Symmetries 2026

Carolyn Zhang

Harvard University -> UBC

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(See also 2605.28967, 2605.28925)

Outline

- Strongly symmetric Lindbladians: phases and instabilities
- Phase transitions into strong-to-weak SSB (SWSSB) phases
  - (1+1d): relation to classical absorbing state transitions
  - Marginal fidelity: efficient proxy for SWSSB
    - General properties
    - Application to non-equilibrium phase transitions
  - (2+1d): modified Toom's rule
  - Outlook

Crash course on the Lindblad master equation

$$\mathscr{L}[\rho] = \dot{\rho} = \boxed{-i[H,\rho]} + \boxed{\sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}} \qquad \rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

coherent part
(unimportant)

local jump operators:
coupling to environment

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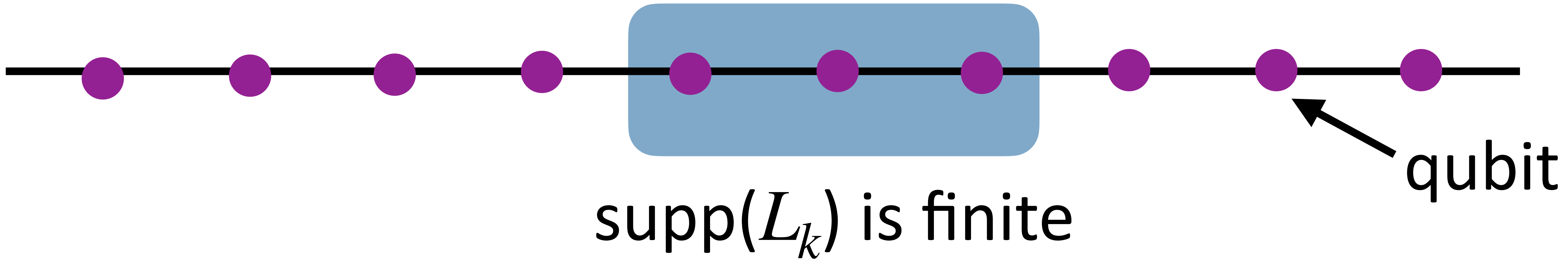

$$\mathscr{H} = \left(\mathbb{C}^2\right)^{\otimes L}$$

Crash course on the Lindblad master equation

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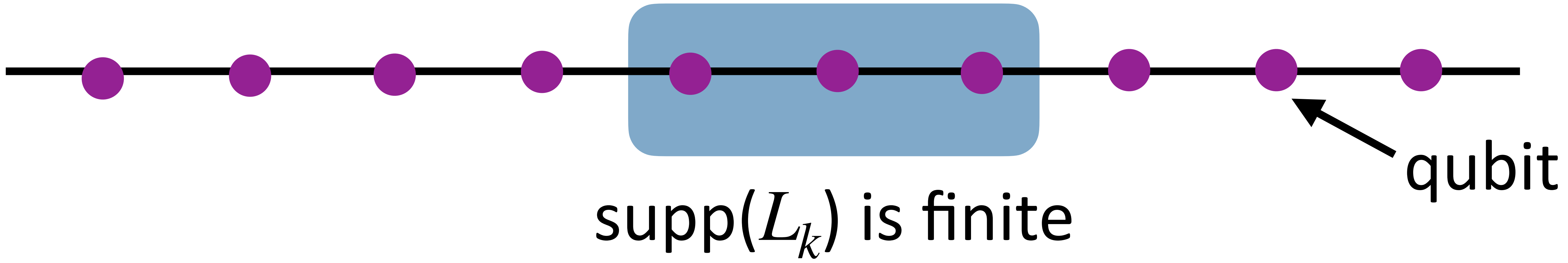

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steady state(s): $\mathscr{L}[\rho_{ss}] = \dot{\rho}_{ss} = 0 \qquad e^{t\mathscr{L}}[\rho_0] \to \rho_{ss}$

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steady state(s): $\mathscr{L}[\rho_{ss}] = \dot{\rho}_{ss} = 0 \qquad e^{t\mathscr{L}}[\rho_0] \to \rho_{ss}$

mixing time : $\min\{t \geq 0 : \|\rho(t) - \rho_{ss}\|_1 \leq \epsilon, \forall \rho(0)\}$

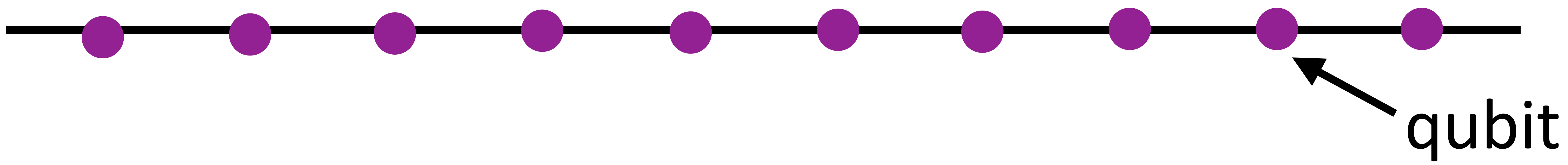
fast : $t_{\text{mix}} \sim \mathcal{O}(\log L)$

slow : $t_{\text{mix}} \sim \mathcal{O}(\text{poly}(L))$ exp(L) : long lived metastable state

Strong symmetries

Global symmetry operators $\{U_g\}$

this talk: $G = \mathbb{Z}_2, U = \prod_r X_r$



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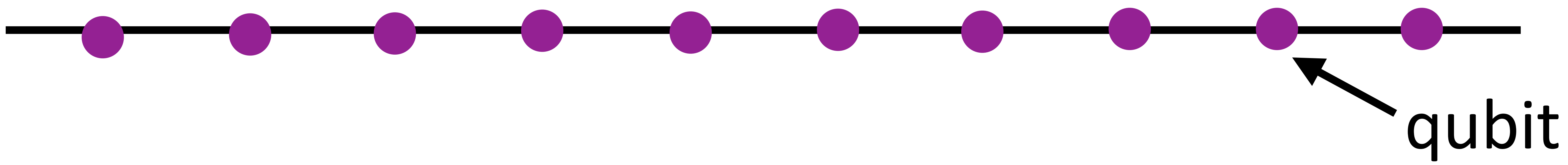

A state is strongly symmetric if $U\rho = \pm\rho$, and weakly symmetric if $U\rho U^\dagger = \rho$

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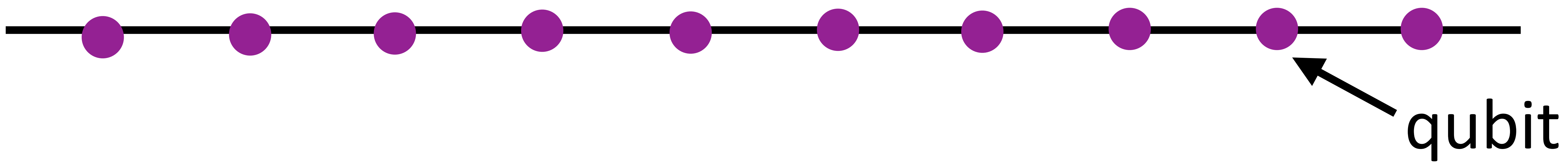

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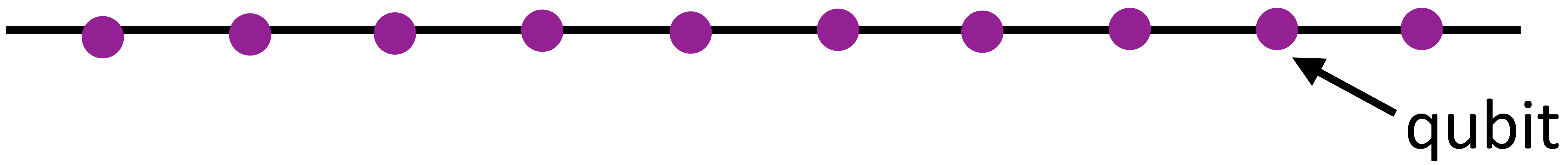

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$\mathscr{L}$ strongly symmetric: $[H,U] = [L_k, U] = 0 \qquad \forall k$

$\exists \rho_{ss}$ in each symmetry sector : $Ue^{t\mathscr{L}}[\rho] = e^{t\mathscr{L}}[U\rho] = \pm\, e^{t\mathscr{L}}[\rho]$

Warm-up: 0+1d

Equilibrium

$$H = \frac{1 - X}{2}$$

$|\mathrm{GS}\rangle = |+\rangle \qquad E = 0$

$|\mathrm{EX}\rangle = |-\rangle \qquad E = 1$

Warm-up: 0+1d

Equilibrium

$$H = \frac{1-X}{2}$$

$|\mathrm{GS}\rangle = |+\rangle \qquad E = 0$

$|\mathrm{EX}\rangle = |-\rangle \qquad E = 1$

Open

$$\mathscr{L}^0[\rho] = \frac{1+X}{2}\rho\frac{1+X}{2} + Z\frac{1-X}{2}\rho\frac{1-X}{2}Z - \rho$$

$\rho_{ss} = |+\rangle\langle+| \qquad E = 0$

$\rho_{ex} = |-\rangle\langle-| - |+\rangle\langle+| \qquad E = -1$

Warm-up: 0+1d

Equilibrium

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$\rho_{ex} = |-\rangle\langle-| - |+\rangle\langle+| \qquad E = -1$

$e^{t\mathscr{L}}[\rho_{ss} + \alpha\rho_{ex}] = \rho_{ss} + \alpha e^{-t}\rho_{ex} \rightarrow$ steer toward ρ_{ss}

unique ss: explicitly break strong X symmetry

Phases with strong Z2 symmetry in 1+1d

$\rho \propto (1 + U)$

Strong-to-weak SSB

$\rho = | + \rangle\langle + |$

Strongly symmetric

$\rho = |\mathrm{GHZ}+\rangle\langle\mathrm{GHZ} + |$

Strong-to-trivial SSB

Phases with strong Z2 symmetry: SWSSB

$\rho \propto (1 + U)$ — Strong-to-weak SSB

$\rho = |+\rangle\langle+|$ — Strongly symmetric

$\rho = |\mathrm{GHZ}+\rangle\langle\mathrm{GHZ}+|$ — Strong-to-trivial SSB

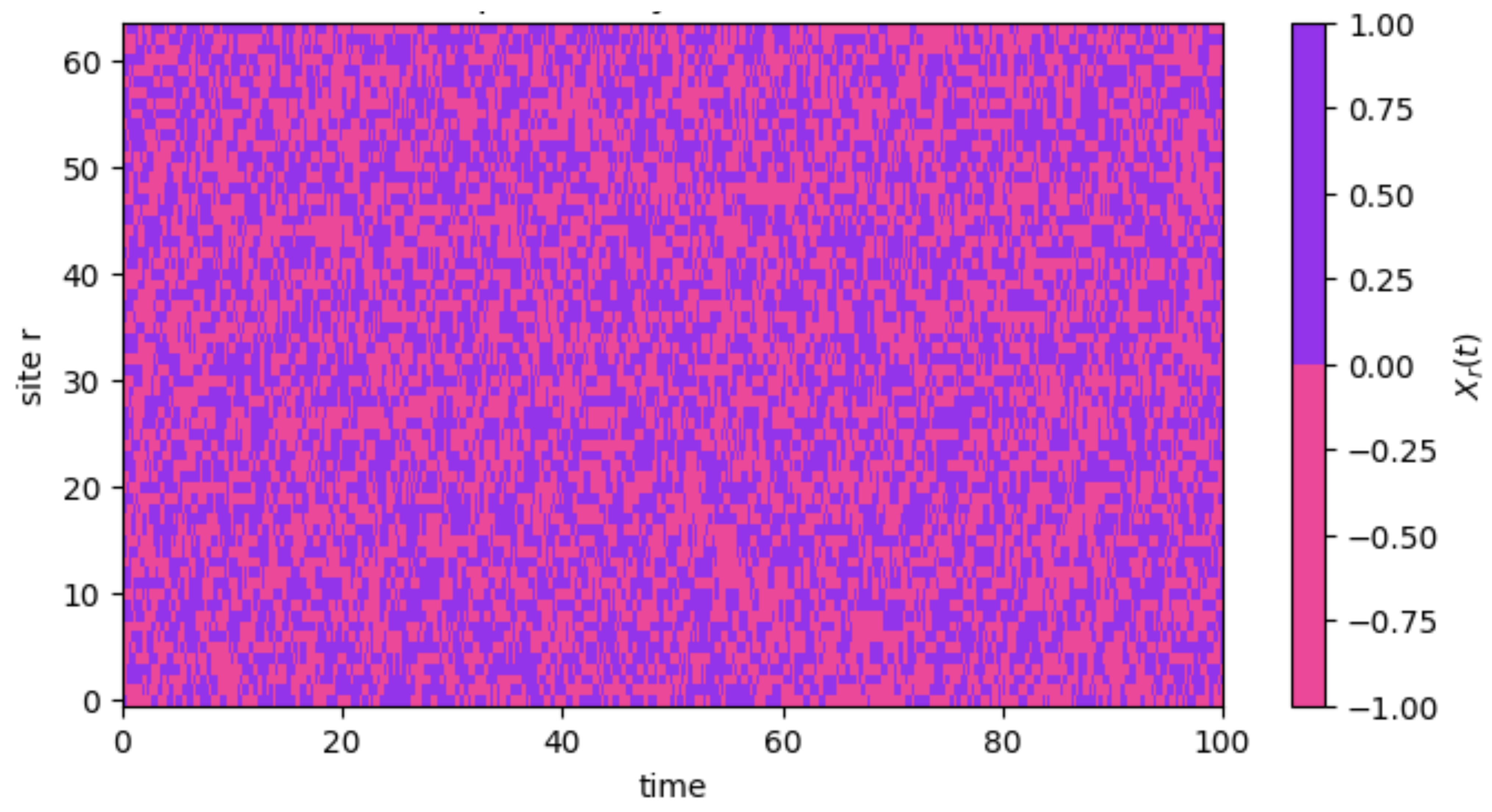
SWSSB : fast, symmetric parent Lindbladian

$$\mathscr{L}_r^{\text{SWSSB}}[\rho] = X_r \rho X_r + Z_{r-1}Z_r \rho Z_{r-1}Z_r - 2\rho$$

$$\Big\downarrow \quad \text{diag in } X \text{ basis}$$

$$(++) \leftrightarrow (--) \qquad (+-) \leftrightarrow (-+)$$

$$(r-1, r)$$



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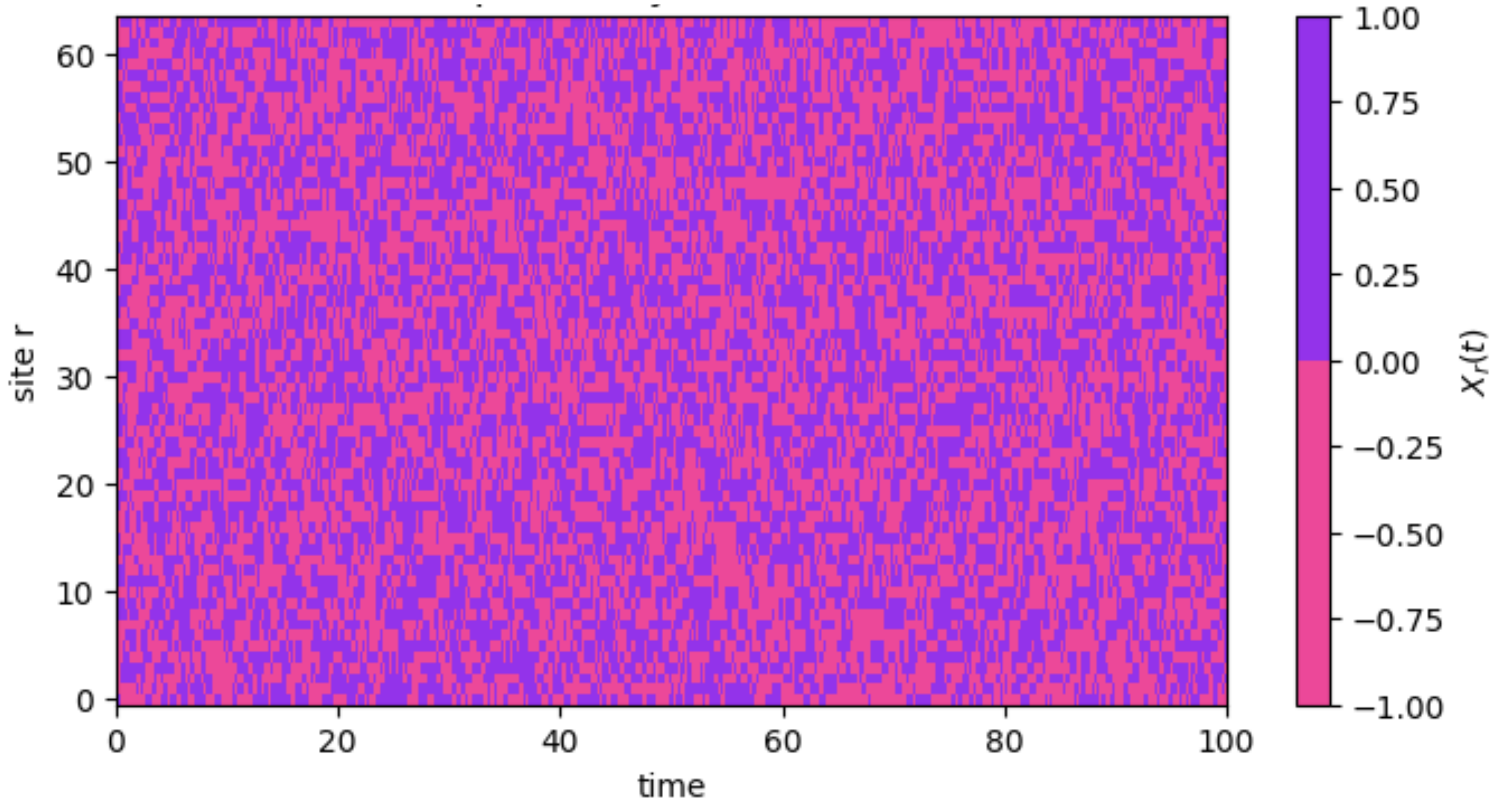
$(r-1, r)$

$\rho_- \propto (1-U)$



Phases with strong Z2 symmetry: symmetric

$\rho \propto (1 + U)$ | $\rho = | + \rangle\langle + |$ | $\rho = |\mathrm{GHZ}+\rangle\langle\mathrm{GHZ}+|$

Strong-to-weak SSB | Strongly symmetric | Strong-to-trivial SSB

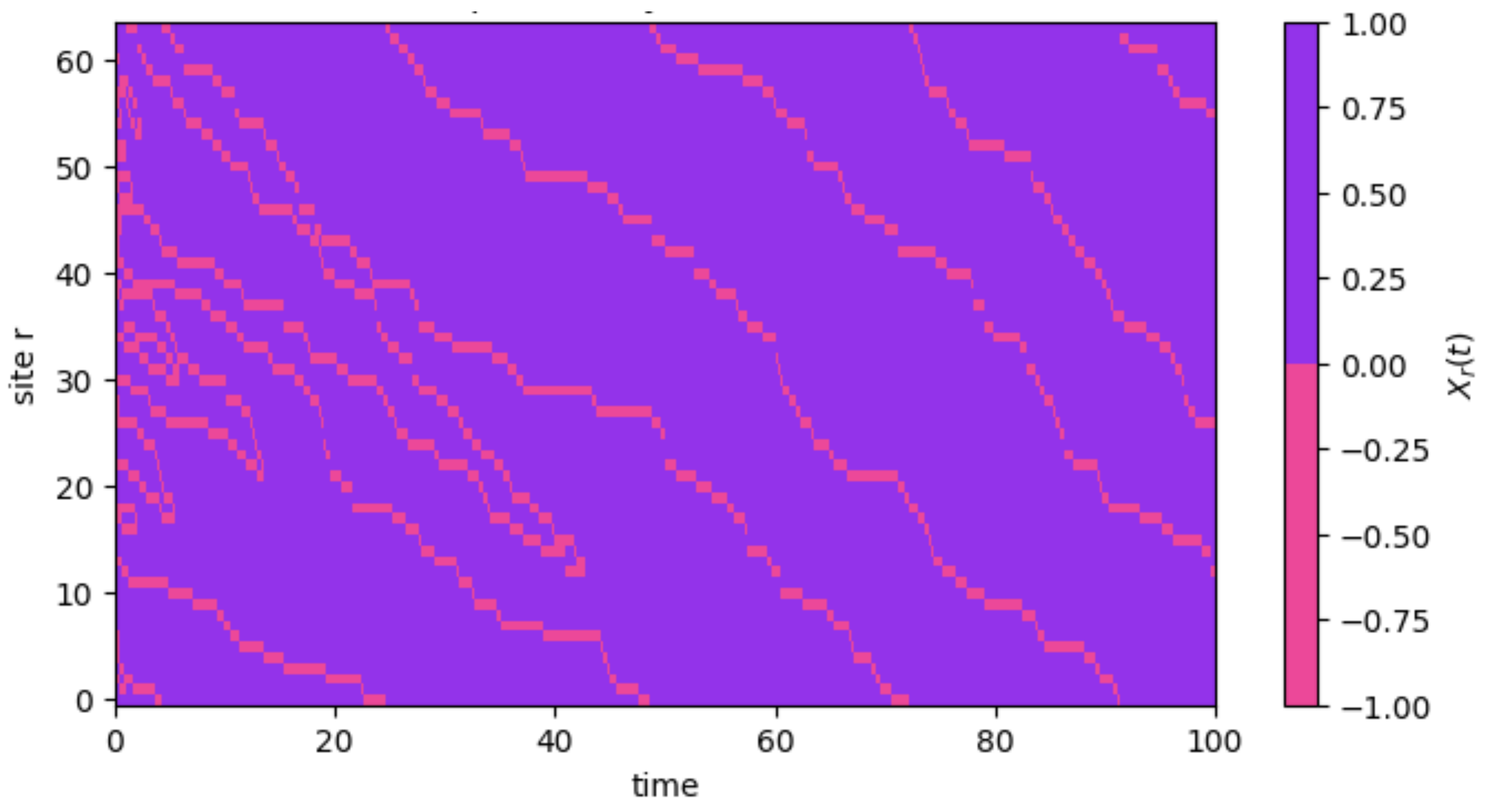
Strongly symmetric : fast, symmetry breaking or slow, symmetric parent Lindbladian

$$\mathscr{L}_r^0[\rho] = \frac{(1+X_r)}{2}\rho\frac{(1+X_r)}{2} + Z_r\frac{(1-X_r)}{2}\rho\frac{(1-X_r)}{2}Z_r - \rho$$

