When Symmetries Twist:

Anomaly Inflow on Monodromy Defects

Based on: **2605.16482** (CC)

also 2507.15466 (CC) + 2412.18652 (CC, Antinucci, Galati, Rizi)

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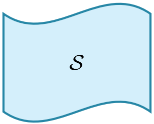
Symmetries 2026

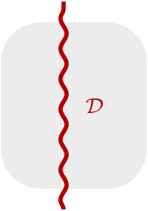
Paris

Science and Technology Facilities Council



Symmetries are powerful

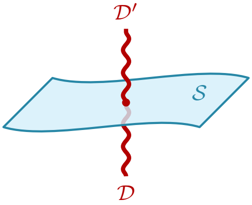
- Rigid yet rich mathematical structure ("Higher" fusion category).
- Selection rules on dynamical processes (Correlation functions, scattering amplitudes...)
- Exact constrains on RG flows and phases of matter.



Defects are everywhere; and hard!

- Strings/Wilson lines (hep); Lattice impurities (cond-mat); ...
- Probe strongly-coupled bulk dynamics (confinement, topological order, ...).
- Resist perturbative control (exceptions: large N, SUSY, free theories... [MANY works!]) $\rightarrow$ New tools needed: Bootstrap [Billó et.al. '16] , Fuzzy sphere [Zhou et. al. ´23] , **Categorical Symmetries**.

Symmetries act on defects!

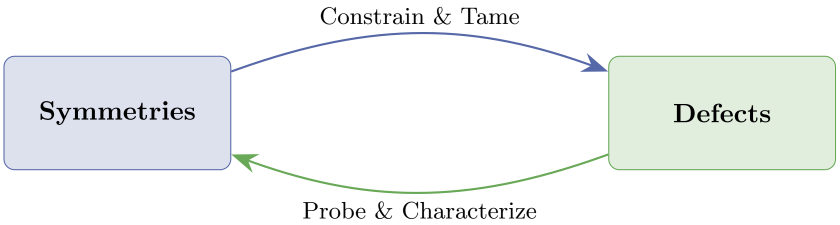


- "Higher" algebraic structures [Barkeshli, Bonderson, Chen, Wang '14; Benini, Cordova, Hsin '18; Bhardwaj, Schafer-Nameki '23; Bartsch, Bullimore, Grigoletto '23; CC '24; ...] .

- New handles on defect properties [Antinucci, CC, Galati, Rizi '24; Komargodski, Popov, Rayhaun '25; ...] from symmetries and (defect) anomalies.

- Topology of symmetry-breaking defects [Debray et. al. '23; Choi et. al. '25; CC '25; ...] ; boundary/defect families.

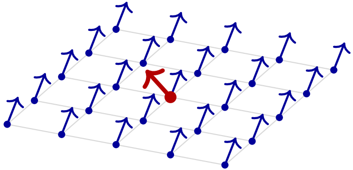
A Virtuous Loop



This Talk:

- (Monodromy) defects as probes of disordered phases. $\Longleftarrow$
- Inflow: anomaly-protected features of gapless monodromy defects. $\Longrightarrow$

SSB phases can be "locally" probed by defects [Assaad, Herbut '13; Popov, Wang '25; ...] (instead of correlators)



A local symmetry-breaking field $\vec{h}$ defines a pinning (line) defect:

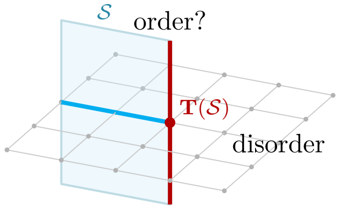
$$\mathcal{D}[\vec{h}] = \exp\left(- \int d\tau \, \vec{h} \cdot \vec{\mathbf{S}} \right) .$$

Bulk SSB phase is encoded in the $\vec{h}$ dependence of the free energy for $\mathcal{D}[\vec{h}]$.

Probing disorder? The Monodromy Defect

Can defects also probe disordered (but still interesting) phases (e.g. SPTs)?

The natural probes are Monodromy defects $\mathbf{T}(\mathcal{S})$ (a.k.a. String order parameters)



While the bulk is disordered, physics on $\mathbf{T}(\mathcal{S})$ can be ordered, or even gapless.

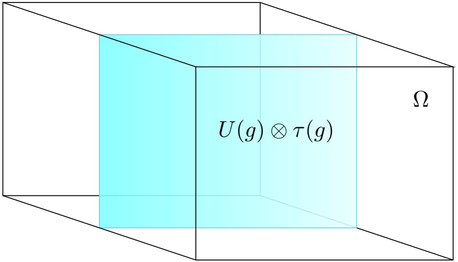
How does this work? $\longrightarrow$ Decorated domain wall (DDW) construction.

