Basepoint anomalies and the vanishing of partition functions

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Based on arXiv:2510.19935 with Felix Christensen and Iñaki García Etxebarria

(see also Iñaki's talk at Strings and Geometry 2026)

Global Categorical Symmetries 2026

MAX PLANCK INSTITUTE
FOR MATHEMATICS IN THE SCIENCES

Outline

Motivation: Freed-Witten anomaly

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Freed-Witten anomaly cancellation [Witten '98, Freed-Witten '99]

In the presence of a D-brane, the worldsheet path integral of Type II string theory suffers from a global anomaly unless

$$[H_3]|_Q + W_3(Q) = 0$$

with Q the D-brane worldvolume, H_3 the NSNS 3-form flux, and W_3 the third integral Stiefel-Whitney class.

We wanted to generalize this condition to D/M-branes in non-perturbative string/M-theory backgrounds (e.g. S-folds [García Etxebarria-Regalado '16, Aharony-Tachikawa-Gomi '16] and Class $\mathcal{S}$ theories [Gaiotto-Moore-Neitzke '13]).

More than just branes. . .

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Inspired by [Hsieh-Tachikawa-Yonekura '20], we take an anomaly-based approach to bypass this difficulty.

- Instead of saying the worldvolume theory on the brane is "inconsistent," whenever the FW condition is not satisfied the partition function necessarily vanishes!
- Goes beyond the study of branes; applies to any QFT with known anomaly theory.
- Systematic perspective unifying several previously known results
  - ABJ anomaly [Fujikawa '79]
  - Shifted quantization of M-theory flux [Witten '96]
  - K-theory classification of D-branes [Minasian-Moore '97, Witten '98]
  - Quantum Gauss' law [Diaconescu-Moore-Freed '03, Belov-Moore '04]
  - Continuum analogue of the LSM theorem [Córdova-Ohmori '19]
  - (Non)-vanishing of the 3d minimal TQFT [Karasik '22]

Prototypical example: ABJ anomaly

Consider a 4d massless Dirac fermion on a closed spin manifold X,

$$S = \int d^4x \sqrt{g}\, i\bar{\psi} \not{D} \psi \,.$$

Classically, $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_A$ act respectively by

$$\psi \to e^{i\alpha}\psi \,, \qquad \psi \to e^{i\beta\gamma^5}\psi \,,$$

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Atiyah-Singer index theorem [Atiyah-Singer '68]

$$\tfrac{1}{2}\, d * j_A = \mathrm{ind}(\mathcal{D}) = \int_X \left(\tfrac{1}{2}\, F_V^2 - \tfrac{1}{24}\, p_1(TX)\right)$$

where $\mathrm{ind}(\mathcal{D}) = n_+ - n_-$ is the difference in the number of zero modes with positive/negative chiralities, and $j_A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$.

The partition function can be explicitly computed [Fujikawa '79],

$$\mathcal{Z}[\omega, A_V] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{-S[\psi,\bar{\psi},\omega,A_V]} = \det(i\not{D}) = \prod_i \lambda_i \,.$$

If we have zero modes ($\lambda_i = 0$ for any i), then $\mathcal{Z}[\omega, A_V] = 0$.

- The AS index theorem implies that there is at least one zero mode whenever $\mathrm{ind}(\mathcal{D}) \neq 0$, in which case $\mathcal{Z}[\omega, A_V] = 0$.
- Suppose $p_1(TX) = 0$, then $\mathcal{Z}[\omega, A_V] = 0$ if $[\frac{1}{2} F_V^2] \neq 0 \in H^4(X;\mathbb{Z})$.

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Index theorems are super powerful, but they are hard and not so easily accessible for generic QFTs!

Anomaly-induced vanishing

We want to rephrase the previous statement in terms of a mixed anomaly between $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_A$.

To do so, let us further couple the theory to a background field A_A for $\mathrm{U}(1)_A$, then one can show that

$$\underbrace{\mathcal{Z}[\omega, A_V - d\alpha, A_A - d\beta]}_{\mathcal{Z}[\omega, A_V, A_A]} = e^{-i\beta \int_X d{*}j_A} \mathcal{Z}[\omega, A_V, A_A]\,,$$

where α, β are the constant rotation angles in $\psi \to e^{i\alpha}\psi$ and $\psi \to e^{i\beta\gamma^5}\psi$.

The anomalous phase $e^{-i\beta \int_X d{*}j_A}$ originates from a 5d anomaly theory,

$$S_{\text{anom}} = -4\pi i \int_Y \underline{A}_A \wedge \left(\tfrac{1}{2}\, \underline{F}_V^2 - \tfrac{1}{24}\, p_1(TY) \right)$$

where $\partial Y = X$, and $\underline{A}_A, \underline{F}_V$ are extensions of A_A, F_V from X to Y.