$\downarrow$ diag in X basis

$$(+),(-) \to (+)$$

no ρ_-



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Strongly symmetric : fast, symmetry breaking or slow, symmetric parent Lindbladian

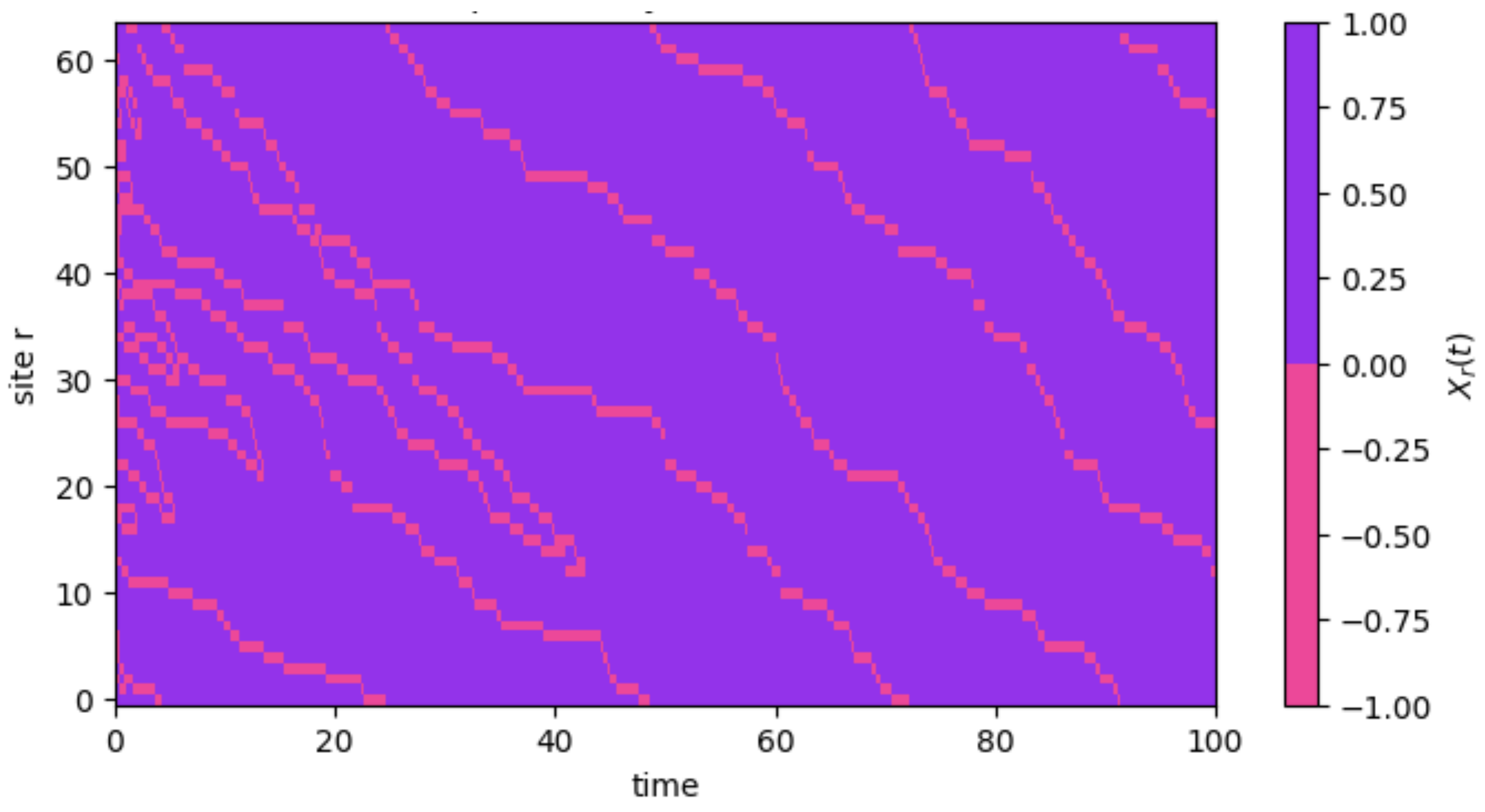
$$\mathcal{L}_r^+[\rho] = \frac{(1+X_r)}{2}\rho\frac{(1+X_r)}{2} + Z_{r-1}Z_r\frac{(1-X_r)}{2}\rho\frac{(1-X_r)}{2}Z_{r-1}Z_r - \rho$$

$\downarrow$ diag in X basis

$$(++),(--) \to (++)$$
$$(-+),(+-) \to (-+)$$

$$\rho_- = \frac{1}{L}\sum_r Z_r|+\rangle\langle+|Z_r$$

(long range correlations!)



Phases with strong Z2 symmetry: STSSB

$\rho \propto (1+U)$	$\rho = \vert + \rangle\langle + \vert$	$\rho = \vert \mathrm{GHZ}+\rangle\langle \mathrm{GHZ}+ \vert$
Strong-to-weak SSB	Strongly symmetric	Strong-to-trivial SSB

STSSB : slow, symmetric parent Lindbladian

$$\mathscr{L}_r^{+}[\rho] = \frac{(1+X_r)}{2}\rho\frac{(1+X_r)}{2} + Z_{r-1}Z_r\frac{(1-X_r)}{2}\rho\frac{(1-X_r)}{2}Z_{r-1}Z_r - \rho$$

$\downarrow$ KW duality on jump ops

$$\mathscr{L}_r^{\mathrm{STSSB}}[\rho] = \frac{(1+Z_rZ_{r+1})}{2}\rho\frac{(1+Z_rZ_{r+1})}{2} + X_r\frac{(1-Z_rZ_{r+1})}{2}\rho\frac{(1-Z_rZ_{r+1})}{2}X_r - \rho$$

Phases with strong Z2 symmetry: STSSB

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$$\rho_- = |\mathrm{GHZ}-\rangle\langle\mathrm{GHZ}-|$$

SWSSB diagnostic: fidelity correlator

Two-point linear correlator: $O_{ij} = \mathrm{Tr}(Z_i Z_j \rho)$

$O_{ij} > 0 \rightarrow$ strong symmetry SSB'd and weak symmetry SSB'd

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Two-point fidelity correlator: $F_{ij}(\rho, Z_i Z_j) = \mathrm{Tr}(Z_i Z_j \sqrt{\rho} Z_i Z_j \sqrt{\rho})$ Weinstein 2025; Liu/Chen/Zhang/Zhou/Zhang 2025

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One-point correlator zero by strong symmetry:

$$F_i(\rho, Z_i) = \mathrm{Tr}(Z_i \sqrt{\rho} Z_i \sqrt{\rho})$$

$$= \mathrm{Tr}(Z_i U \sqrt{\rho} Z_i \sqrt{\rho})$$

$$= -\mathrm{Tr}(Z_i \sqrt{\rho} Z_i \sqrt{\rho}) \rightarrow F_i(\rho, Z_i) = 0$$

$F_{ij}(\rho, Z_i Z_j) > 0 \rightarrow$ strong symmetry SSB'd but not necessarily weak symmetry

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Strong-to-weak SSB	Strongly symmetric	Strong-to-trivial SSB
$O_{ij} = 0$	$O_{ij} = 0$	$O_{ij} = 1$
$F_{ij}(\rho, Z_i Z_j) = 1$	$F_{ij}(\rho, Z_i Z_j) = 0$	$F_{ij}(\rho, Z_i Z_j) = 1$

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Stability Theorem: Lessa/Ma/Zhang/Bi/Cheng/Wang 2025

If ρ has $F_{ij}(\rho, O_i O_j^\dagger) > 0$ for some charged O_i, O_j then $F_{ij}(\mathscr{E}[\rho], \tilde{O}_i \tilde{O}_j^\dagger) > 0$

for $\mathscr{E}$ any symmetric finite depth quantum channel.

SWSSB diagnostic: fidelity correlator

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for $\mathscr{E}$ any symmetric finite depth quantum channel.

$$\rho \propto (1 + U) \quad \cancel{\longrightarrow} \quad \rho = |+\rangle\langle+| \qquad \rho = |\mathrm{GHZ}+\rangle\langle \mathrm{GHZ}+|$$

Strong-to-weak SSB — Strongly symmetric — Strong-to-trivial SSB

(need time to develop LR linear correlator)

SWSSB diagnostic: fidelity correlator

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✔

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Strong-to-weak SSB | Strongly symmetric | Strong-to-trivial SSB

✔

Phase transitions?

$\rho = |+\rangle\langle+|$

Strongly symmetric

$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\text{SWSSB}}$

0 —— 1

$\rho \propto 1 + U$

SWSSB

Phase transitions?

$\rho = |+\rangle\langle+|$

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0 ———————— 1

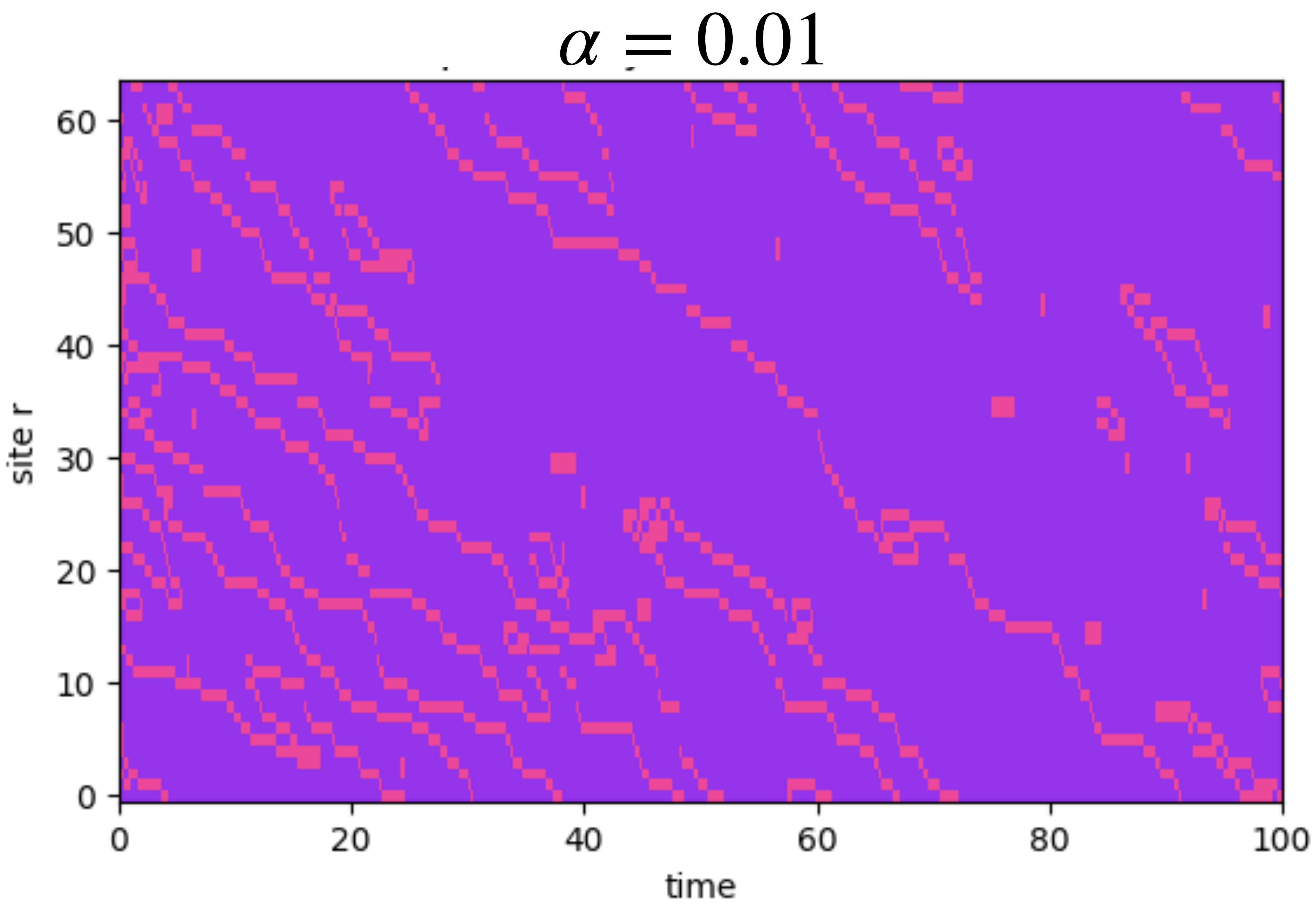
$\rho \propto 1 + U$

SWSSB

$\mathscr{L}^{+}$ is unstable to $\mathscr{L}^{\text{SWSSB}}$!

😵 defects spawn spontaneously

😵 defects cleaned diffusively



Generic in any dimension

$1+1d$: STSSB phase unstable, strongly symmetric phase unstable

$\geq 2+1d$: STSSB phase stable, strongly symmetric phase unstable

ferromagnetic phase

$$\rho \propto \frac{1+\prod_r X_r}{2} e^{-\beta H}$$

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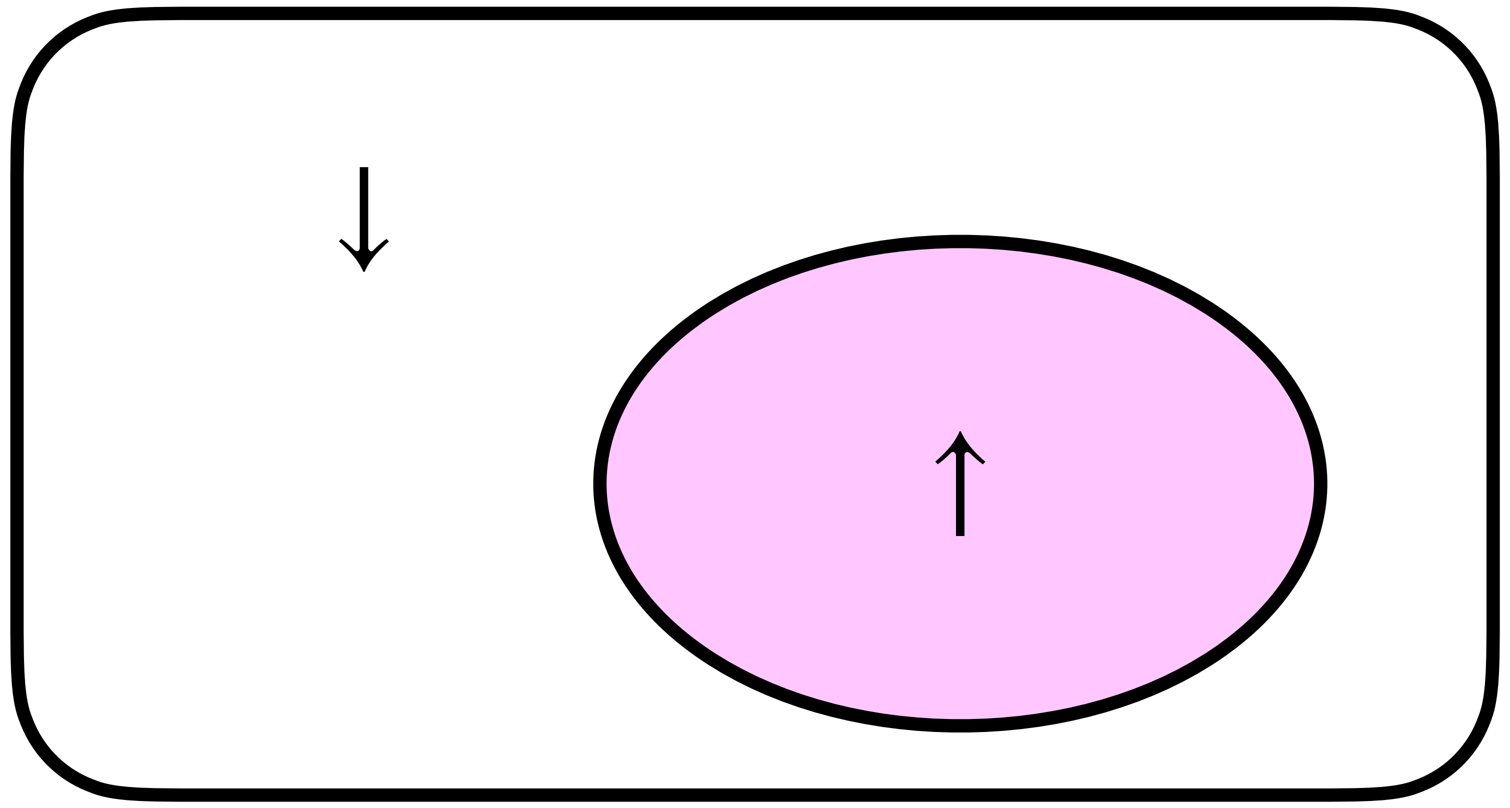
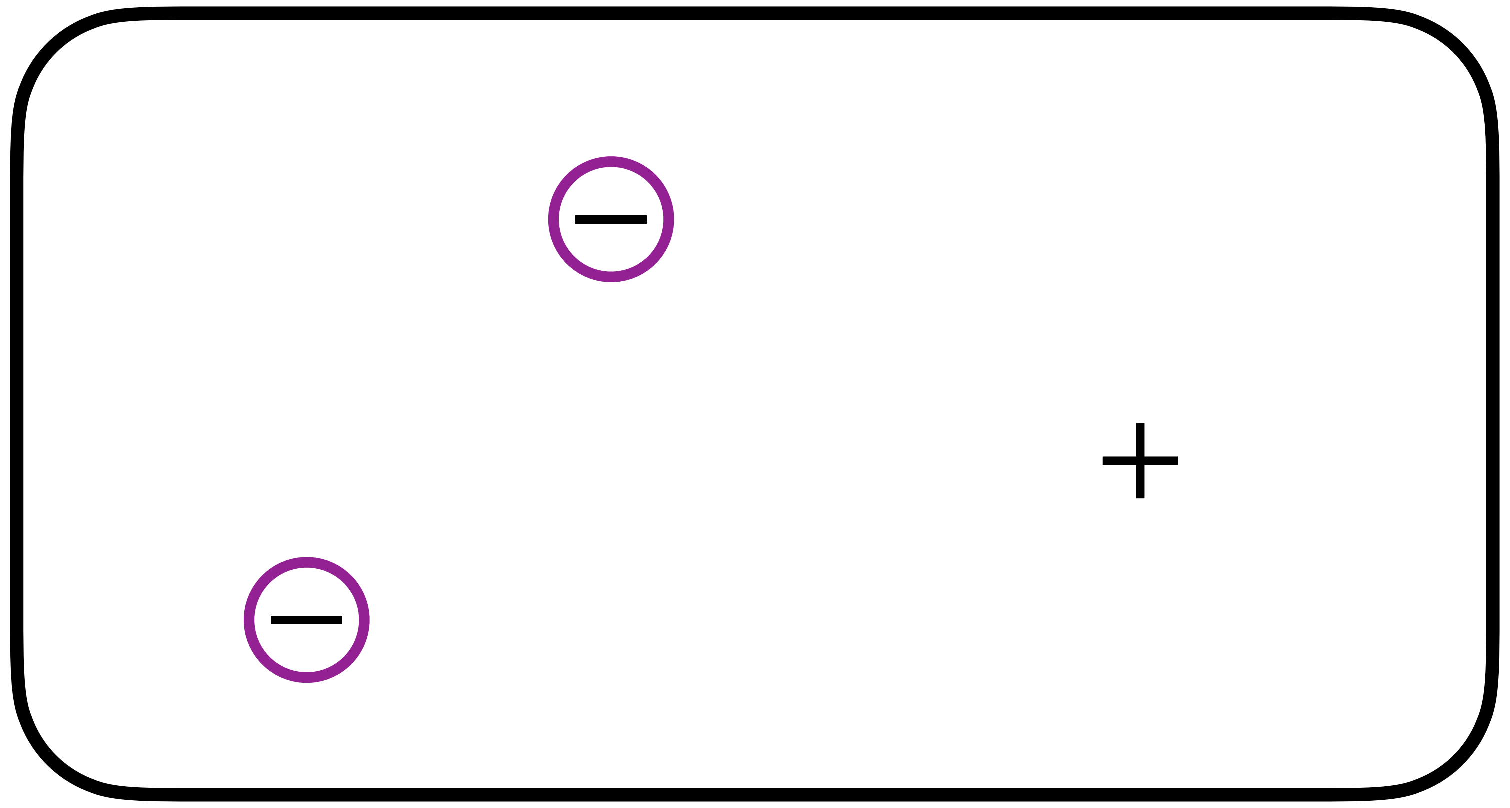


STSSB: efficient domain wall erosion



Symmetric: point-wise annihilation :(

Generic in any dimension

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$\geq 2+1d$: STSSB phase stable, strongly symmetric phase unstable

ferromagnetic phase

$$\rho \propto \frac{1+\prod_r X_r}{2} e^{-\beta H}$$

Can prove: any projected finite temp Gibbs state has $F_{ij}(\rho, Z_i Z_j) \geq e^{-\beta \|V\|/2} > 0$

$$V = Z_i Z_j H Z_i Z_j - H$$

Generic in any dimension

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$$V = Z_i Z_j H Z_i Z_j - H$$

Need to go beyond Gibbs states/Gibbs samplers to get phase transitions!