For this talk we consider $\mathcal{S} = G$ a group. Let:

$$\Omega \in \mathrm{SPT}_G^{d+1} \, ,$$

denote the bulk SPT.

Inserting a symmetry defect $U(g)$ in the bulk SPT decorates its worldvolume by a lower-dimensional SPT $\tau(g)$.



The decorated system is still gapped, and disordered.

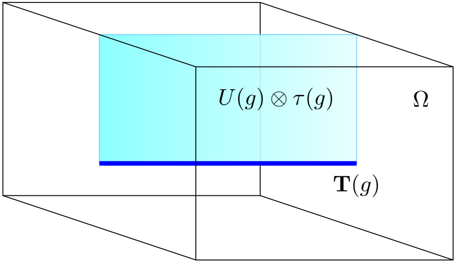
The decoration is computed by a twisted-compactification (transgression) in a flat g background. This defines a map

$$\tau(g) : \mathrm{SPT}_G^{d+1} \longrightarrow \mathrm{SPT}_{C(g)}^d \,.$$

Concretely, we can compute the Ω partition function on $X \times S_g^1$ with background holonomy g on S^1

$$Z_{\tau(g)}[X] = Z_\Omega[X \times S_g^1] \,, \qquad P \exp\left(i \int_{S^1} A \right) = g \,.$$

The dressing becomes important in describing $\mathbf{T}(g)$, as it must provide a termination for both $U(g)$ and $\tau(g)$.



For a nontrivial $\tau(g)$ there are three options:

1. $\mathbf{T}(g)$ (spontaneously) breaks the $C(g)$ symmetry: the system is ordered on the defect and disordered in the bulk.
2. $\mathbf{T}(g)$ saturates the anomaly by topological order (requires $d \geq 5$). Alternatively, the symmetry is **extended** on $\mathbf{T}(g)$.
3. $\mathbf{T}(g)$ is gapless.

A) Type III SPT in (2+1)d ($G = \mathbb{Z}_2^A \times \mathbb{Z}_2^B \times \mathbb{Z}_2^C$):

$$\Omega = \pi \int A \cup B \cup C \,.$$

Transgression ($\mathbb{Z}_C$ defect):

$$\tau(C) = \pi \int A \cup B \,.$$

Nontrivial SPT: edge modes on $\mathbf{T}(C)$ must break the $\mathbb{Z}_2^A \times \mathbb{Z}_2^B$ symmetry (ordered phase with two vacua on monodromy defect).

B) (2+1)d SPT for $G = U(1)$ (IQH state):

$$\Omega = \frac{1}{4\pi} \int A dA \, .$$

transgression:

$$\tau(\theta) = \frac{\theta}{2\pi} \int dA \, .$$

For $\theta \neq \pi$ this is a trivial 2d SPT. However, we cannot trivialize it in a strictly $U(1)$-preserving manner, as the Wilson line:

$$\exp\left(i \frac{\theta}{2\pi} \int A \right) ,$$

is ill-quantized.

The resolution is that the symmetry Γ on the monodromy defect is an extension.

$$\Gamma = [U(1)]_N \text{ when } \theta = 2\pi p/N\,, \qquad \Gamma = \mathbb{R} \text{ otherwise}\,,$$

and the monodromy Hilbert space hosts $\theta/2\pi \mod 1$ units of charge.

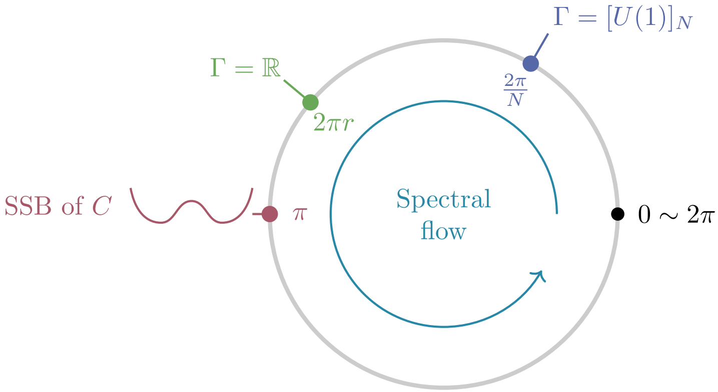
The defect displays an **anomaly in the space of defect couplings** [Cordova, Freed, Lam, Seiberg '19; Choi et. al. '25; CC '25; Komargodski, Popov, Rayhaun '25; ...] : as

$$\theta \to \theta + 2\pi$$

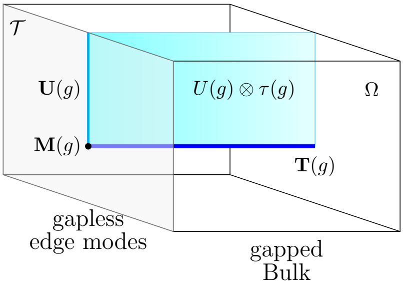
adiabatically, one unit of charge is deposited on $\mathbf{T}(\theta)$.