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Replace Y with the mapping cylinder $X \times [0,1]$, and set

$$\underline{A}_A = A_A - \frac{\beta}{2\pi}\,dt\,, \qquad \underline{A}_V = A_V - \frac{\alpha}{2\pi}\,dt\,,$$

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Exactly what the AS index theorem told us, without needing to compute $\mathcal{Z}[\omega, A_V, A_A]$ explicitly!

Rigid gauge transformations

A slick way to see why β is the axial rotation angle:

- Integration by parts: $e^{-i\beta \int_X d{*}j_A} \sim e^{i\int_X d\beta \wedge {*}j_A}$ ($d * j_A$ is not actually exact)
- Coupling term: $e^{-i\int_X A_A \wedge {*}j_A} \subset \mathcal{Z}[\omega, A_V, A_A]$
- Combine into $e^{-i\int_X (A_A - d\beta)\wedge {*}j_A}$

The action $\psi \to e^{i\beta\gamma^5}\psi$ of the global $U(1)_A$ symmetry is implemented by rigid gauge transformations $A_A \to A_A - d\beta$ for some constant β (up to 2π normalization factors).

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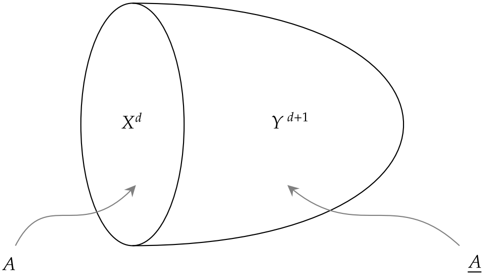
Remark

A vanishing partition function means generic correlators are zero, unless we also insert suitable operators which source the current (in this case, some fermion bilinear).

Outline

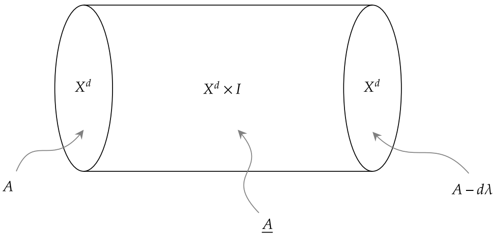
Three ways to geometrize anomaly

1. As a bulk invertible field theory $e^{2\pi i \mathcal{A}[\underline{A}]}$ [Freed '14]



- Defined on any bulk Y^{d+1} with $\partial Y^{d+1} = X^d$
- $\mathcal{A}[\underline{A}]$ depends only on the extension of A to the bulk
- Useful for studying anomaly inflow [Callan-Harvey '85]

2. As a mapping cylinder



- Place the same theory $\mathcal{A}[\underline{A}]$ on $X^d \times I$
- $\underline{A}$ interpolates between A and $A - d\lambda$ to model gauge transformation
    - E.g. $\underline{A} = A - d(t\lambda)$
- For non-flat λ, the bulk field $\underline{A} \supset -t \wedge d\lambda$ changes with t

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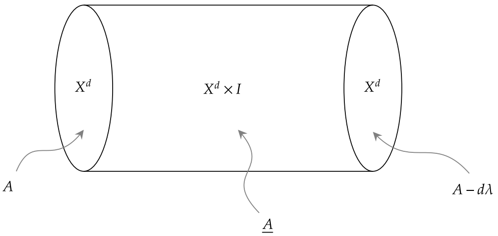

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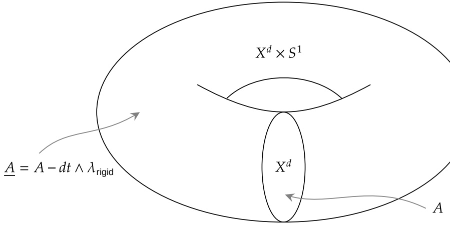
  - E.g. $\underline{A} = A - d(t\lambda)$
- For non-flat λ, the bulk field $\underline{A} \supset -t \wedge d\lambda$ changes with t
- Standard 't Hooft anomaly is the parallel transport of $\mathcal{A}[\underline{A}]$ along I
  - $\mathcal{Z}[A - d\lambda] = e^{2\pi i \mathcal{A}[A,\lambda]} \mathcal{Z}[A]$
- $\mathcal{Z}[A]$ doesn't have to vanish even if $e^{2\pi i \mathcal{A}[\underline{A}]} \neq 1$