Weakening the SWSSB phase

Design dynamics such that

$$\left.\begin{array}{l} \bullet\ |+\rangle\langle+| \to \rho_0 = |+\rangle\langle+| \\[2ex] \bullet\ \text{any other even parity state} \to \rho_+ \propto \dfrac{1+\prod_r X_r}{2} - |+\rangle\langle+| \end{array}\right\} U = +1$$

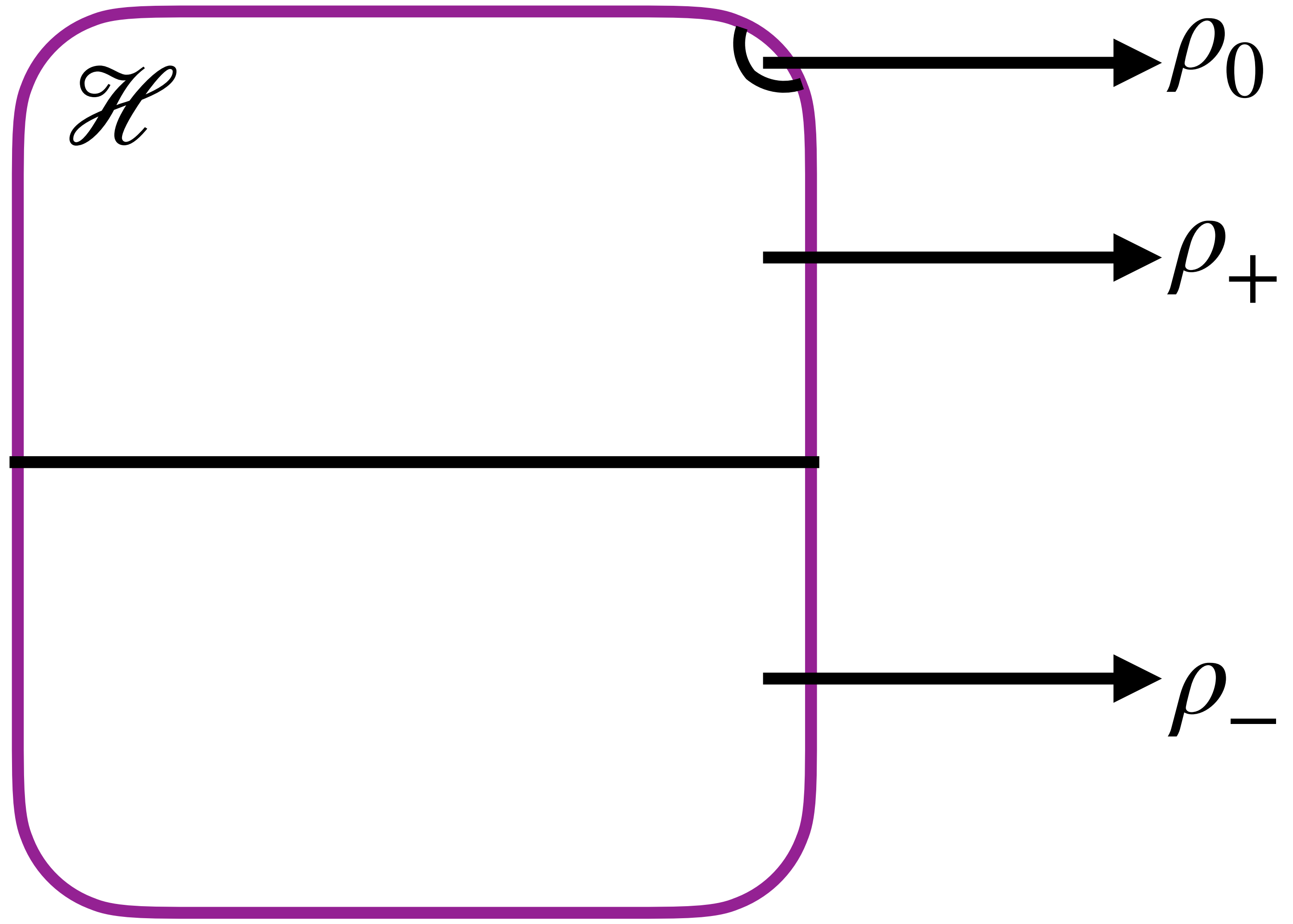
$$\left.\begin{array}{l} \bullet\ \text{any odd parity state} \to \rho_- \propto \dfrac{1-\prod_r X_r}{2} \end{array}\right\} U = -1$$

Steady state space $=$ span(ρ_0, ρ_-, ρ_+)

"SWSSB + symmetric" in even sector, SWSSB in odd sector

Weakening the SWSSB phase

Design dynamics such that



SWSSB at late times

with prob $1 - 2^{-L}$

Steady state space $=$ span(ρ_0, ρ_-, ρ_+)

"SWSSB + symmetric" in even sector, SWSSB in odd sector

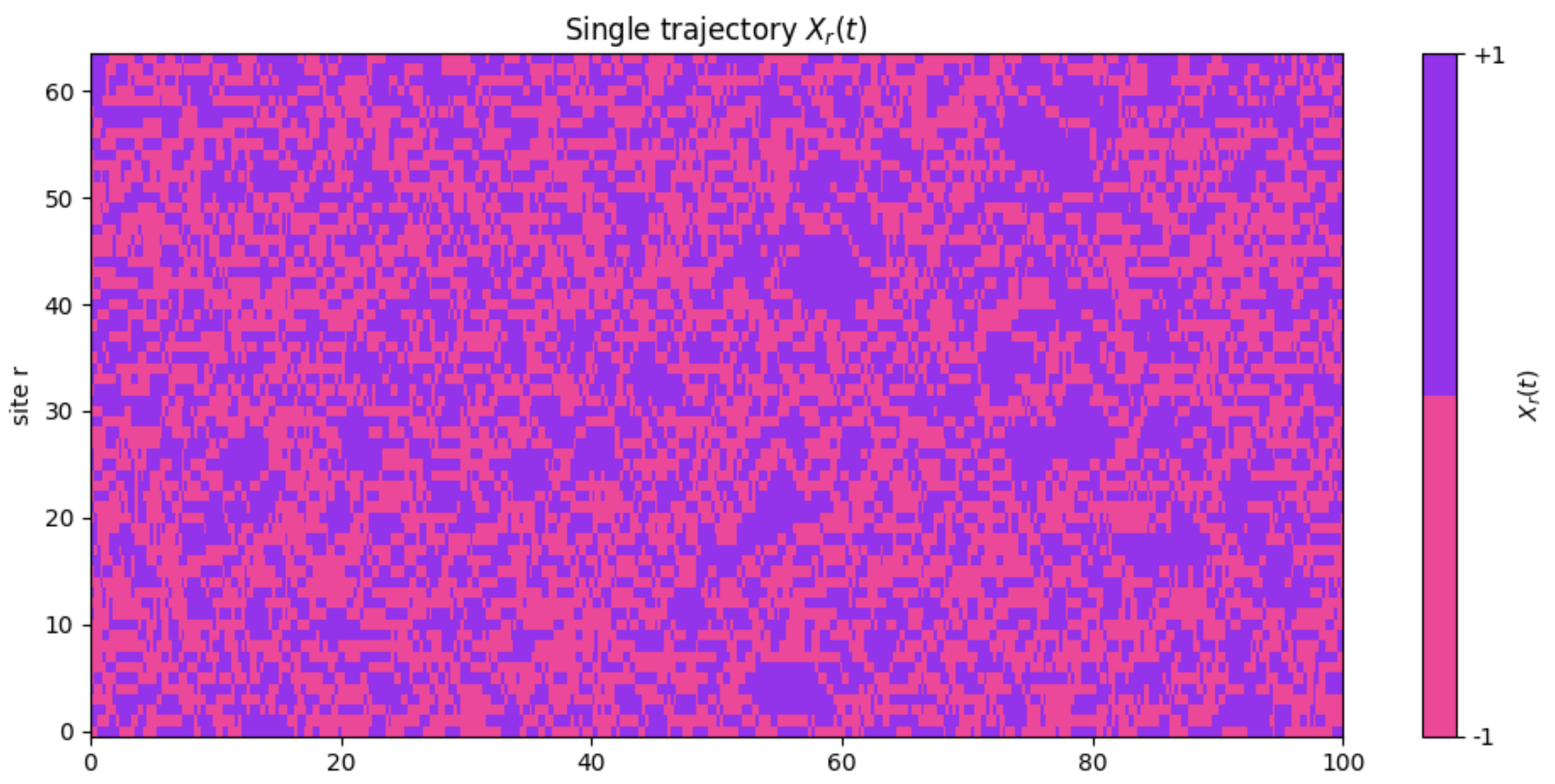
Weakening the SWSSB phase

Three site update rule (after X dephasing) :

$$(+ + +) \to (+ + +)$$

scramble $(- + +), (+ - +), (+ + -), (- - -)$

scramble $(- - +), (- + -), (+ - -)$



Weakening the SWSSB phase

Three site update rule (after X dephasing) :

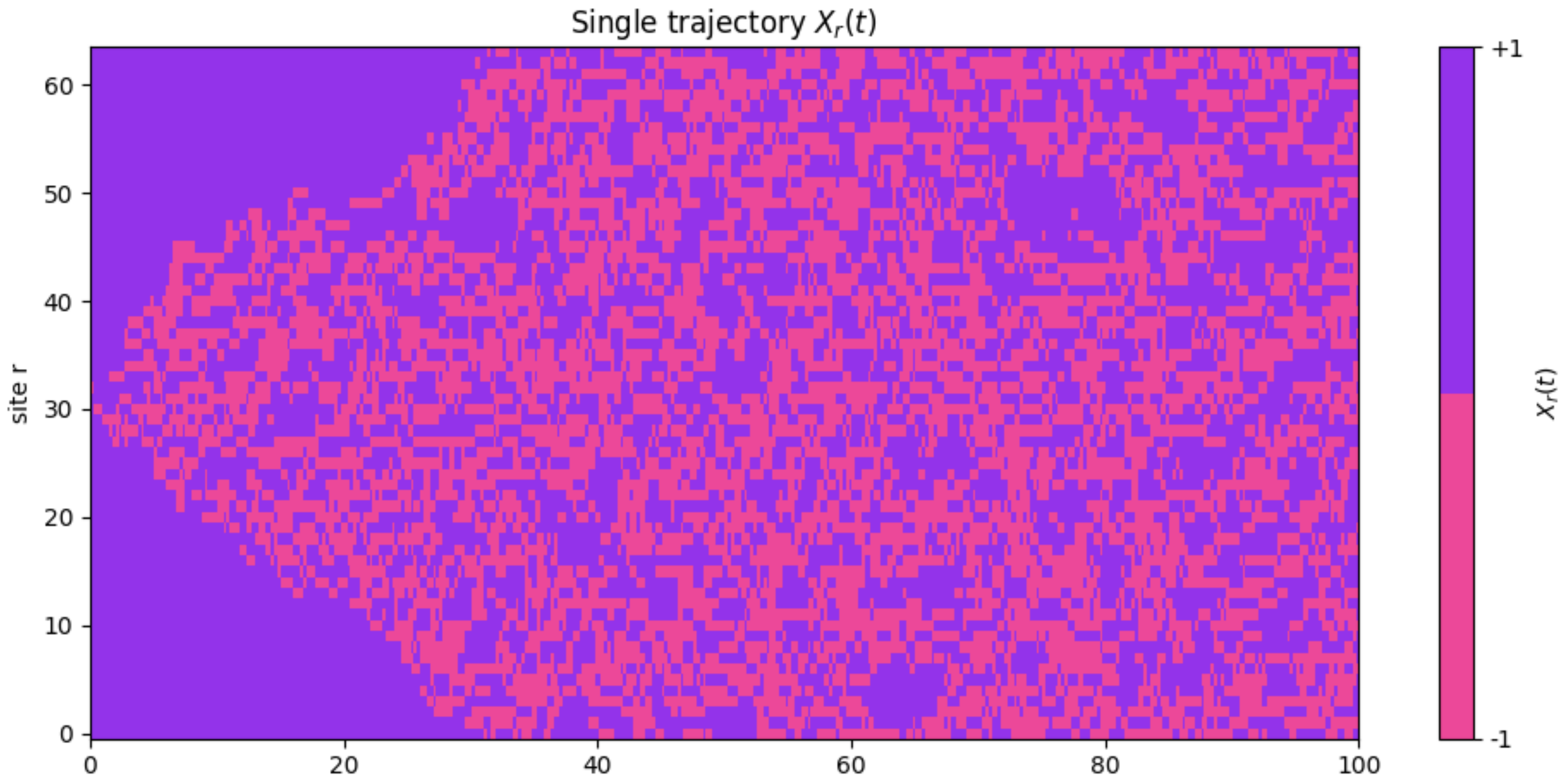
$$(+++) \to (+++)$$

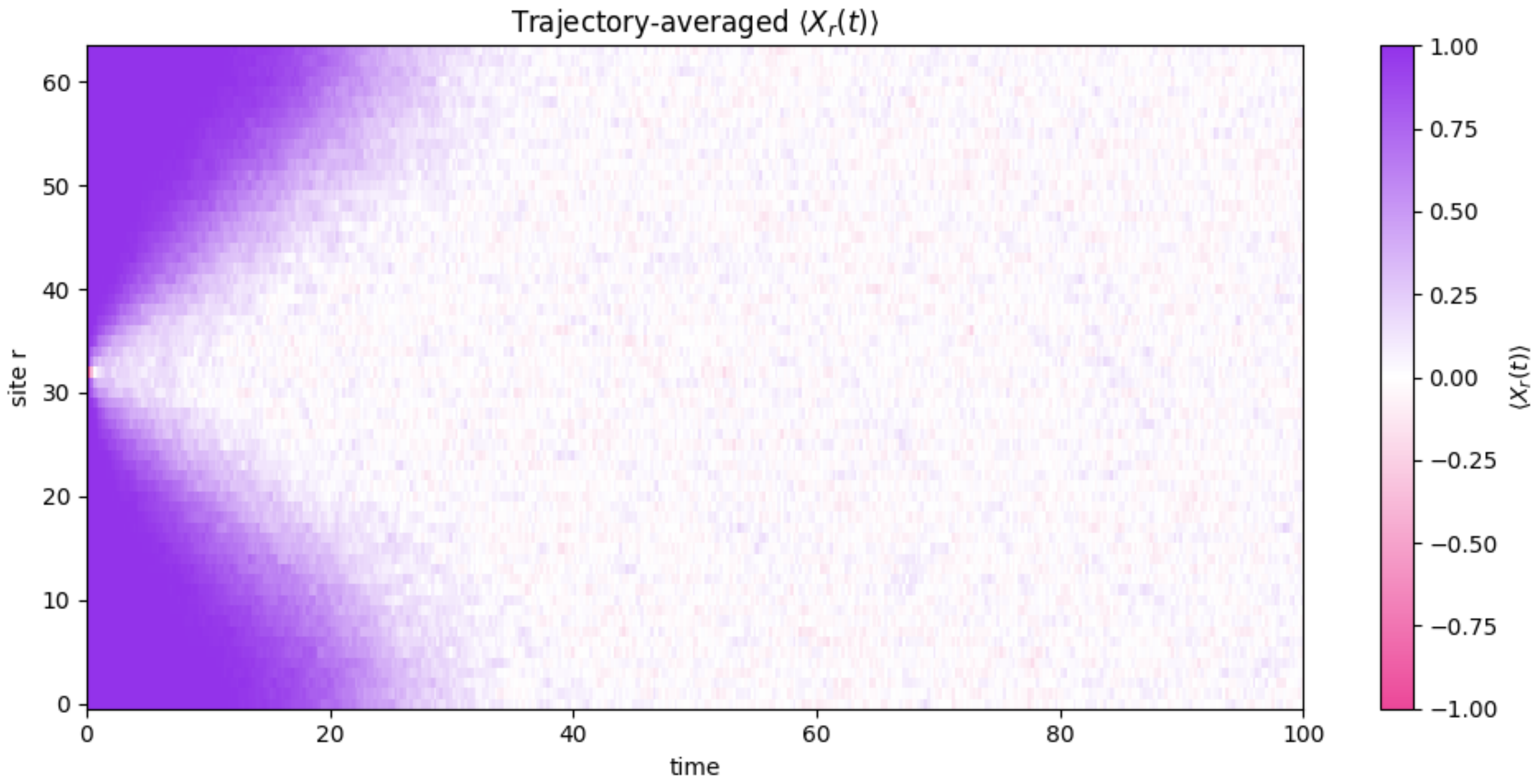
$$\text{scramble } (-++), (+-+), (++-), (---)$$

$$\text{scramble } (--+), (-+-), (+--)$$

No more spawning defects spontaneously! ☺

$\mathcal{O}(L)$ mixing time





A genuine phase transition

$$\rho_0 = |+\rangle\langle+|$$

$$\rho_W \propto \sum_r Z_r |+\rangle\langle+| Z_r$$

Strongly symmetric

$$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\text{SWSSB},+}$$

0 1

$$\rho_+ \propto \frac{1+U}{2} - \rho_0$$

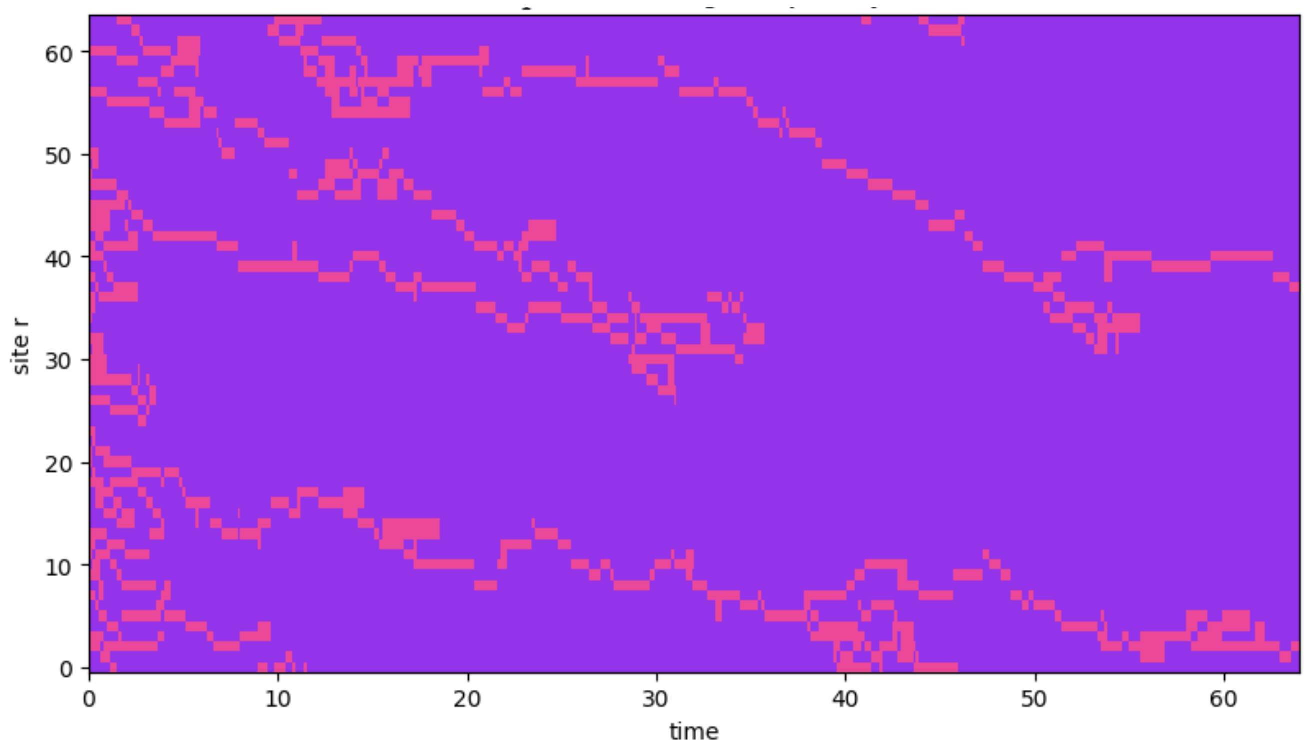
$$\rho_0 = |+\rangle\langle+|$$

$$\rho_- \propto \frac{1-U}{2}$$

SWSSB + symmetric

A genuine phase transition

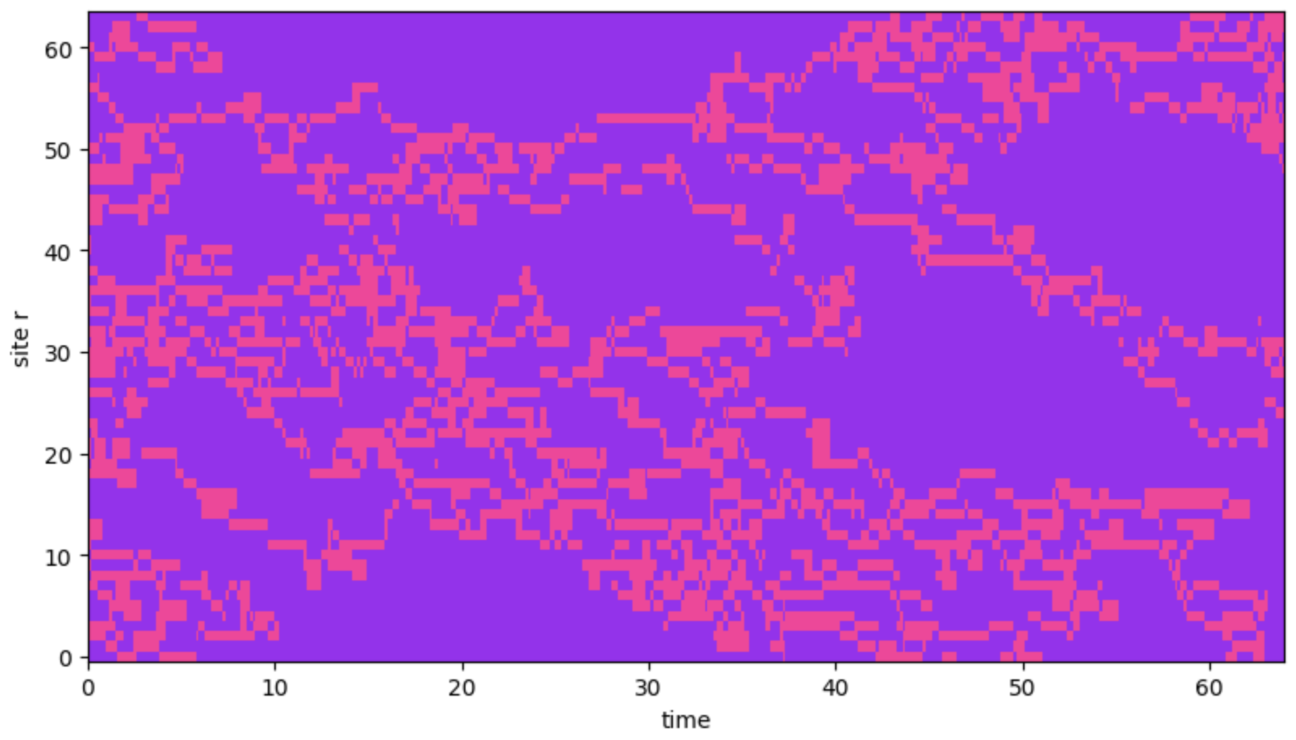
def late time defect density: $n_{-} = \frac{1}{L}\sum_r \frac{1-\langle X_r \rangle}{2}$