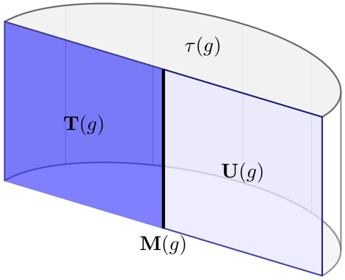
Matched by the fact that, at $\theta = \pi$, $\tau(\pi)$ is a nontrivial SPT for $U(1) \rtimes C$ and $\mathbf{T}(\pi)$ must SSB C.

This is analogous to the spectral flow for QM on a circle.



We realize an anomalous gapless system $\mathcal{T}$ as edge modes of the SPT Ω





$\mathbf{U}(g)$ becomes domain wall betwenn $\mathcal{T}$ and $\mathcal{T} \otimes \tau(g)$. Terminating $\mathbf{U}(g)$ requires a second, topological domain wall $\mathbf{T}(g)$ to define the monodromy defect.

Properties of $\mathbf{T}(g)$ translate into protected features of the gapless defect $\mathbf{M}(g)$:

1. $\mathbf{T}(g)$ SSB the symmetry $\Rightarrow$ $\mathbf{M}(g)$ is a non-simple defect
$$\mathbf{M}(g) = \bigoplus_m \mathbf{M}_m(g)\,.$$
2. $\mathbf{T}(g)$ is Topologically ordered $\Rightarrow$ $\mathbf{M}(g)$ hosts protected (chiral) edge modes.
3. $\mathbf{T}(g)$ is gapless $\Rightarrow$ $\mathbf{M}(g)$ is ill-defined (rotation symmetry around $\mathbf{M}(g)$ is broken by gapless doamin wall).

Example: chiral monodromy defect.

(Early studies [Bianchi et. al. '21; Giombi, Helfenberger, Ji, Khanchandani '21; ...] See also [Boyle Smith, Caramati, Cuomo, Tortora, Wang to appear] .) Consider free (Dirac) fermion:

$$S = i \int \overline{\Psi} \gamma^\mu \partial_\mu \Psi \,,$$

Symmetry $U(1)_V \times U(1)_A / \mathbb{Z}_2$ with action

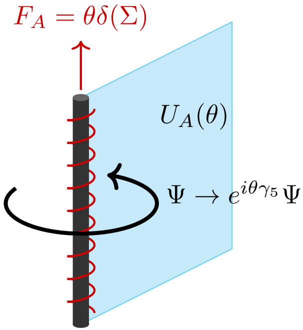
$$\begin{aligned} U(1)_V : & \qquad \Psi \longrightarrow e^{i\alpha_V} \Psi, \\ U(1)_A : & \qquad \Psi \longrightarrow e^{i\beta_A \gamma_5} \Psi \end{aligned}$$

Anomaly:

$$P_6 = 2\pi k \left[\frac{1}{3} \left(\frac{F_A}{2\pi} \right)^2 + \frac{F_A}{2\pi} \left(\frac{F_V}{2\pi} \right)^2 + \frac{F_A}{2\pi} \wedge \widehat{A}(R) \right] + 48 \widetilde{k}_g F_A \wedge \widehat{A}(R)$$

$k = 1$, $\widetilde{k_g} = 0$.

Study monodromy for (anomalous) axial symmetry:

$\mathbf{M}(\theta) =$ 

For which (trivial axial background):

$$\tau(\theta) = \frac{\theta}{2\pi}\left[\frac{1}{4\pi}F_V \wedge F_V + \widehat{A}(R)\right]$$

Similar to previous example, for $\theta \neq \pi$ trivial SPT, but cannot be trivialized in a $U(1)_V$-preserving manner. Two resolutions:

(A) Keep the fractional response.