3. As a mapping torus



- If we restrict $\lambda = \lambda_{\text{rigid}}$ to be flat, then $A - d\lambda_{\text{rigid}} = A$
  - Glue two ends of the interval to form a mapping torus
- $\underline{A} = A$ on any t-slice
- Tracks rigid gauge transformations $A \to A - d\lambda_{\text{rigid}}$

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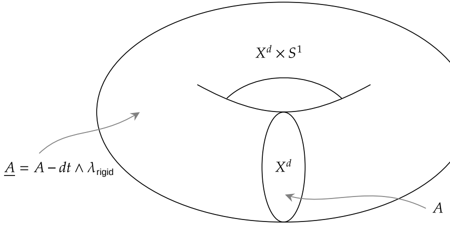

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  - Glue two ends of the interval to form a mapping torus
- $\underline{A} = A$ on any t-slice
- Tracks rigid gauge transformations $A \to A - d\lambda_{\text{rigid}}$
- 't Hooft anomaly reduces to a holonomy of $\mathcal{A}[\underline{A}]$ around S^1
  - $\mathcal{Z}[A - d\lambda_{\text{rigid}}] = e^{2\pi i \mathcal{A}[A, \lambda_{\text{rigid}}]}\mathcal{Z}[A]$
- $\mathcal{Z}[A]$ necessarily vanishes if $e^{2\pi i \mathcal{A}[A, \lambda_{\text{rigid}}]} \neq 1$

Interlude: differential cohomology

Why can $e^{2\pi i \mathcal{A}[A,\lambda_{\text{rigid}}]}$ ever be non-trivial?

- If we only cared about local data, then the phase is indeed trivial, but as we saw in the ABJ example, the partition function vanishes precisely when we have a non-zero instanton number (i.e. $F \neq dA$).

Global topological information matters!

Best captured using differential cohomology [Cheeger-Simons '85, Hopkins-Singer '02].

A differential cochain $\check{A} \in \check{C}^{p+2}(X^d)$ is a triple

$$(\mathsf{C}_{p+2}, A_{p+1}, F_{p+2}) \in C^{p+2}(X^d;\mathbb{Z}) \times C^{p+1}(X^d;\mathbb{R}) \times \Omega^{p+2}(X^d)$$

describing the characteristic class, connection, curvature.

A gauge field for a p-form symmetry is a differential cocycle $\check{A} \in \check{Z}^{p+2}(X^d)$ satisfying

$$\delta \mathsf{C}_{p+2} = 0\,, \qquad dF_{p+2} = 0\,, \qquad \delta A_{p+1} = F_{p+2} - \mathsf{C}_{p+2}\,.$$

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Topologically trivial: trivial characteristic class ($\mathsf{C}_{p+2} = 0$)
Flat: trivial curvature ($F_{p+2} = 0$)

We define $\check{H}^{p+2}(X^d) = \frac{\check{Z}^{p+2}(X^d)}{\check{C}^{p+1}_{\text{flat}}(X^d)}$ by identifying gauge transformations that leave F_{p+2} invariant, i.e.

$$\mathsf{C}_{p+2} \sim \mathsf{C}_{p+2} - \delta\Lambda_{p+1} \quad \text{and} \quad A_{p+1} \sim A_{p+1} + \Lambda_{p+1} + \delta\lambda_p$$

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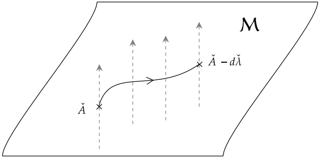
We also have flat differential cocycles $\check{\lambda}_{\text{rigid}} \in \check{Z}^{p+1}_{\text{flat}}(X^d) \subset \check{C}^{p+1}_{\text{flat}}(X^d)$, meaning $\delta\lambda_p = -\Lambda_{p+1}$ (equivalently, $\beta(\lambda_p) = \Lambda_{p+1}$).

Since $d\check{\lambda}_{\text{rigid}} = 0$, we have $\check{A} - d\check{\lambda}_{\text{rigid}} = \check{A}$.

- $\check{Z}^{p+1}_{\text{flat}}(X^d)$ gives the automorphisms of $\check{A} \in \check{C}^{p+2}(X^d)$
- $\check{H}^{p+1}_{\text{flat}}(X^d) \cong H^p(X^d; \mathbb{R}/\mathbb{Z})$
- Corresponds to the space of rigid gauge transformations

Moduli space perspective

Naïvely, $\mathcal{Z}[\check{A}]$ is a function over the moduli space $\mathcal{M} = \check{Z}^{p+2}(X^d)$.