$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\mathrm{SWSSB},+}$

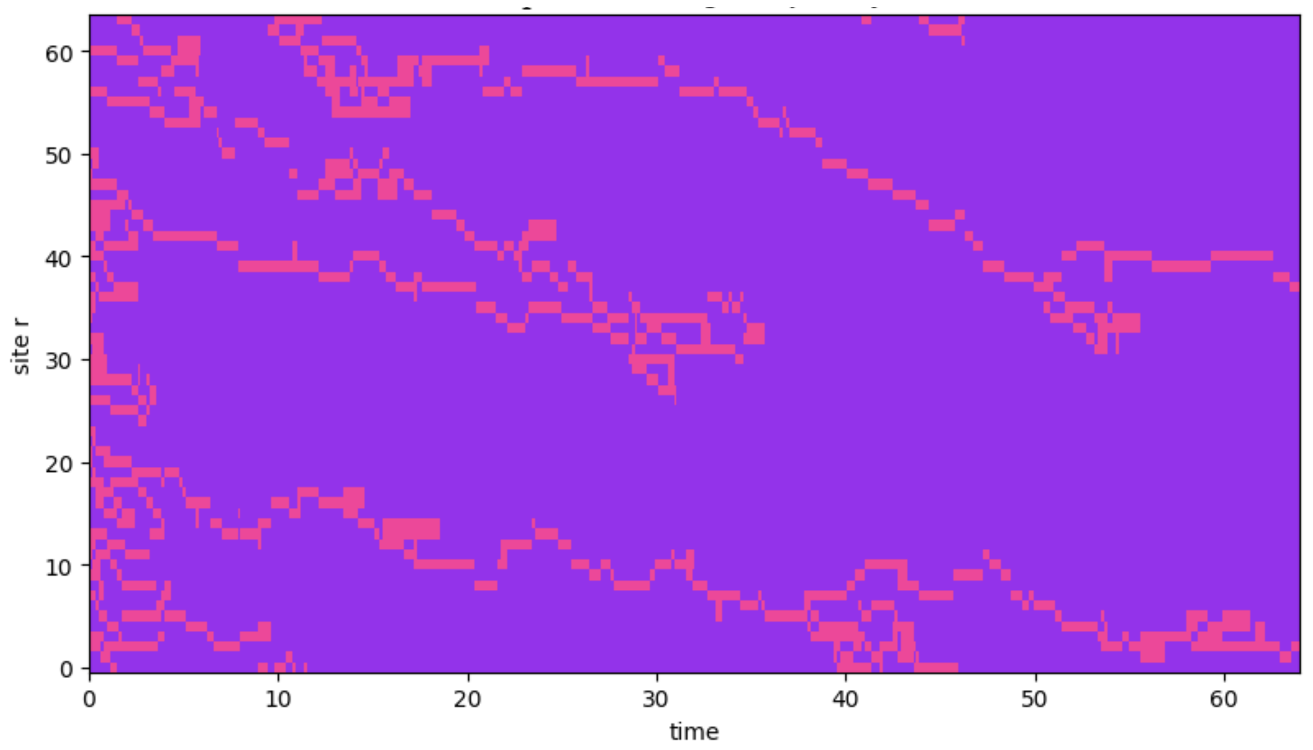
0 — 1

$\alpha_c \sim 0.44$



A genuine phase transition

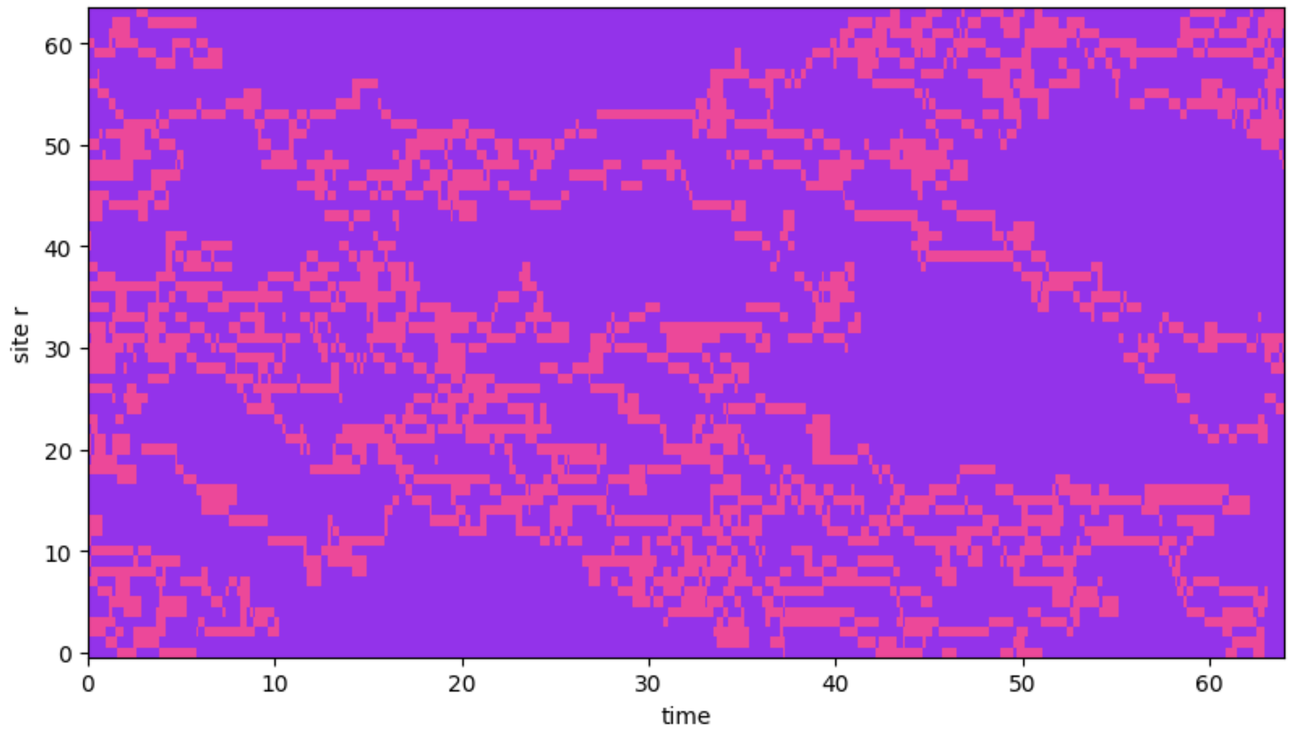
def late time defect density: $n_{-} = \frac{1}{L} \sum_{r} \frac{1 - \langle X_r \rangle}{2}$



$$(1-\alpha)\mathscr{L}^{+} + \alpha \mathscr{L}^{\mathrm{SWSSB},+}$$

0 $\qquad\qquad\qquad\qquad$ 1

$$\lim_{L\to\infty} n_{-}(L) = 0 \qquad\qquad \lim_{L\to\infty} n_{-}(L) > 0$$



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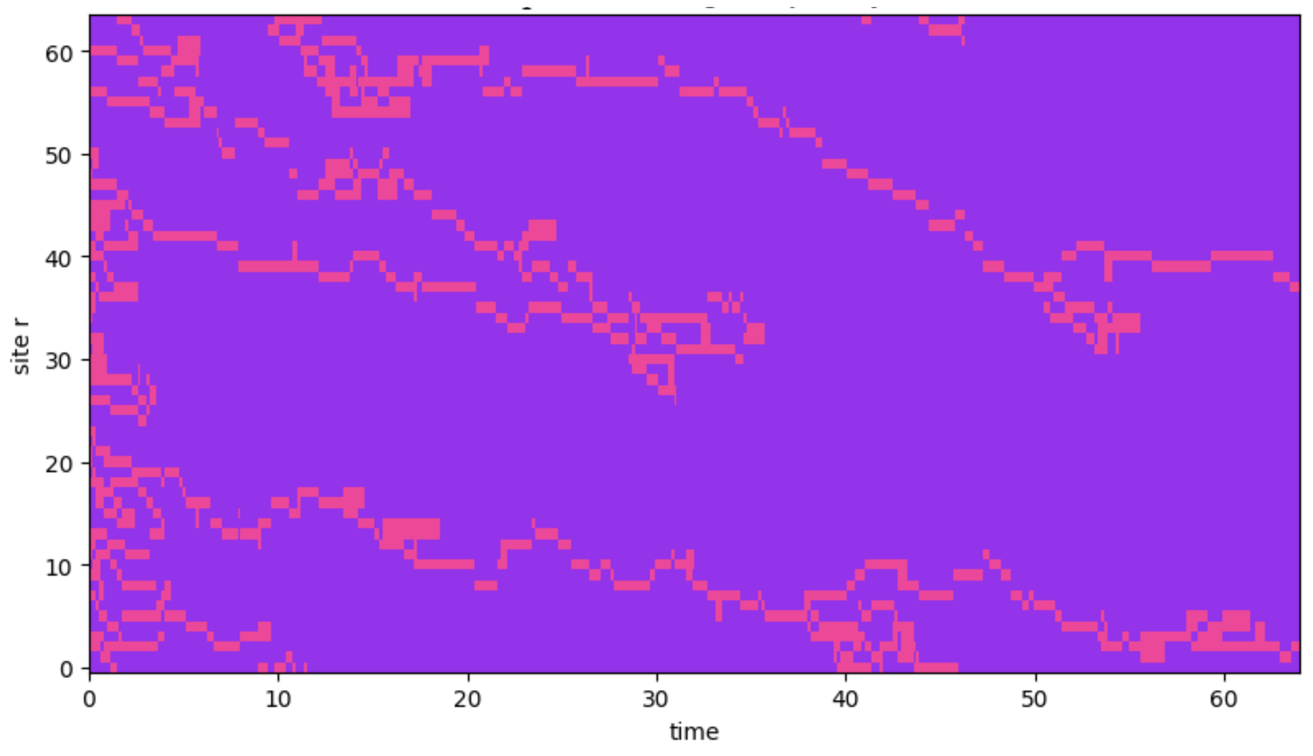


$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\text{SWSSB},+}$

0

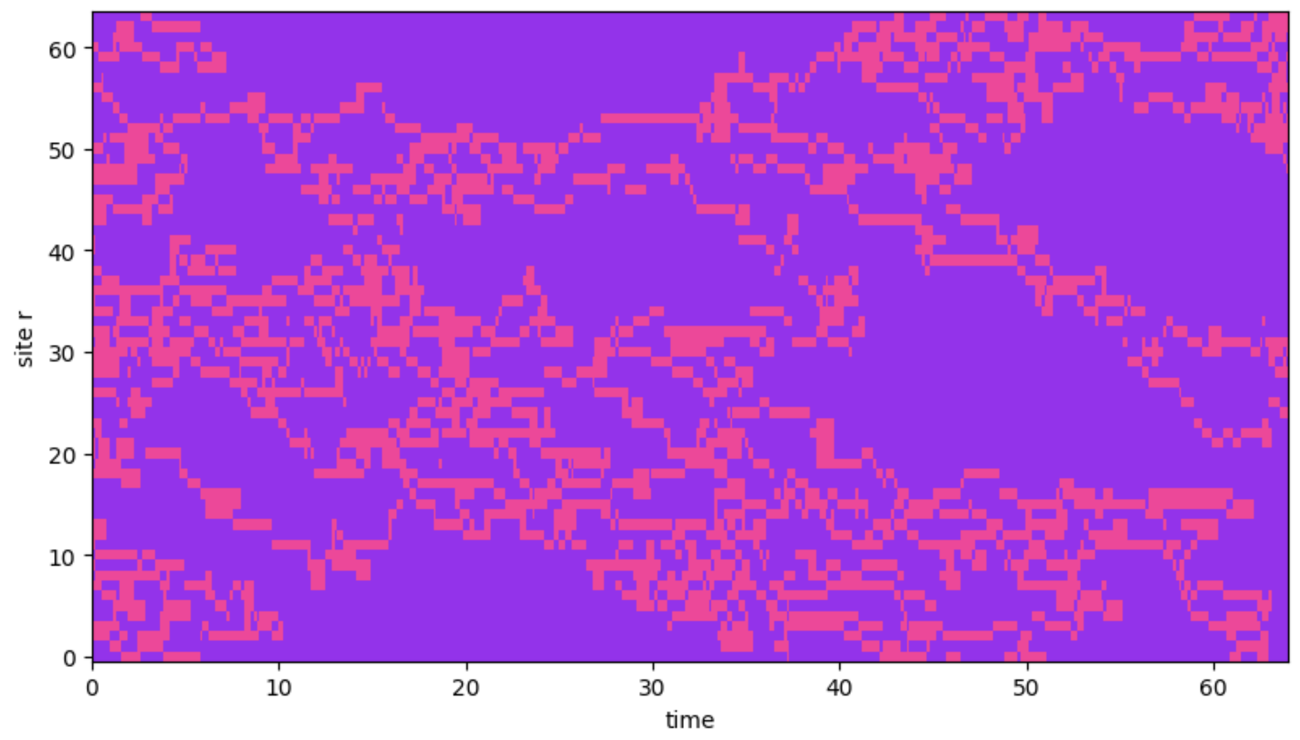
$\alpha_c \sim 0.44$

1

gapless

metastable SWSSB in even sector

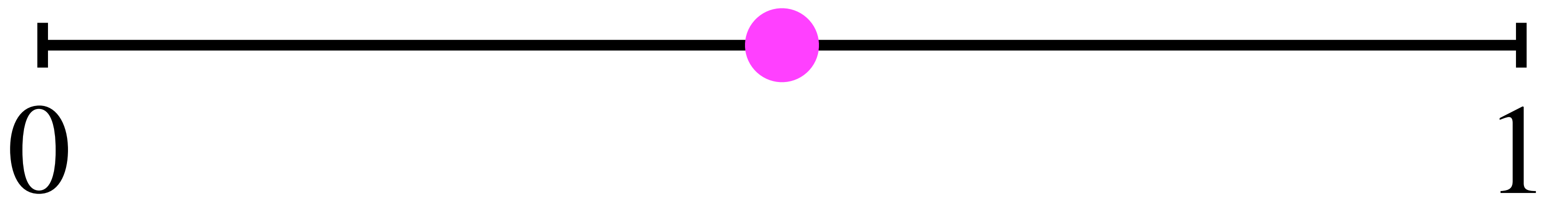
exact SWSSB steady states



A genuine phase transition

def late time defect density: $n_{\_} = \dfrac{1}{L}\sum_r \dfrac{1-\langle X_r \rangle}{2}$

$$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\mathrm{SWSSB},+}$$



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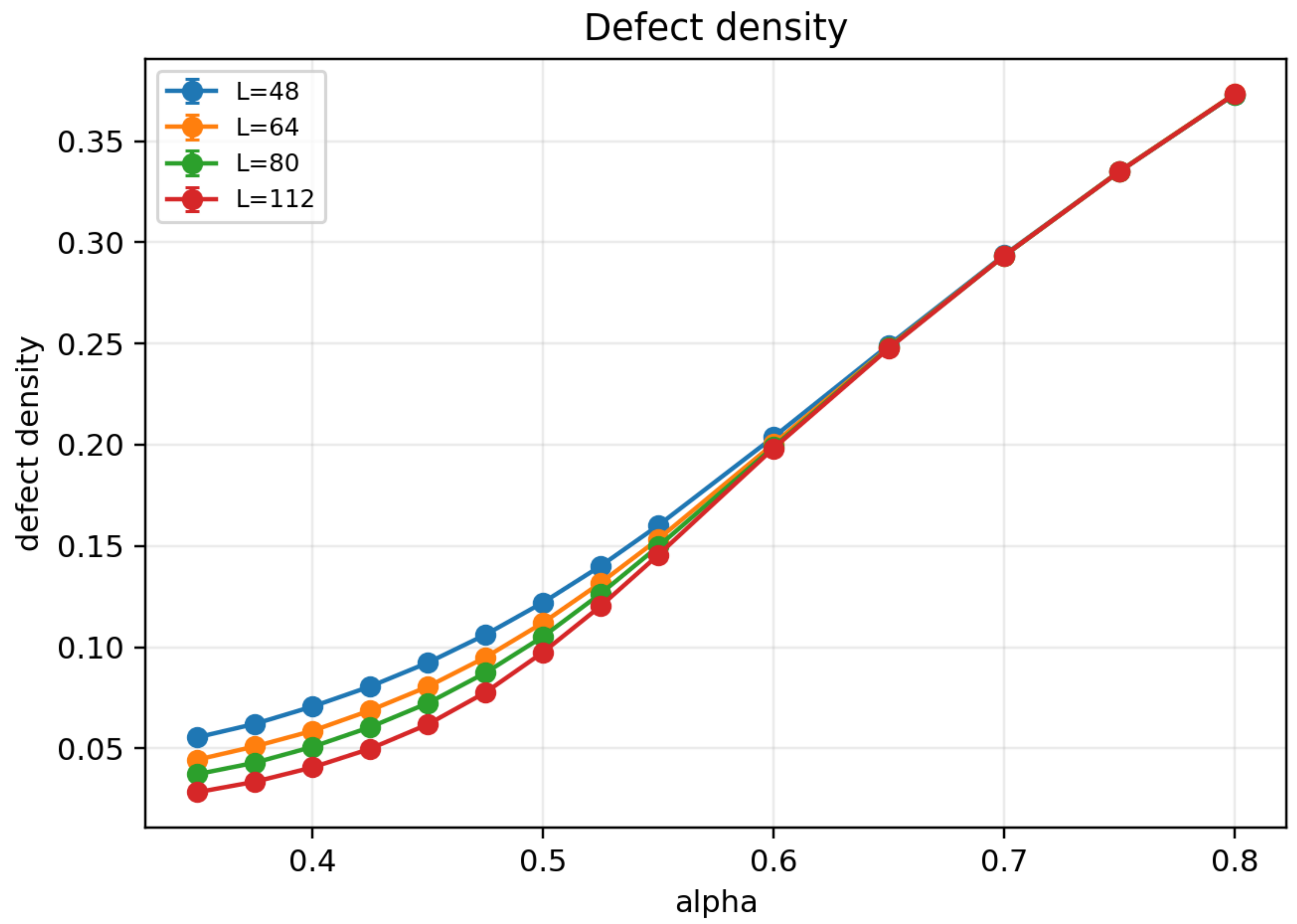
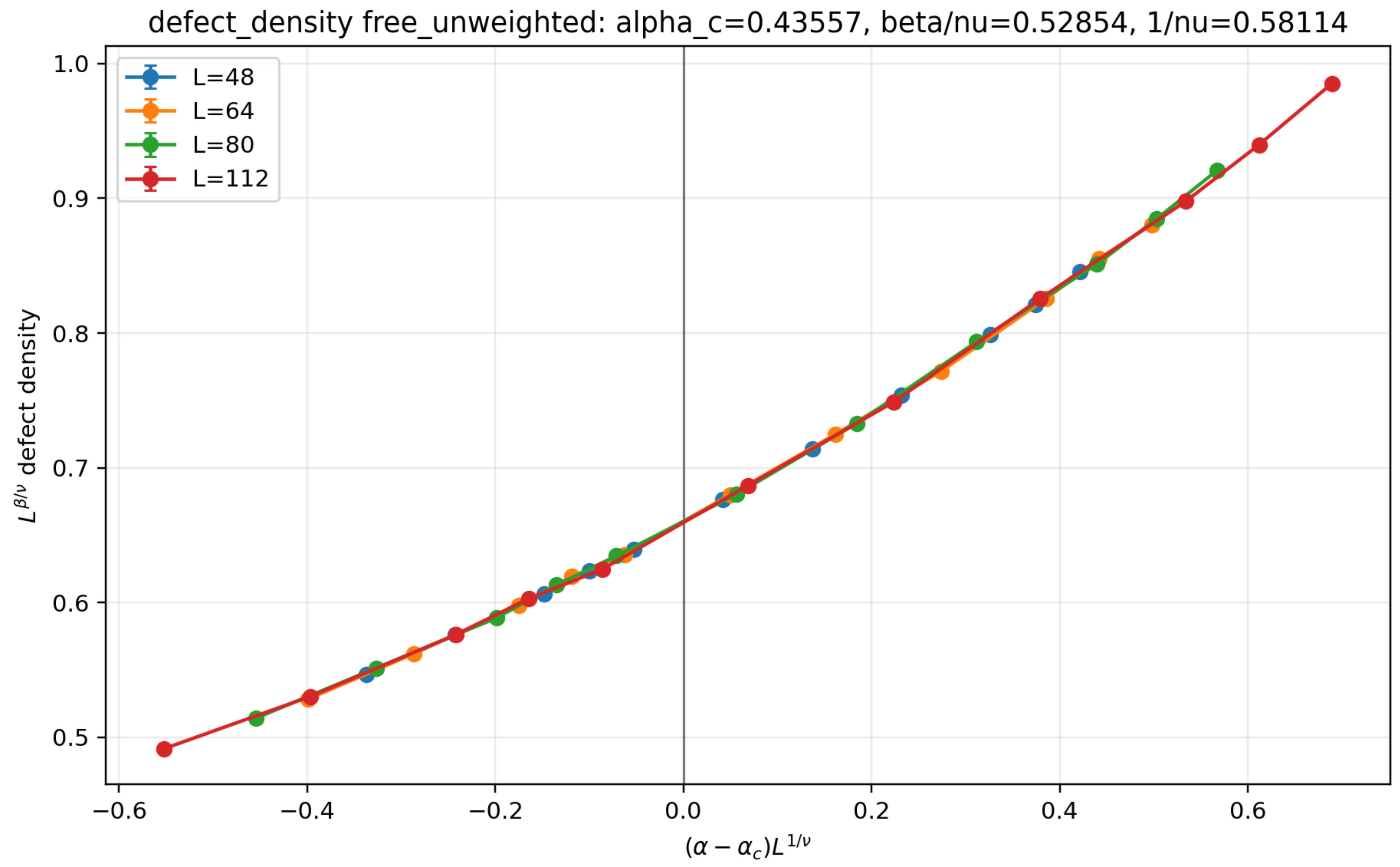