(natural for the free fermion)

- Massless system: response saturated by gapless bulk modes.
- $U(1)_V$ is *fractionalized* on the twisted Hilbert space $\mathcal{H}_\theta$.
- Ill-quantized CS $\Rightarrow$ fractional charge on $\mathbf{T}(\theta)$, seen in a monopole background.

(B) Match by an FQH state.

(natural if $U(1)_V$ weakly gauged [Choi, Cordova, Hsin, Lam, Shao '21; Kaidi, Ohmori, Zheng '21; ...])

- For $\nu = \frac{\theta}{2\pi} = \frac{p}{N}$ rational, dress $\mathbf{M}(\theta)$ by a (2+1)d TQFT with

$$\sigma_H = c_- = \frac{\theta}{2\pi} \, .$$

- Symmetry fractionalization $B = \kappa\, F_V/2\pi$: anyons carry charge $\kappa q/N$, and $\mathbf{M}(\theta)$ gains (decoupled) chiral edge modes.
- Canonical candidate is:

$$\mathbf{T}(p/N) = \mathcal{A}_{N,p}(F_V/2\pi) \otimes \mathbf{T}_0 \, .$$

$\mathcal{A}_{N,p}$ minimal $\mathbb{Z}_N$ TQFT of [Hsin, Lam, Seiberg '18] .

Due to nontrivial $\pi_1(U(1)) = \mathbb{Z}$ we can have a spectral pump on $\mathbf{M}(\theta)$. Indeed:

$$\mathbf{T}(2\pi) = [\mathrm{IQH}]_k \otimes \mathbf{T}(0)\,,$$

where $[\mathrm{IQH}]_k$ is a $\sigma_H = c_- = k$ (Spin$_c$) integer quantum Hall state. The pump gives (1+1)d chiral edge modes with $c_- = k$ on the $\mathbf{M}(1)$ worldvolume.

For free fermion axial symmetry ($k = k_g = 2$), this can be verified from the b-function (conformal anomaly) on the defect [Bianchi et. al. '21] :

$$b(\theta = 2\pi\nu) = \nu^2(2 - \nu^2)\,,$$

which satisfies:

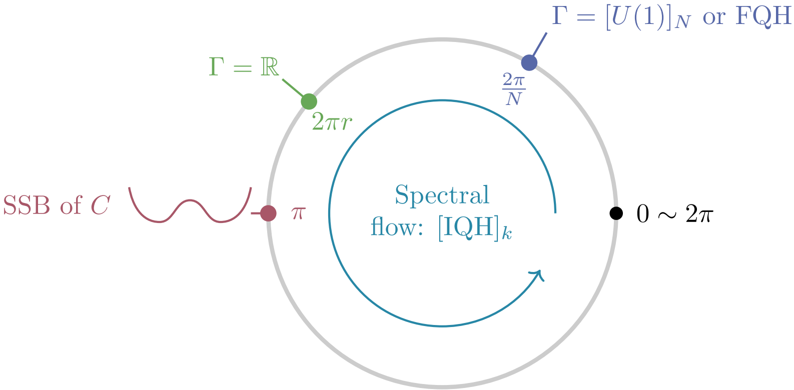
$$b(2\pi) - b(0) = 1\,.$$

So that (inclding the pump from the bulk anomaly)

$$\Delta c_L = \Delta b + \Delta k_g/2 = 2\,, \qquad \Delta c_R = \Delta b - \Delta k_g/2 = 0\,.$$

The nontrivial spectral pump implies level crossing at $\theta = \pi$, where $\tau(\pi)$ is the nontrivial T-invariant topological insulator.

In the free case $\mathbf{M}(\pi)$ is non-simple ($\mathbf{T}(\pi)$ SSB T):



Curiosity: (3+1)d TI admits gapped BC given by T-Pfaffian [Bonderson et. al. '13; Wang et. al. '13; Chen et. al. '13; Metlitski et. al. '13; Fidkowski et. al. '13; ...] . Can this give rise to a T-preserving $\mathbf{M}(\pi)$?

Open questions

- Higher spectral pumps? $(\pi_n(G))$
- Monodromy for non-invertible symmetries? See [Shao, Zhong '25; Bashmakov '26] .
- Renyi / conical defects? Impact of gravitational anomaly on EE?
- Crystalline defects (i.e. disclinations and discloations on lattice), characterization of protected modes.
- Scattering experiments on $\mathbf{M}(g)$: can amplitudes probe topological data? See e.g. discussion on fermion-monopole scattering in hep-th [van Beest et.al. '23; Loladze, Okui, Tong '25; Ueda et.al. '25; CC, Antinucci, Galati, Rizi '26 (see next talk!)] .

Thank you!