcal{B}$-topological defects of bulk/boundary system $\mathcal{A}$.

- recovering central change