$\mathcal{M}$ is equipped with an action by $\mathcal{G} = \check{C}^{p+1}_{\text{flat}}(X^d)$, so $\mathcal{Z}[\check{A}]$ is a section of a $\mathcal{G}$-equivariant line bundle $\mathcal{L}$ over $\mathcal{M}$. The topology of $\mathcal{L}$ is characterized by the 't Hooft anomaly $e^{2\pi i \mathcal{A}[\check{A},\check{\lambda}]}$ [Seiberg-Tachikawa-Yonekura '18].

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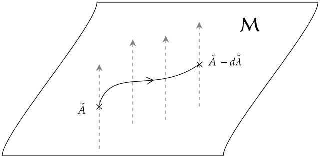

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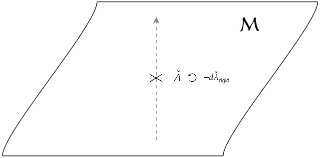
Gauging the symmetry means lifting $\mathcal{L}$ to an ordinary line bundle over $\mathcal{M}/\mathcal{G} \cong \check{H}^{p+2}(X^d)$, i.e. identifying gauge transformations.

- Trivial anomaly: $\mathcal{Z}[\check{A}]$ is a function over $\mathcal{M}/\mathcal{G}$
- Non-trivial anomaly: $\mathcal{Z}[\check{A}]$ is a section of a non-trivial line bundle

$\mathcal{M}/\mathcal{G}$ is a well-behaved space only if the $\mathcal{G}$-action admits no fixed points! Should replace with $\mathcal{M} /\!\!/ \mathcal{G}$ in general.

The fixed points correspond precisely to the rigid gauge transformations, i.e. $\mathcal{G}_{\text{fixed}} = \check{Z}^{p+1}_{\text{flat}}(X^d) \subset \mathcal{G}$.



Instead of moving along some path, we sit at a fixed basepoint on $\mathcal{M}$.

- An "internal symmetry" of $\mathcal{M}$
- We call $e^{2\pi i \mathcal{A}[\check{A},\check{\lambda}_{\text{rigid}}]}$ the basepoint anomaly
- Partition function has to vanish over this point
- Better to promote the moduli space to a moduli stack

An aside: quantum Gauss' law

So far we have been working with global symmetries.

- Non-trivial basepoint anomaly $\implies \mathcal{Z}[\check{A}] = 0$

We can apply the philosophy in reverse for gauge symmetries.

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- Gauge-invariance means $\mathcal{Z}[\check{A} - d\check{\lambda}] \overset{!}{=} \mathcal{Z}[\check{A}] \neq 0$ for any $\check{\lambda}$

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- Gauge-invariance means $\mathcal{Z}[\check{A} - d\check{\lambda}] \overset{!}{=} \mathcal{Z}[\check{A}] \neq 0$ for any $\check{\lambda}$
- Implies the anomaly must be trivialized!
  - Classical Gauss' law: $e^{2\pi i \mathcal{A}[\check{A}, \check{\lambda}]} = 1$
  - Quantum Gauss' law: $e^{2\pi i \mathcal{A}[\check{A}, \check{\lambda}_{\text{rigid}}]} = 1$ [Belov-Moore '04]
- Useful when viewing $\mathcal{Z}[\check{A}]$ as a wavefunction in the SymTFT

Outline

A dichotomy

Our construction relies only on the knowledge of the anomaly theory, but not the partition function itself.

Two main classes of anomaly theories:

1. BF theories (mixed anomaly)
2. Chern-Simons theories (self anomaly)

Can be generalized beyond quadratic theories!

Generalized Maxwell theory

$$S = -\frac{1}{4}\int_{X^d} F_{p+1} \wedge *F_{p+1}$$

- $\mathrm{U}(1)_e^p$ symmetry (bkg B_{p+1}) & $\mathrm{U}(1)_m^{d-p-2}$ symmetry (bkg C_{d-p-1})
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- Reduction on mapping torus:

$$\mathcal{A}[\check{B}, \check{C}, \check{b}, \check{c}] = \int_{X^d}\Big((-1)^d[c_{d-p-2}]_{\mathbb{R}/\mathbb{Z}} \cup [\mathrm{H}_{p+2}]_{\mathbb{Z}} + [\mathrm{G}_{d-p}]_{\mathbb{Z}} \cup [b_p]_{\mathbb{R}/\mathbb{Z}}\Big)$$

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Maxwell partition function is non-vanishing only if $[H_{p+2}] = [G_{d-p}] = 0$.

D3-brane

Recall FW condition: $[H_3] + W_3 = 0$.

The bosonic part of the D3-brane worldvolume theory is (partly) described by a standard 4d Maxwell theory ($d = 4$, $p = 1$).