Universality class of the transition



Close to parity-conserving branching annihilating random walks:

$$\beta \sim 0.93 \quad \beta/\nu_\perp \sim 1/2 \quad 1/\nu_\perp \sim 0.54$$ i.e. Jensen PRE 1994

absorbing $\sim$ symmetric, active $\sim$ SWSSB

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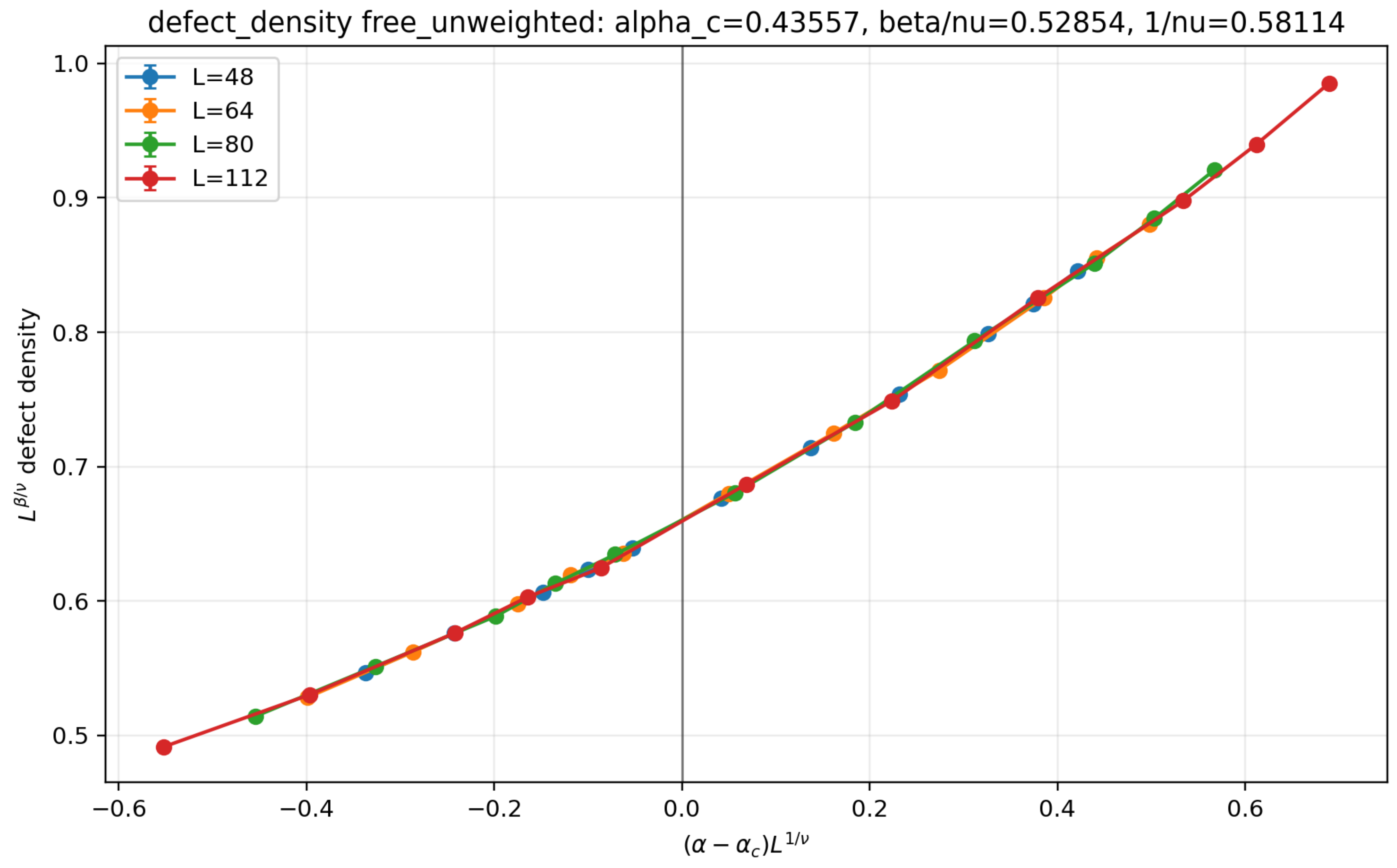

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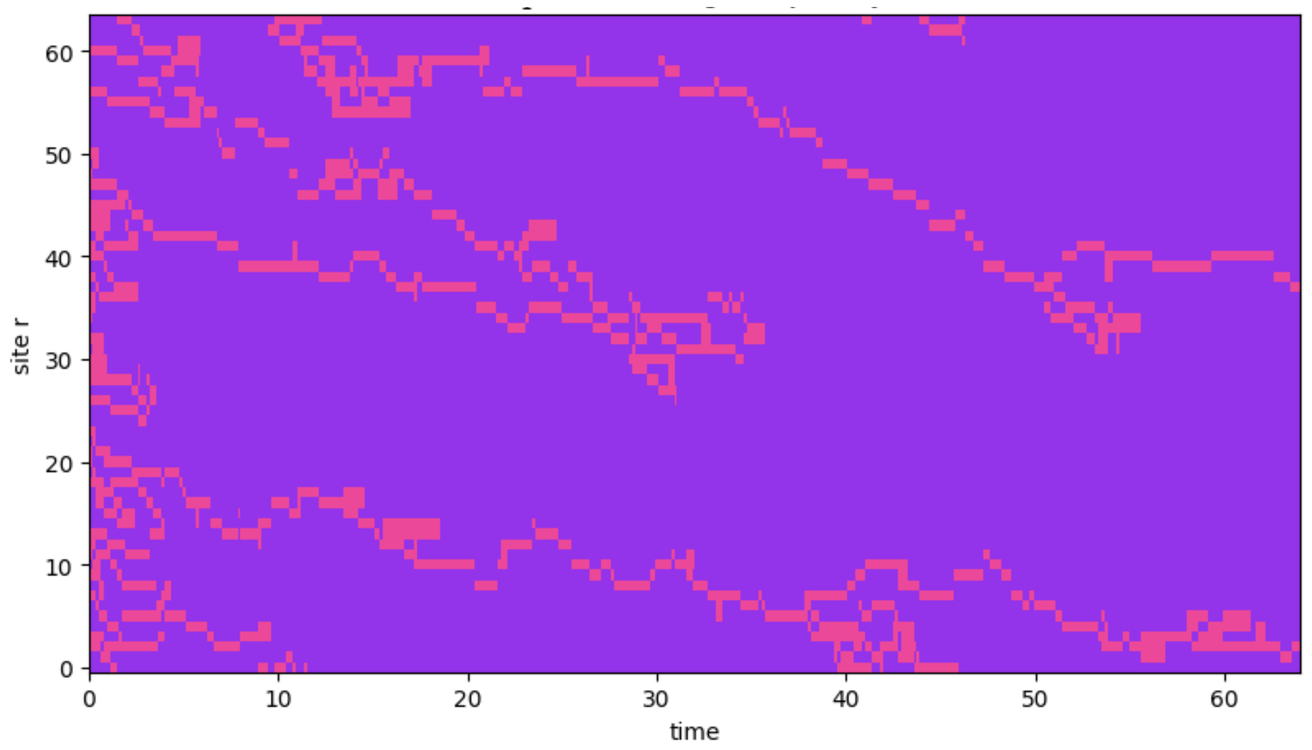
Generic for $2A \to \varnothing$ annihilation, $A \to 3A$ branching

Diagnosing the SWSSB/“active” phase

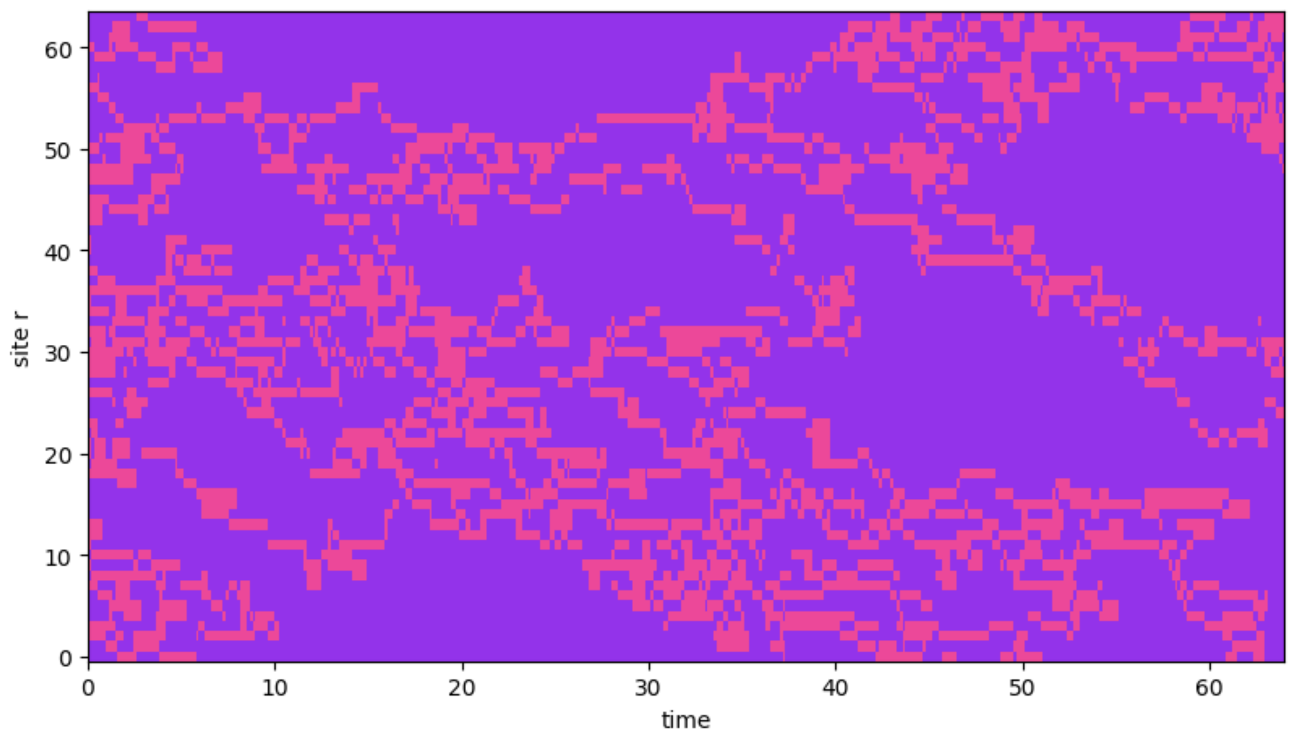
def late time defect density: $n_- = \frac{1}{L} \sum_r \frac{1 - \langle X_r \rangle}{2}$



$(1-\alpha)\mathscr{L}^+ + \alpha \mathscr{L}^{\mathrm{SWSSB},+}$

0 $\qquad$ 1

$\lim_{L\to\infty} n_-(L) = 0 \qquad \lim_{L\to\infty} n_-(L) > 0$



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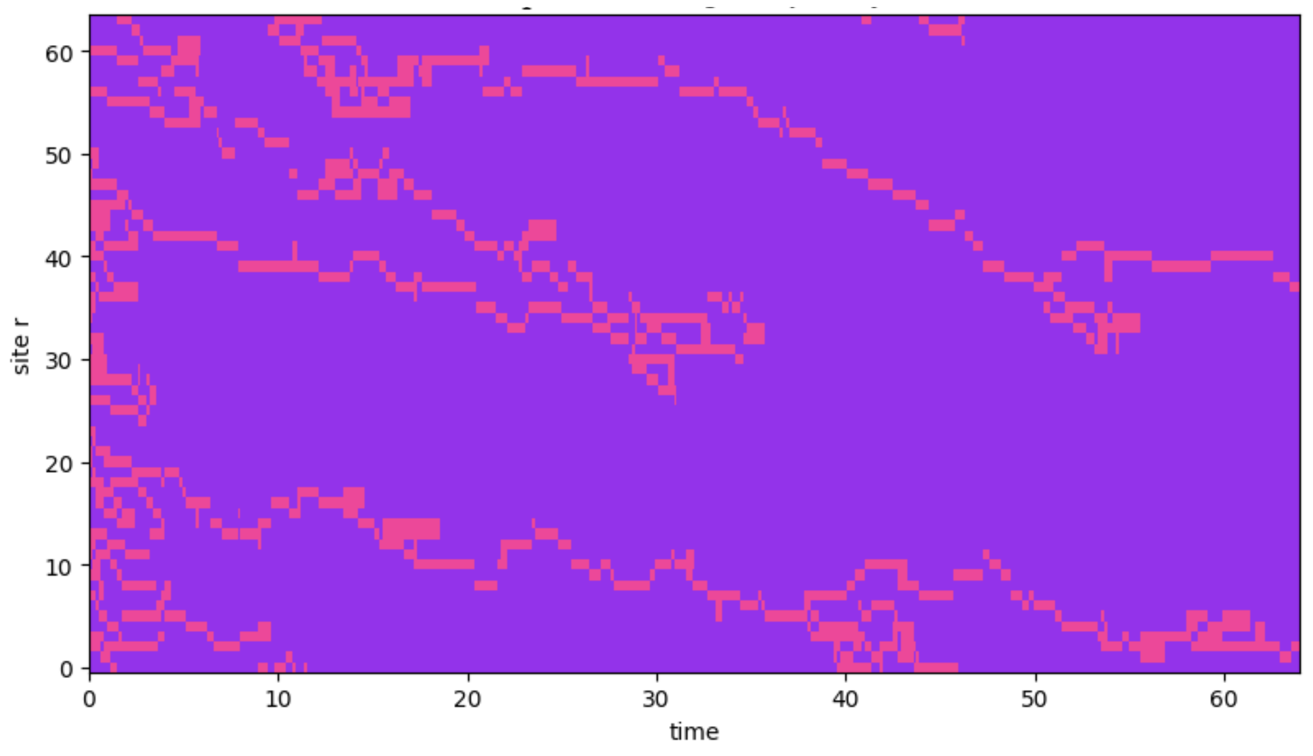

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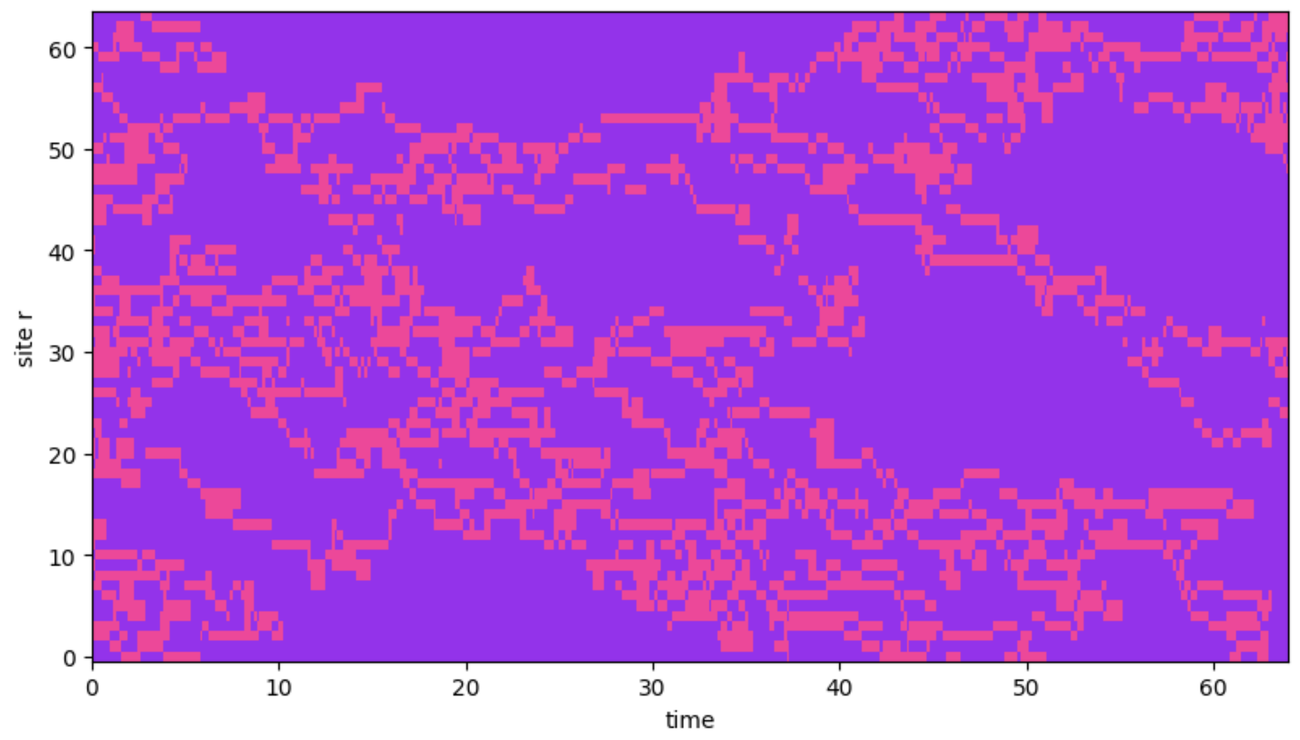

In principle can have product state with $n_- > 0$!

Need both $n_- > 0$ and fluctuations!

What about the fidelity correletor $\mathrm{Tr}(Z_iZ_j\sqrt{\rho}Z_iZ_j\sqrt{\rho})$?

Diagnosing the SWSSB/“active” phase

For ρ diagonal in the X basis:

$$\boxed{F_{ij}(\rho, Z_i Z_j) = \sum_{\{X_r\}} \sqrt{p(\{X_r\})p(\{X_r^{ij}\})}}$$

flip signs of X_i, X_j

Bhattacharyya distance
of classical probabilities

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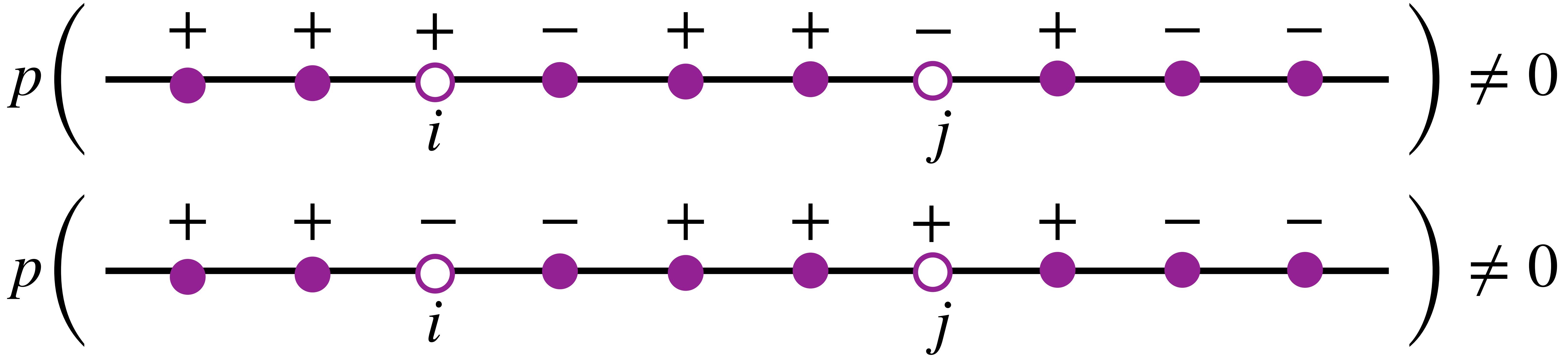
Fixing $\{X_{r\neq i,j}\}$, can X_i, X_j fluctuate?



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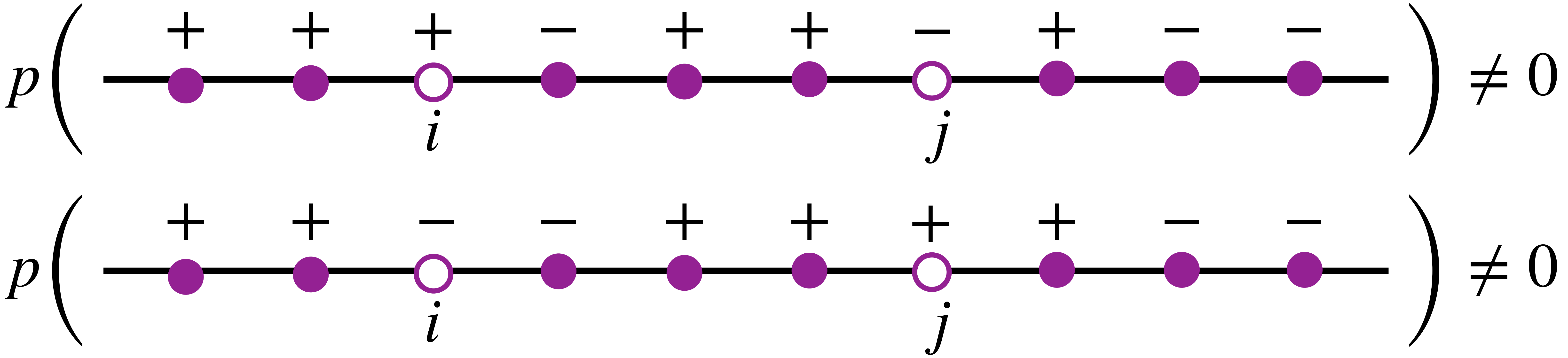

Need active
sites!

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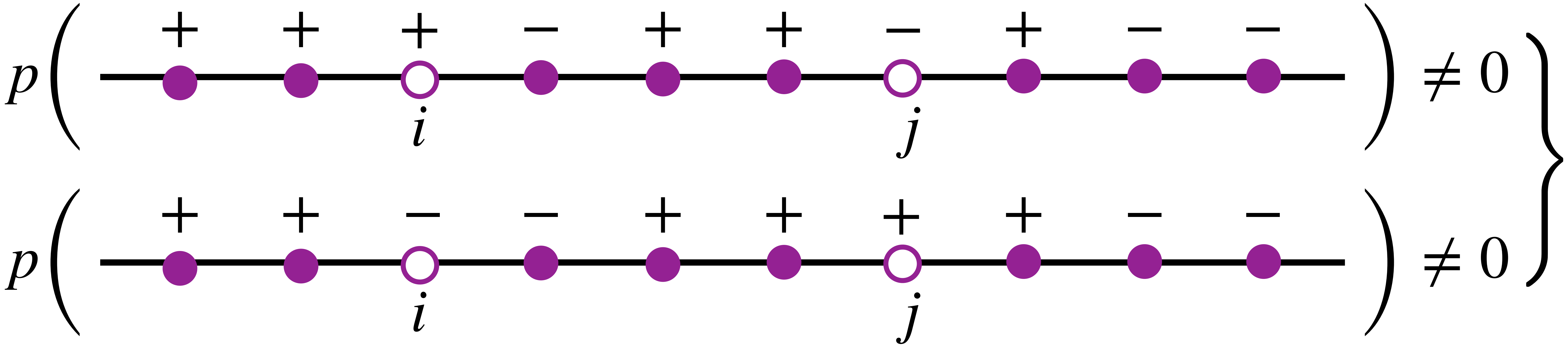
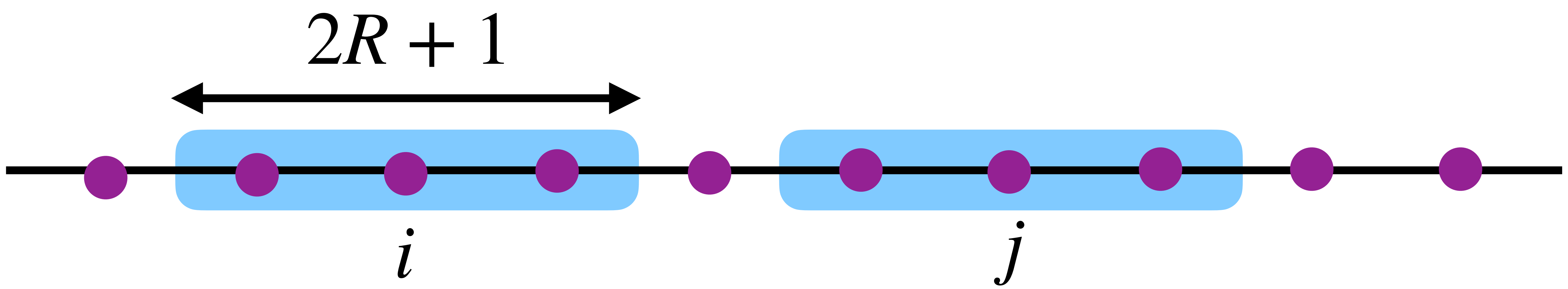


Need active
sites!

However, need $\mathcal{O}(2^V)$ samples to measure :(

Marginal radius-R fidelity

$$F^R_{ij}(\rho, Z_iZ_j) = \mathrm{Tr}\left(Z_iZ_j\sqrt{\rho_{R,ij}}Z_iZ_j\sqrt{\rho_{R,ij}}\right) \qquad \rho_{R,ij} = \mathrm{Tr}_{\overline{R(i)\cup R(j)}}(\rho)$$



Only requires tomography on a size $2(2R+1)$ region

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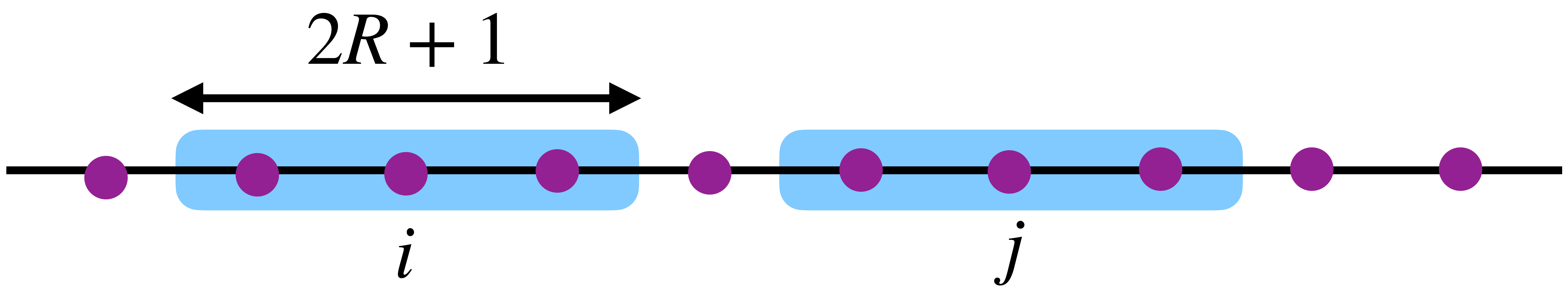


Only requires tomography on a size $2(2R+1)$ region

Similar to how we compute linear connected correlation function:

$$\langle Z_iZ_j\rangle = \mathrm{Tr}(Z_iZ_j\rho) = \mathrm{Tr}(Z_iZ_j\rho_{R,ij})$$

Properties of marginal fidelity

Monotonicity:

$F_{ij}^{R}(\rho, Z_iZ_j) \leq F_{ij}^{R'}(\rho, Z_iZ_j)$ if $R > R' : F_{ij}^{0} \geq F_{ij}^{1} \geq F_{ij}^{2} \geq \cdots F_{ij}$

In particular if there is SWSSB, $F_{ij}^{R}(\rho, Z_iZ_j) > 0$, and if $F_{ij}^{R}(\rho, Z_iZ_j) = 0$ there is no SWSSB.

Stability

Inherits stability theorems from true fidelity as long as $R > \mathcal{O}(\text{time of evolution})$

Complexity

Strong symmetry $+ F_{i}^{R}(\rho, Z_i) > 0 \rightarrow$ long range conditional mutual information

$+$ Can proves fast convergence in R for broad classes of states

Summary

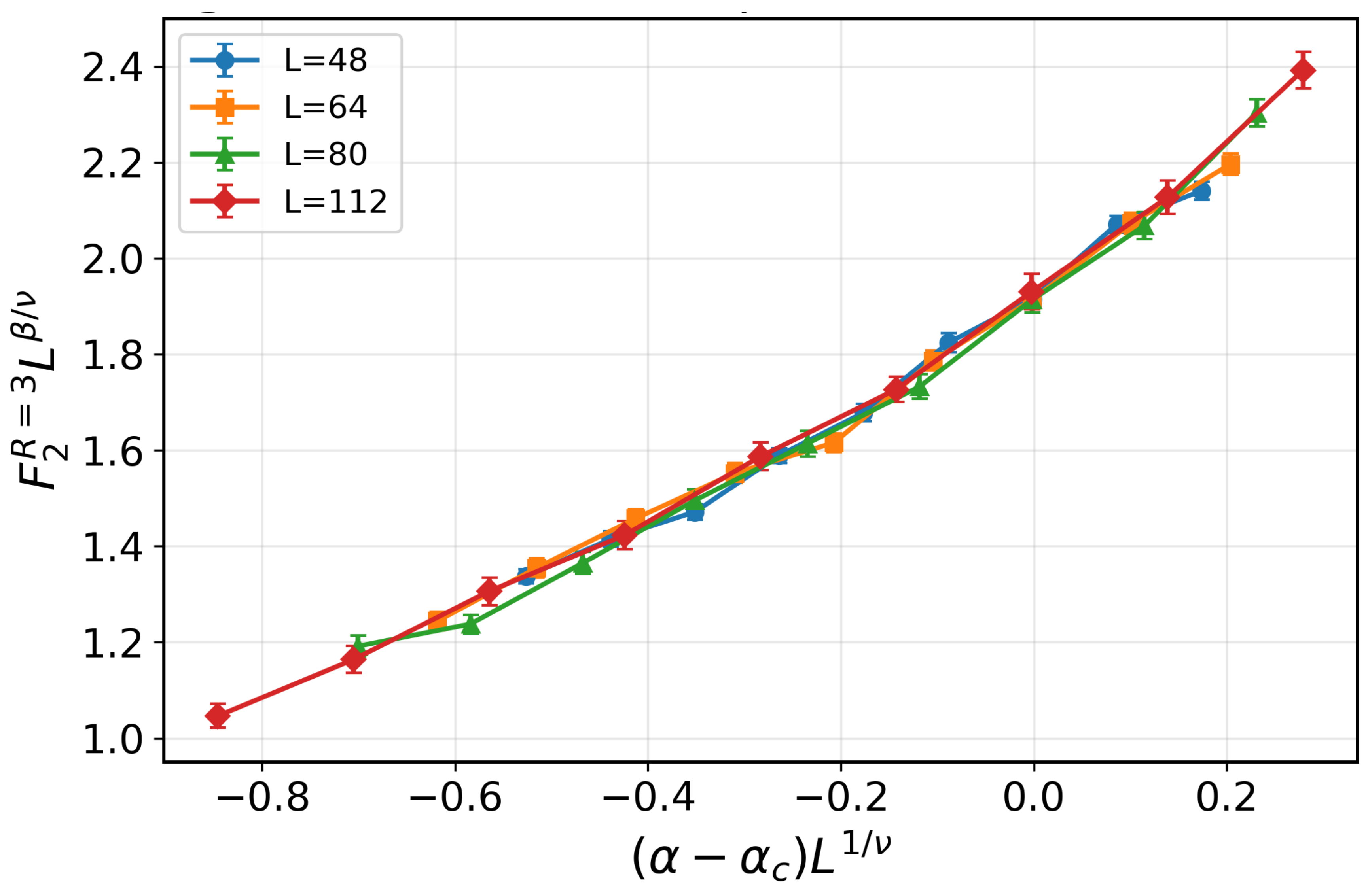
Marginal fidelity correlator is usually a good proxy for true fidelity correlator.

It satisfies several nice information theoretic properties.

Can it be applied generally to detect transitions into an active phase?

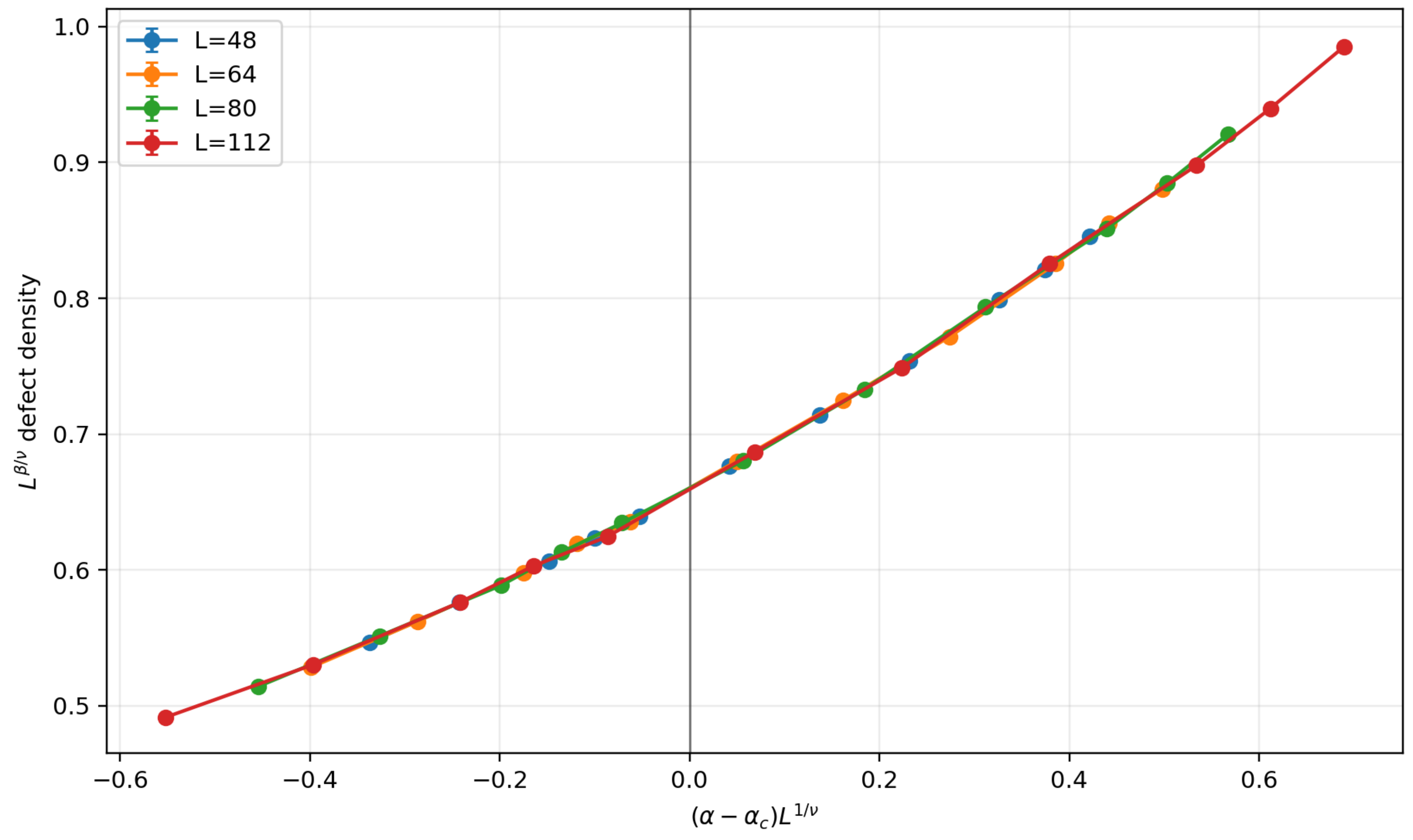
Marginal 2-point fidelity at the transition

$F^2_{ij}(\rho, Z_iZ_j)$



$\alpha_c = 0.46, \beta/\nu_\perp = 0.52, 1/\nu_\perp = 0.56$

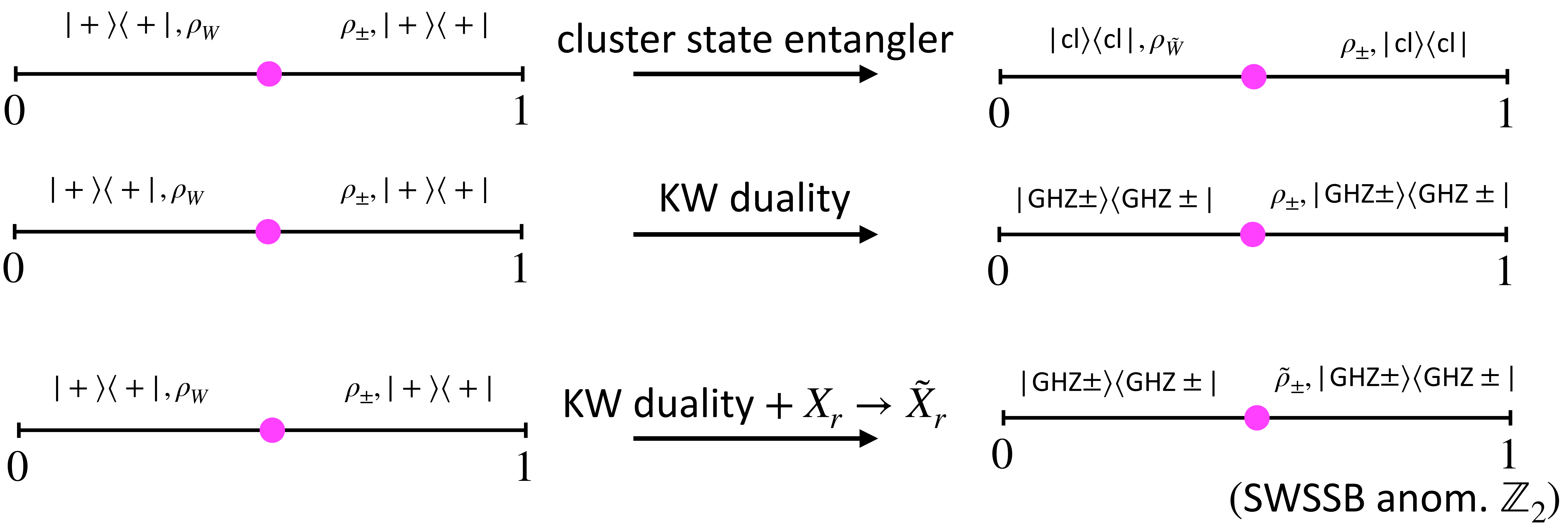
n_- (defect density)



$\alpha_c = 0.44, \beta/\nu_\perp = 0.53, 1/\nu_\perp = 0.58$

Re: literature on PC-BARW: $\beta/\nu_\perp \sim 1/2 \quad 1/\nu_\perp \sim 0.54$

Other SWSSB transitions



2+1d phase transitions

Problem: no symmetric phase with $2A \to \varnothing$ annihilation, $A \to 3A$ branching in $\geq$ 2+1d!