Our non-vanishing condition $[H_3] = 0$ is precisely the bosonic FW condition.

- S-dual statement: $[G_3] = 0$
- What about W_3? More on this later...

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$\mathfrak{u}(N)$ gauge theory also has a BF-type anomaly.

- FW condition for coincident D3-branes: $[H_3] + \beta(w_2^B) = 0$ [Kapustin '99]

Chiral gauge theories

Lagrangians for chiral gauge theories are tricky to write down, but their anomaly theories are Chern-Simons theories, schematically

$$\mathcal{A}[\underline{C}_{p+1}] \approx \frac{1}{2}\int_{Y^{d+1}} \underline{C}_{p+1} \wedge d\underline{C}_{p+1} \,.$$

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Anomaly is not well-defined in $\mathbb{R}/\mathbb{Z}$ because of the $\frac{1}{2}$. Need quadratic refinement [Hopkins-Singer '02], i.e.

$$\mathcal{A}[\underline{G}_{p+2}] = \frac{1}{2}\int_{Z^{d+2}} \underline{G}_{p+2} \wedge (\underline{G}_{p+2} - \Lambda_{p+2})$$

up to gravitational counterterms. Λ_{p+2} is an integral lift of the Wu class $v_{p+2} \in H^{p+2}(Z^{d+2};\mathbb{Z}_2)$, e.g. $v_1 = w_1$, $v_2 = w_2 + w_1^2$, $v_3 = w_2 w_1$.

Tangential structure comes into play for vanishing of chiral theories!

M5-brane (or 6d (2,0) SCFT)

Non-Lagrangian theory of chiral 2-form, but its anomaly theory is

$$\mathcal{A}[\underline{G}_4] = \frac{1}{2}\int_{Z^8} \underline{G}_4 \wedge (\underline{G}_4 - \Lambda_4)\,,$$

where $\Lambda_4 \bmod 2 = w_4 + w_3 w_1 + w_2^2 + w_1^4$.

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Make $\partial Z^8 = X^6 \times S^1$ our mapping torus, then reduce over S^1:

$$\mathcal{A}[\check{C}, \check{c}] = -\int_{X^6} [c_2]_{\mathbb{R}/\mathbb{Z}} \cup \left([\mathrm{G}_4]_{\mathbb{Z}} - \tfrac{1}{2}\Lambda_4\right)$$

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M5-brane partition function is non-vanishing only if $[G_4] - \frac{1}{2}\Lambda_4 = 0$.

On spin 6-manifolds, $v_4 = w_4 = \frac{1}{2} p_1 \mod 2$, so M5-brane partition function is non-vanishing only if $[G_4] - \frac{1}{4} p_1 = 0$.

Shifted quantization of M-theory flux [Witten '96]

The M-theory flux is quantized as

$$\int_{\mathcal{C}^4} \left(G_4 - \frac{1}{4} p_1 \right) \in \mathbb{Z}$$

on any 4-cycle $\mathcal{C}^4$.

From our perspective, a vanishing M5-brane partition function can be "cured" by attaching M2-branes to cancel this charge.

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The FW condition $[H_3] + W_3 = 0$ implies a twisted Spin^c structure for the D3-brane. The M5-brane analogue is a twisted String structure.

K-theory from M-theory

Compactify our M5-brane basepoint anomaly on T^2, i.e. take the 6d worldvolume to be $X^6 = T^2 \times X^4$ to obtain a D3-brane on X^4.

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Basepoint anomaly of the D3-brane

The D3-brane partition function is non-vanishing only if

$$[H_3] + W_3 = 0\,, \qquad [G_3] + W_3 = 0\,,$$
$$[E_4] - \tfrac{1}{2}\left(e_4 + W_4 + [I_4] + W_2^2\right) = 0\,,$$

where $E_4 \approx \frac{1}{2}B_2^2$, e_4 is the Euler class, and $I_4 \bmod 2 = w_2^2$. On orientable manifolds, $W_4 = W_2 = 0$.

First two conditions from Maxwell term $\int_{X^4} C_2 \wedge (B_2 + F_2)$; third condition from theta-term $\int_{X^4} \frac{1}{2} C_0 (B_2 + F_2)^2$.

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Doesn't rely on a worldsheet analysis as in the original FW derivation!

Outlook

For the sake of illustration, I discussed “easy” examples that reproduce known results, but our approach applies more generally whenever the anomaly theory is known, particularly in non-perturbative settings.

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What is a homological version of our vanishing theorem in terms of defects rather than background gauge fields?

- See partial discussion in our paper and also Iñaki's talk at S&G '26
- Generalization for categorical symmetries?