Cardy and Täuber 1998

$$(1-\alpha)\mathscr{L}^{+} + \alpha\mathscr{L}^{\text{SWSSB},+}$$

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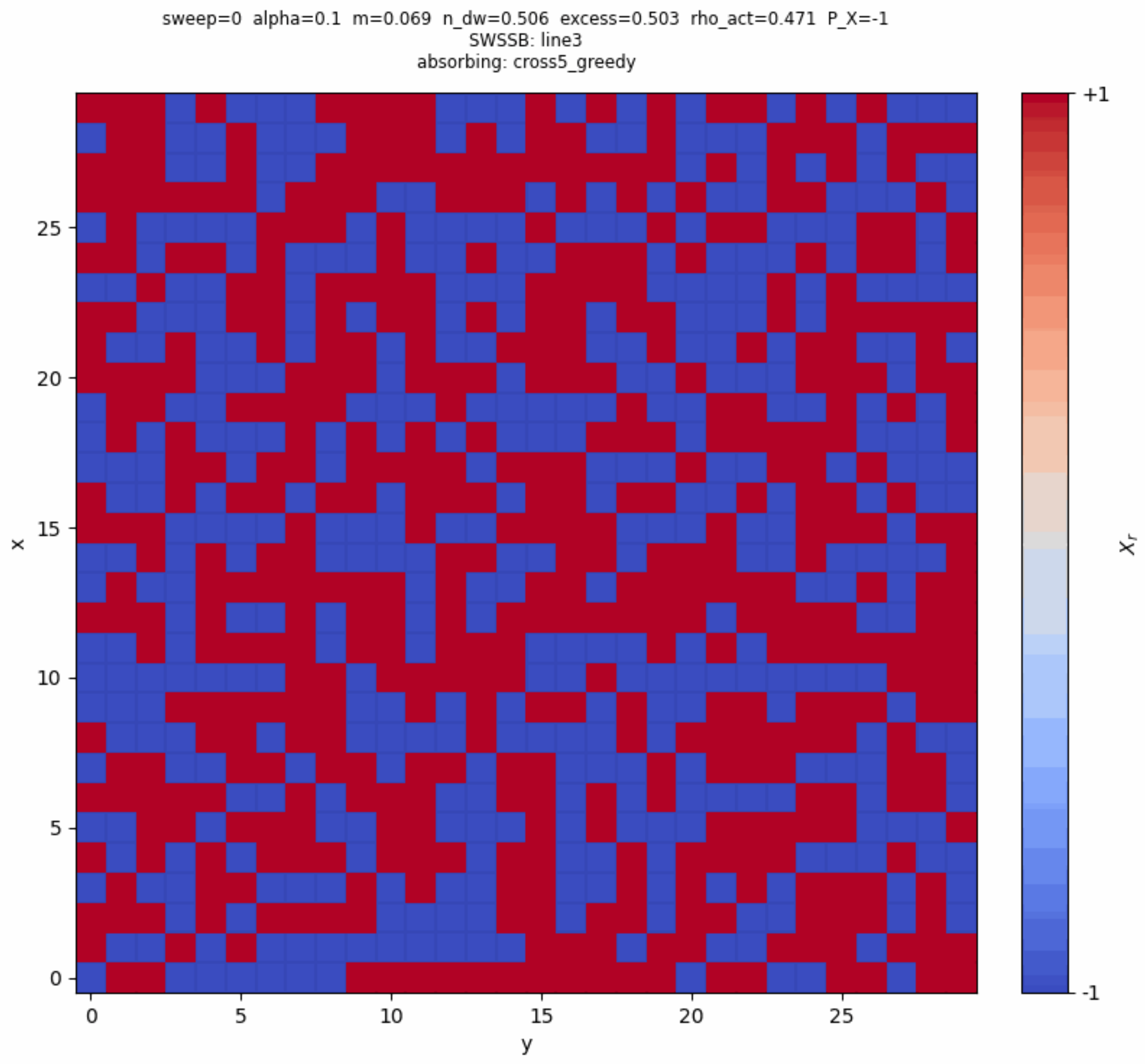
Cardy and Täuber 1998



Aggressive 5-site annihilation rule, with 10% chance of 1->3 branching

Solution: add another steady state

Design dynamics such that

$$\left.\begin{array}{l} \bullet\ |+\rangle\langle+| \to \rho_0 = |+\rangle\langle+| \\[2ex] \bullet\ |-\rangle\langle-| \to \rho_0^- = |-\rangle\langle-| \\[4ex] \bullet\ \text{any other even parity state} \to \rho_+ \propto \dfrac{1+\prod_r X_r}{2} - \rho_0 - \rho_0^- \end{array}\right\} U = +1$$

(even number of sites)

$$\left.\bullet\ \text{any odd parity state} \to \rho_- \propto \dfrac{1-\prod_r X_r}{2}\right\} U = -1$$

Steady state space $=$ span$(\rho_0, \rho_0^-, \rho_-, \rho_+)$

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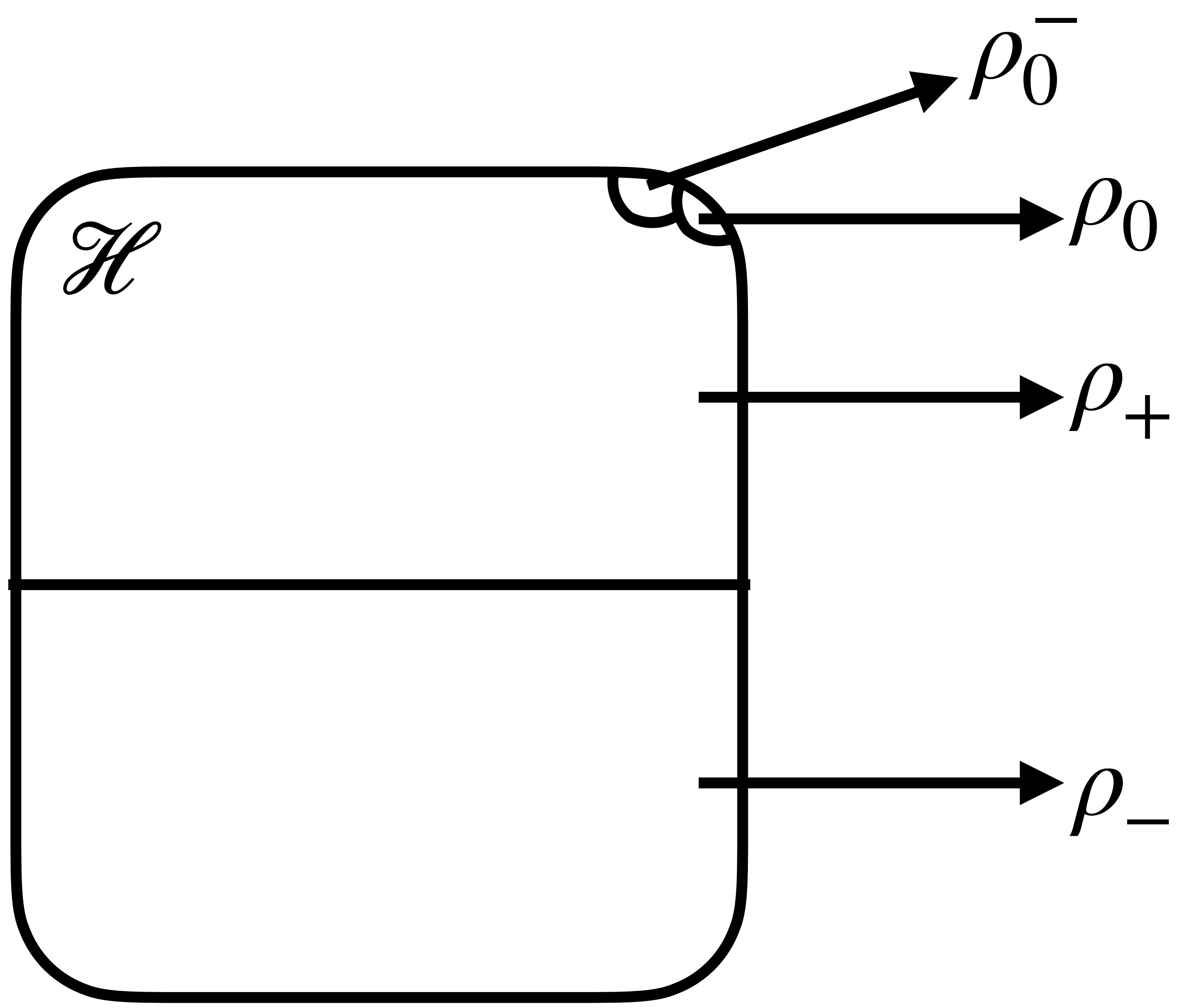
Steady state space $=$ span$(\rho_0, \rho_0^-, \rho_-, \rho_+)$

Main idea: use domain wall erosion rather than pairwise particle annihilation

Solution: add another steady state

Design dynamics such that



SWSSB at late times

with prob $1 - 2 \cdot 2^{-L}$

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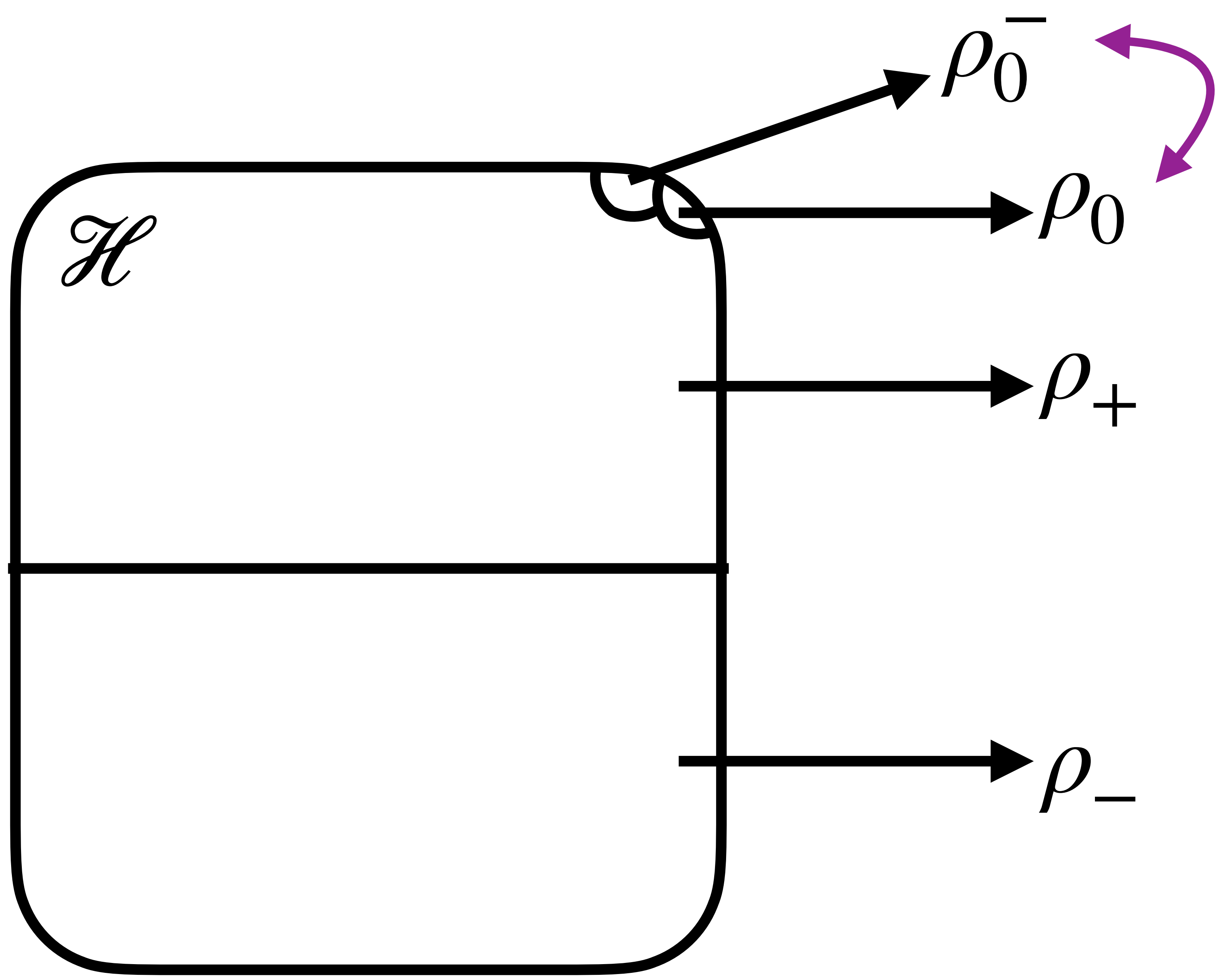


SWSSB at late times

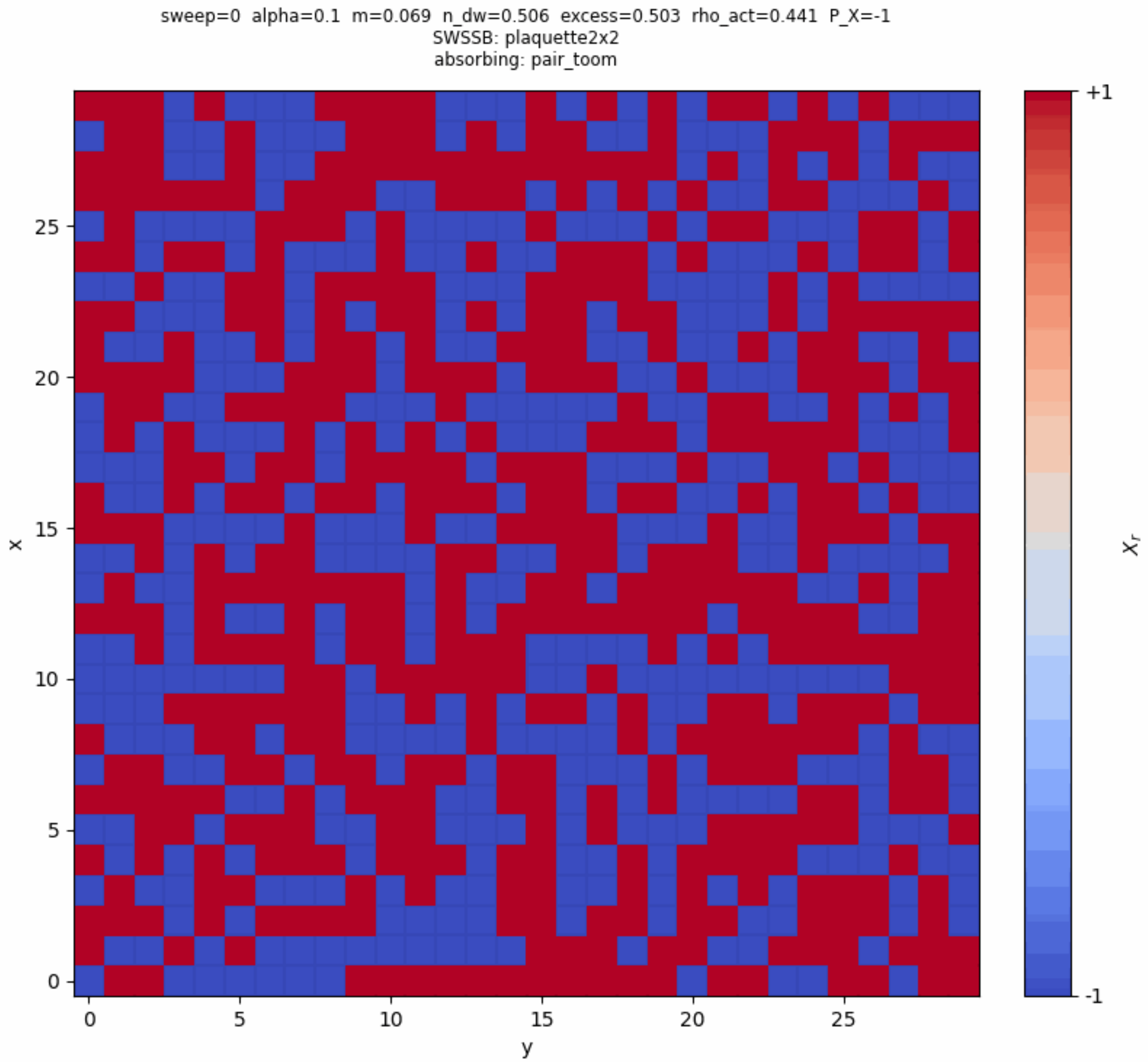
with prob $1 - 2 \cdot 2^{-L}$

New ingredient: weak $\mathbb{Z}_2$ symmetry $\prod_r Z_r$

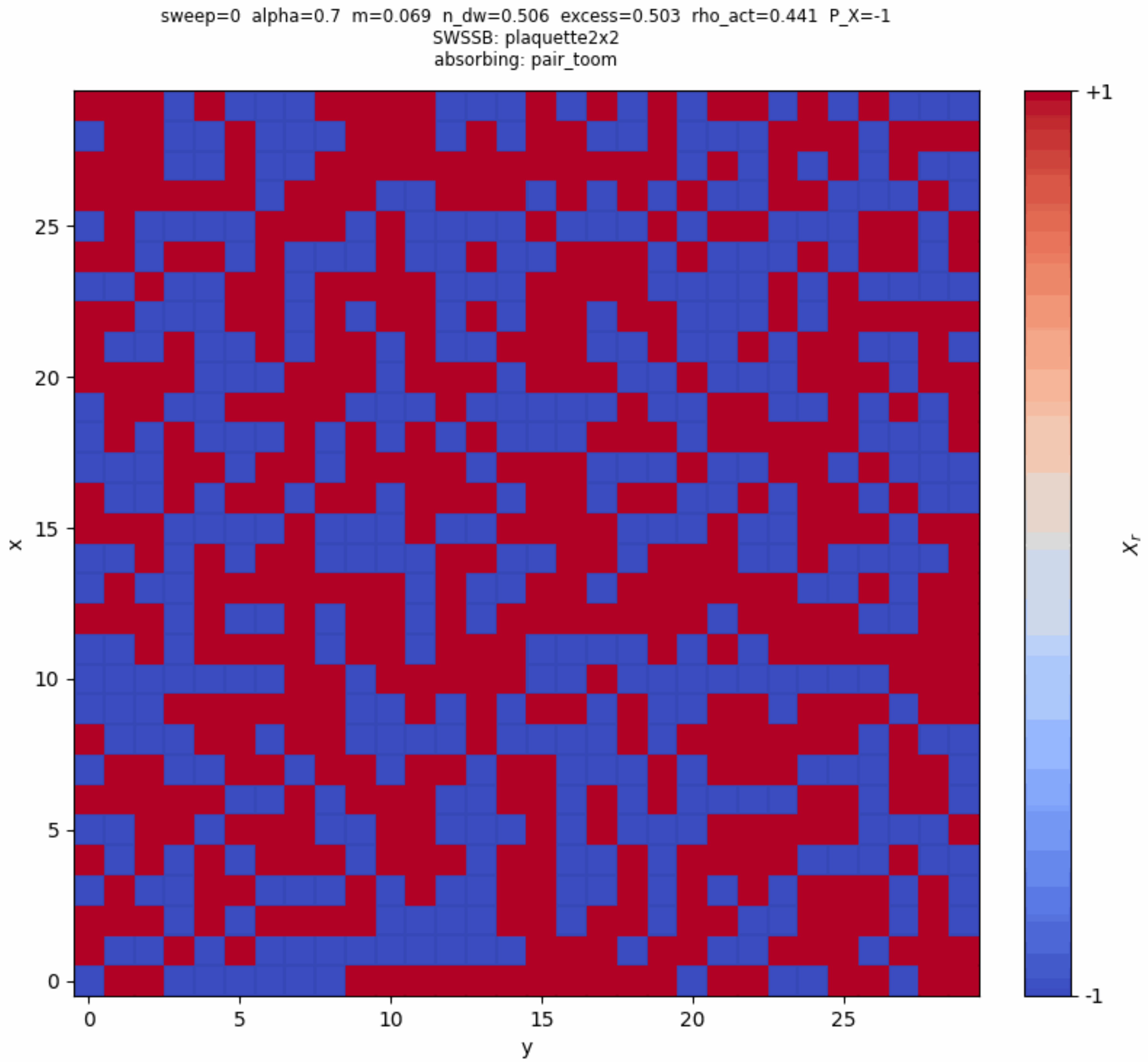
Steady state space $= \text{span}(\rho_0, \rho_0^-, \rho_-, \rho_+)$

Main idea: use domain wall erosion rather than pairwise particle annihilation

Symmetric phase



SWSSB phase

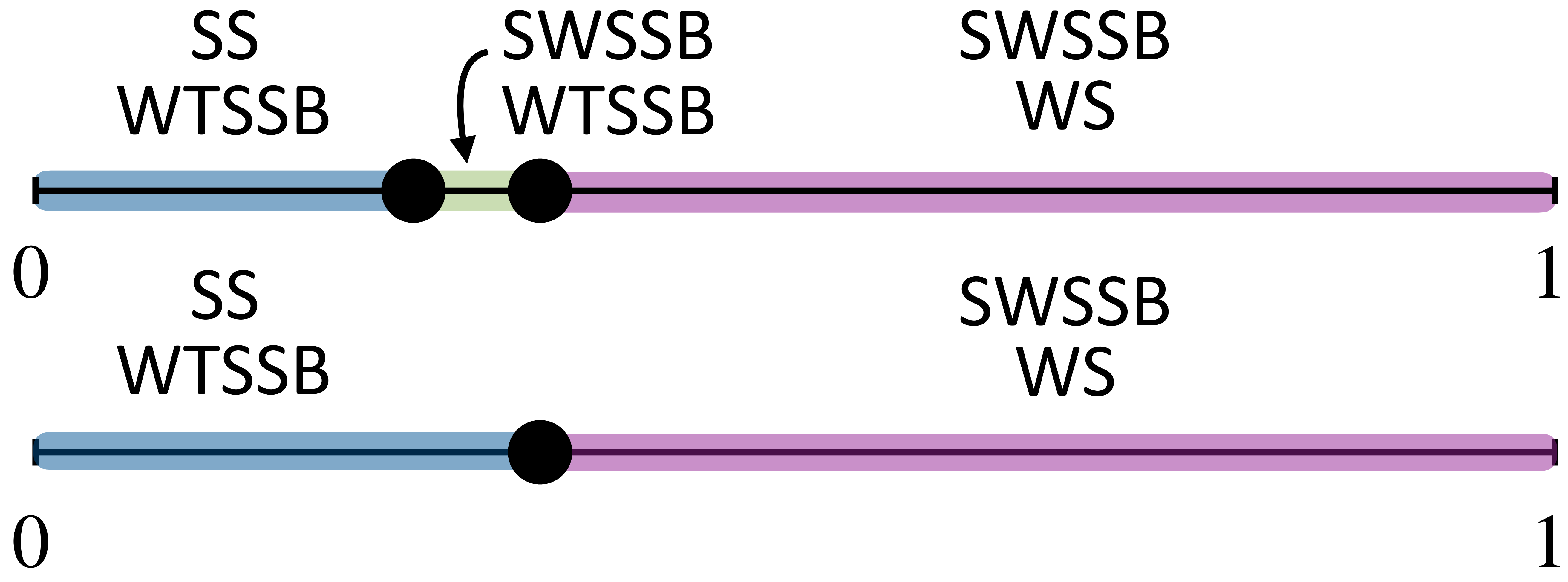

sweep=0 alpha=0.7 m=0.069 n_dw=0.506 excess=0.503 rho_act=0.441 P_X=-1
SWSSB: plaquette2x2
absorbing: pair_toom
x
y
0
5
10
15
20
25
+1
-1
X_r

Weak Z2 symmetry: multiple transitions?

ρ_{WTSSB}

$\propto |+\rangle\langle+| + |-\rangle\langle-|$

(even number of sites)



ρ_{WS}

$\propto (1 + \prod_r X_r)$

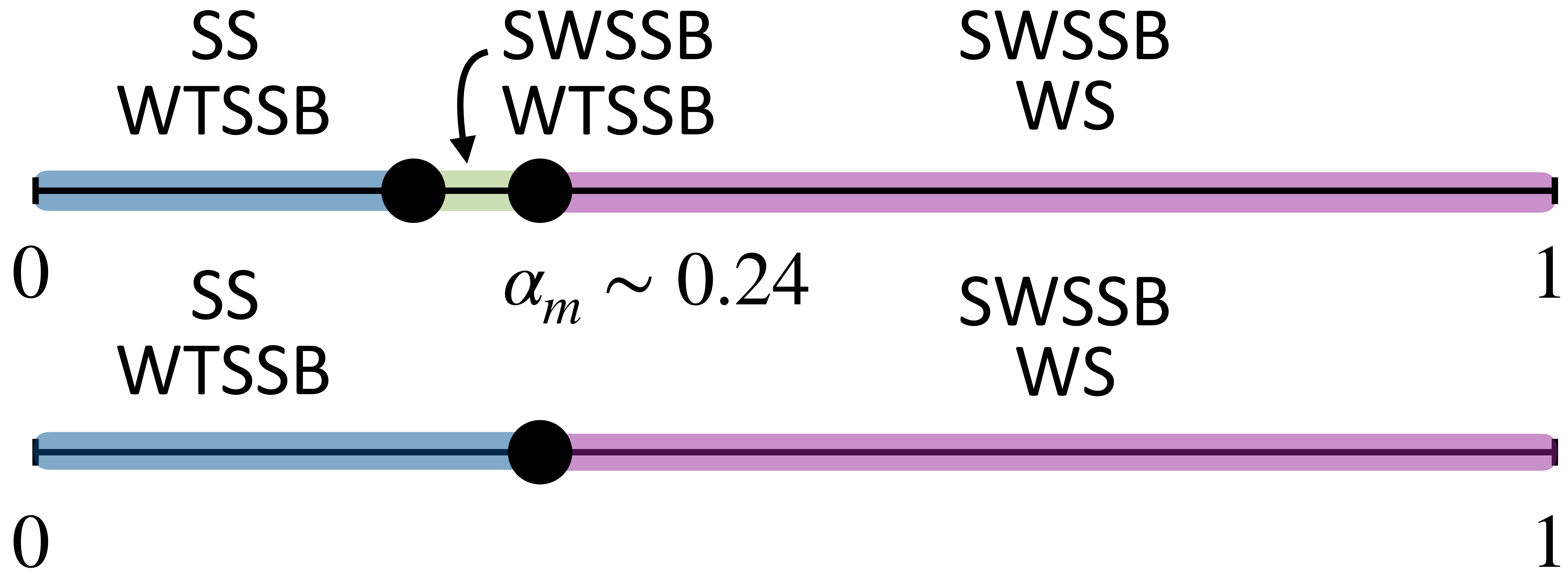
(even number of sites)

Weak Z2 symmetry: multiple transitions?

ρ_{WTSSB}

$\propto |+\rangle\langle+| + |-\rangle\langle-|$

(even number of sites)



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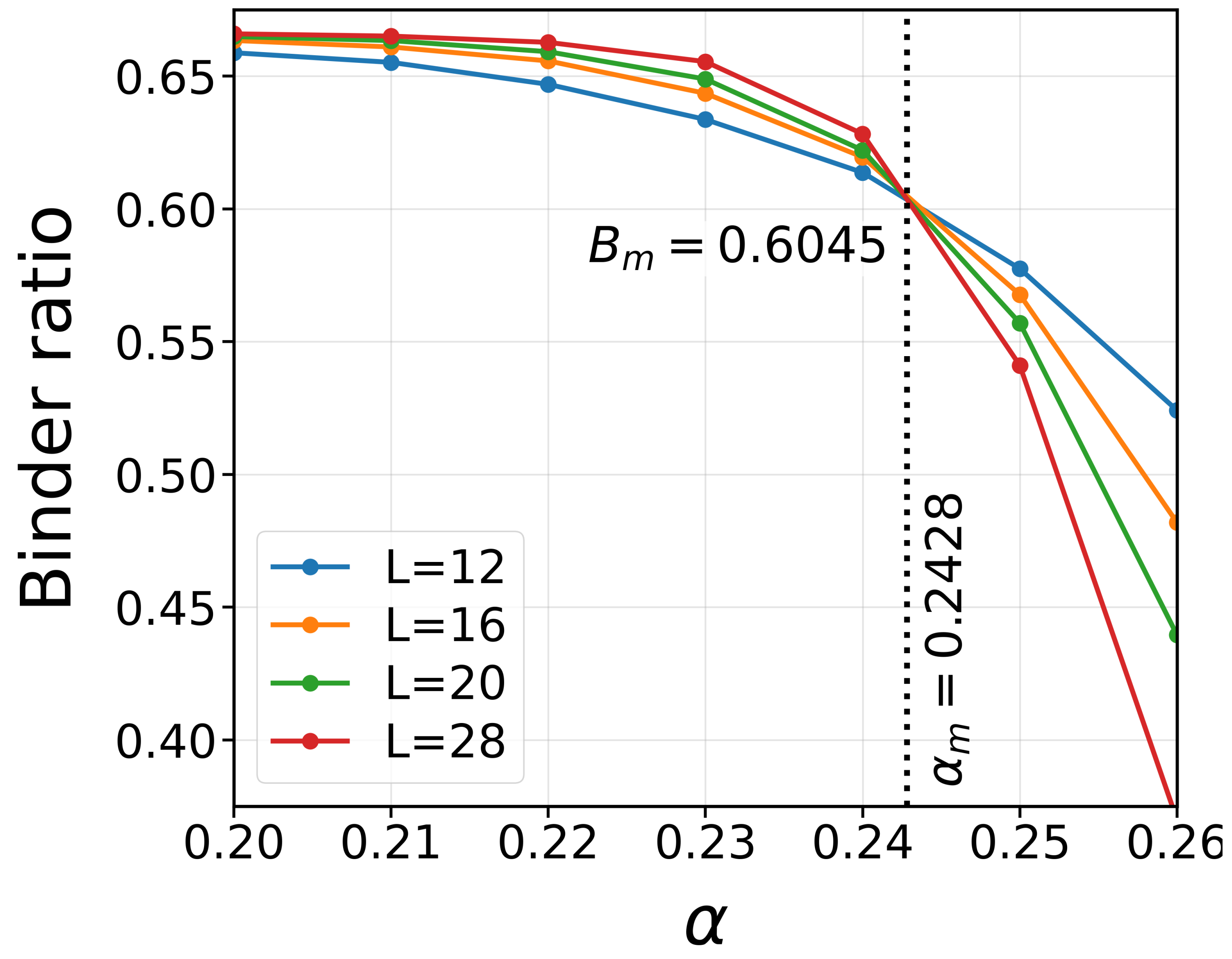
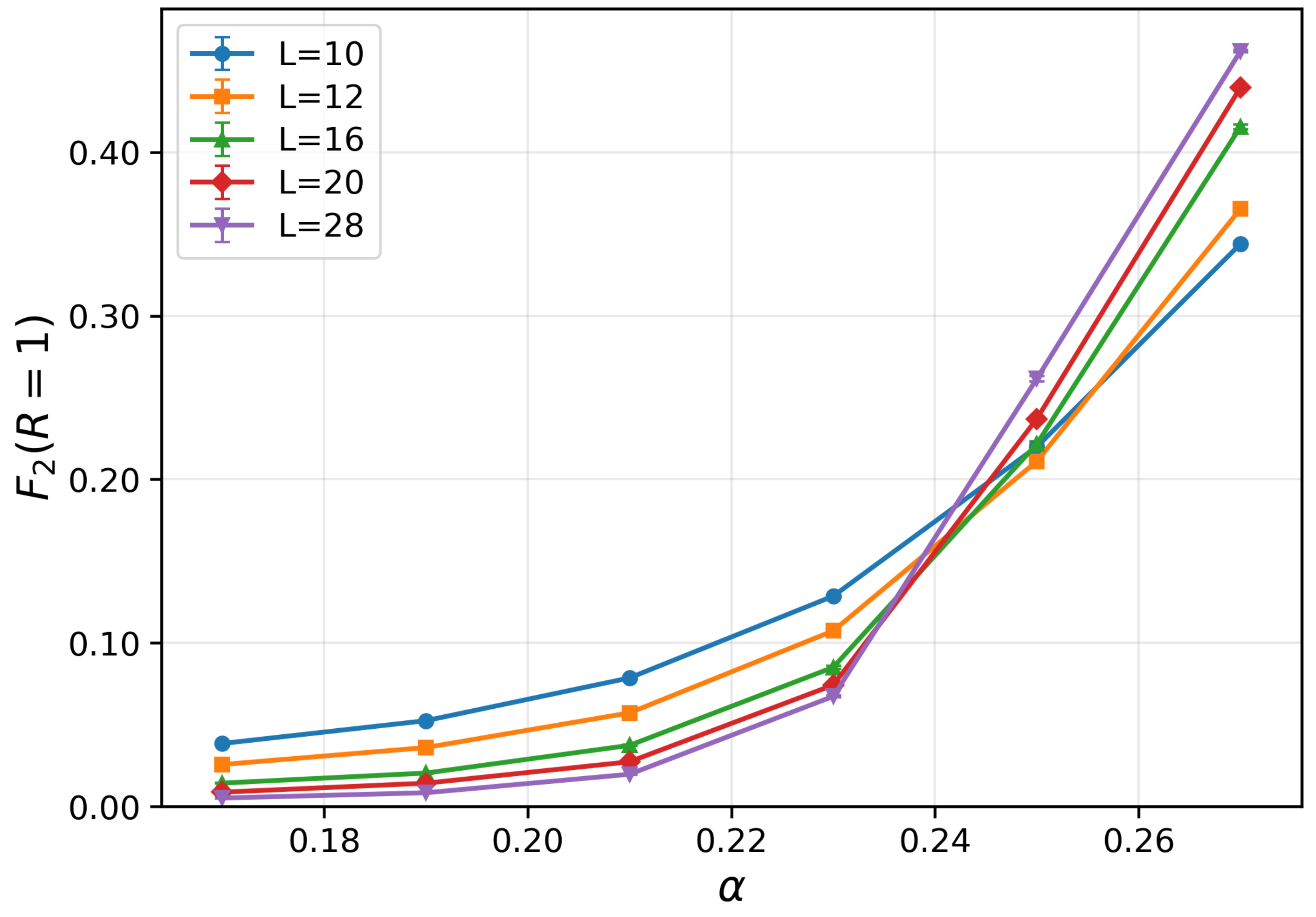
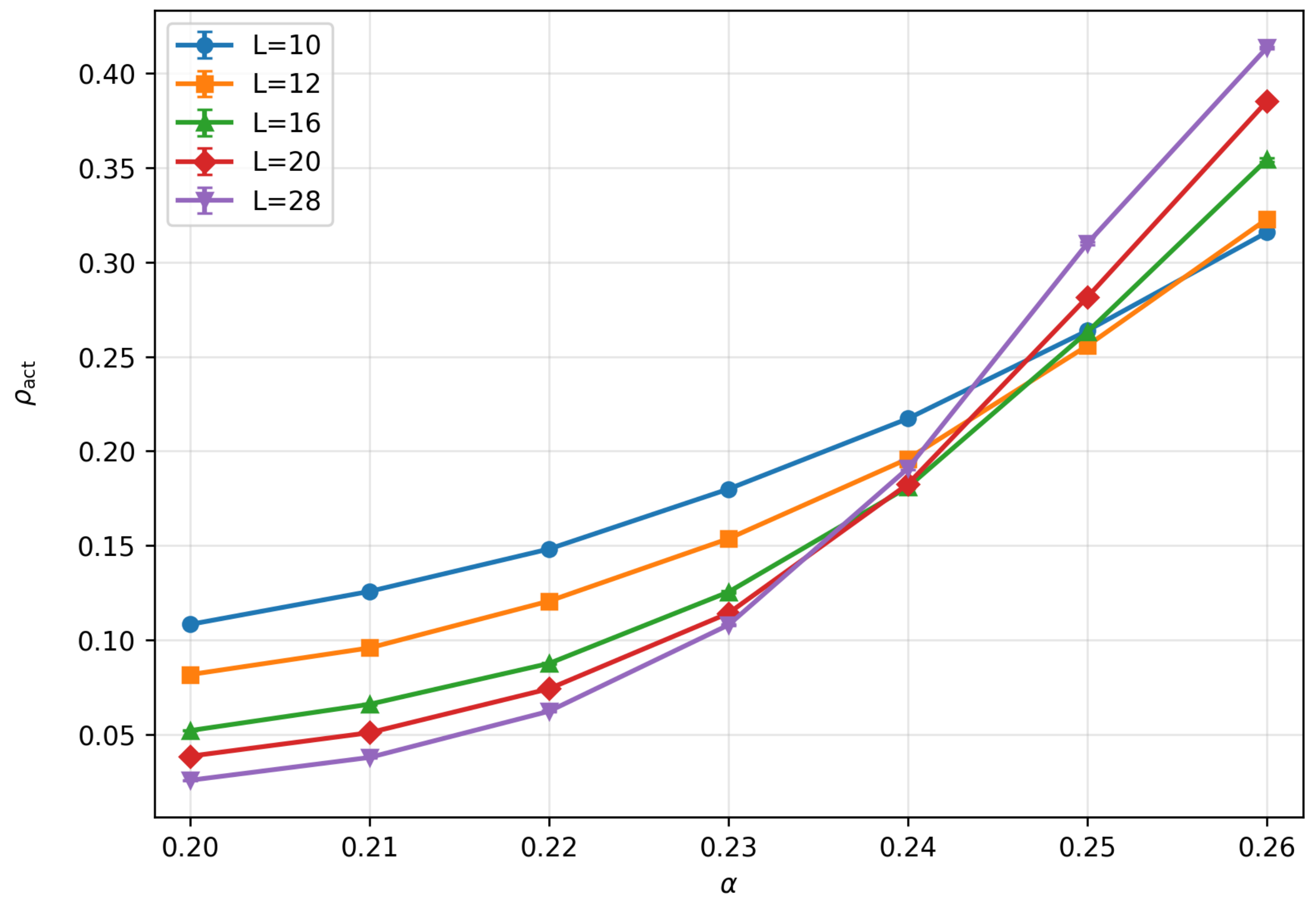


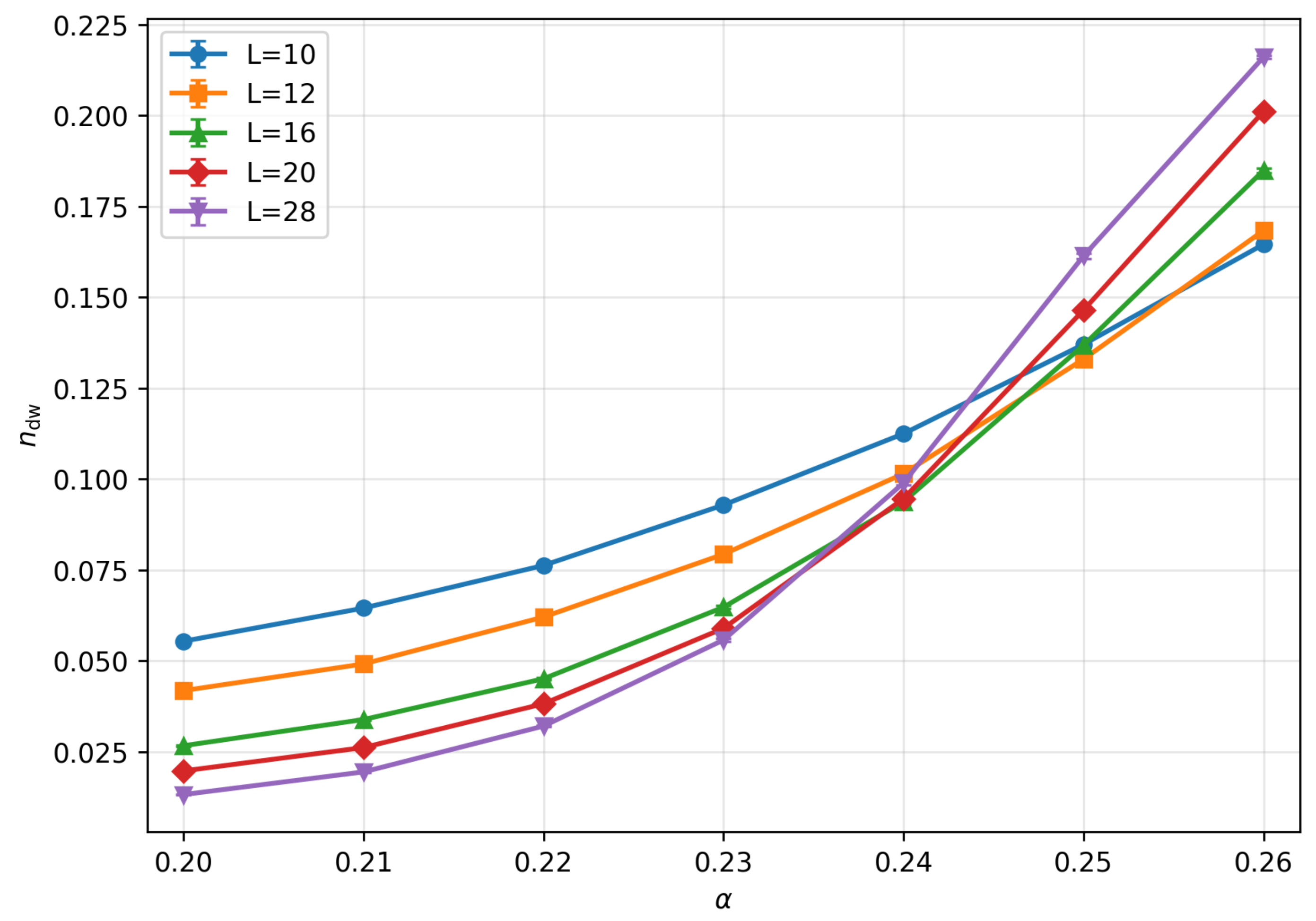
Weak symmetry transition detected by:

$$U_4 = 1 - \frac{\langle m^4 \rangle}{3\langle m^2 \rangle^2}$$

How to detect the SWSSB?







$F^R_{ij}(\rho, Z_i Z_j) =$ marginal fidelity (at radius 1)

$\rho_{\text{act}} =$ density of plaquettes not $(++++), (----)$

$n_{\text{dw}} =$ density of domain walls $(1/2L^2) \sum_{r,r'} (1 - X_r X_{r'})/2$

Cannot use n_- bc of weak $\mathbb{Z}_2$!

Summary

Absorbing state transition?

SWSSB transition!

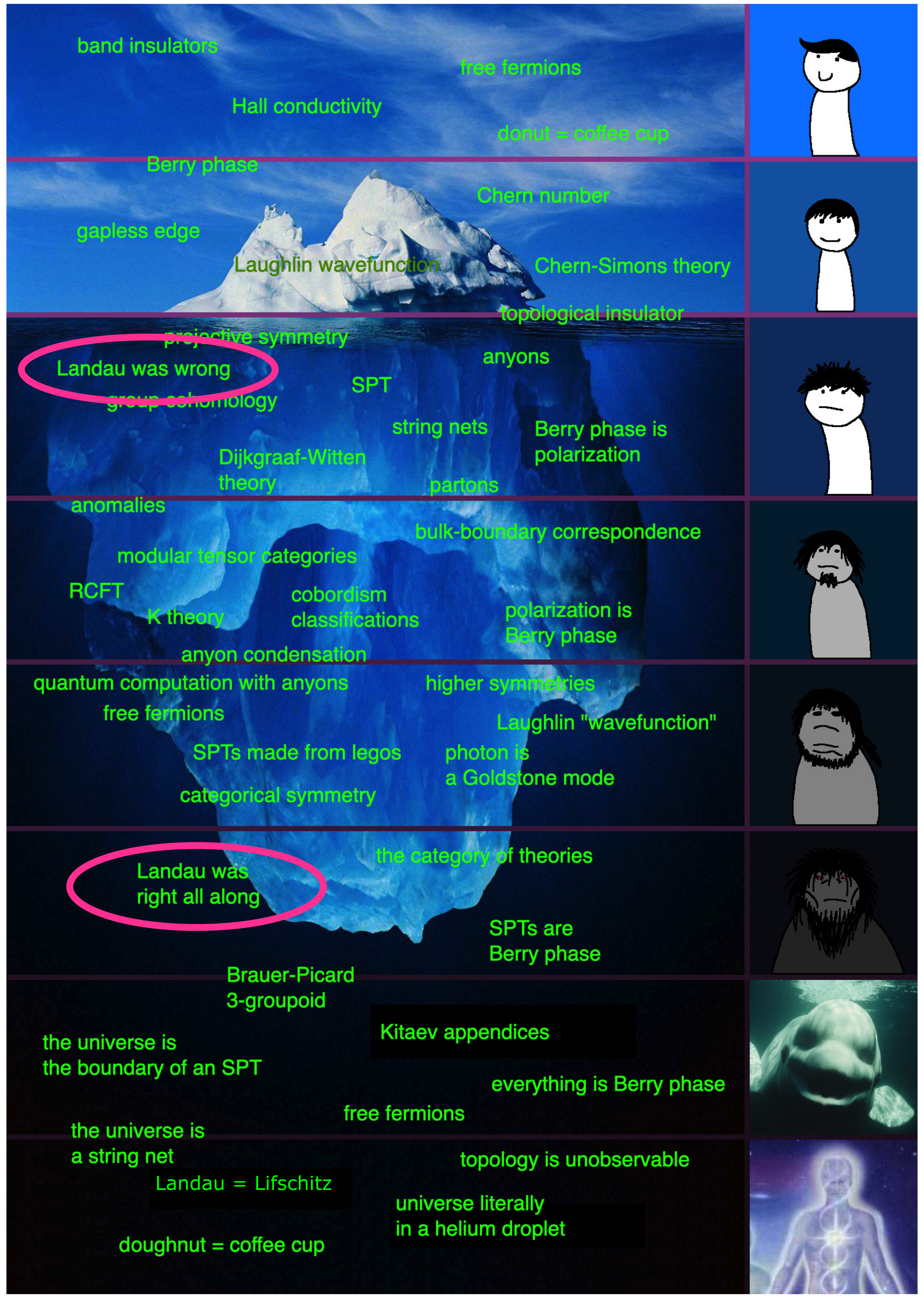


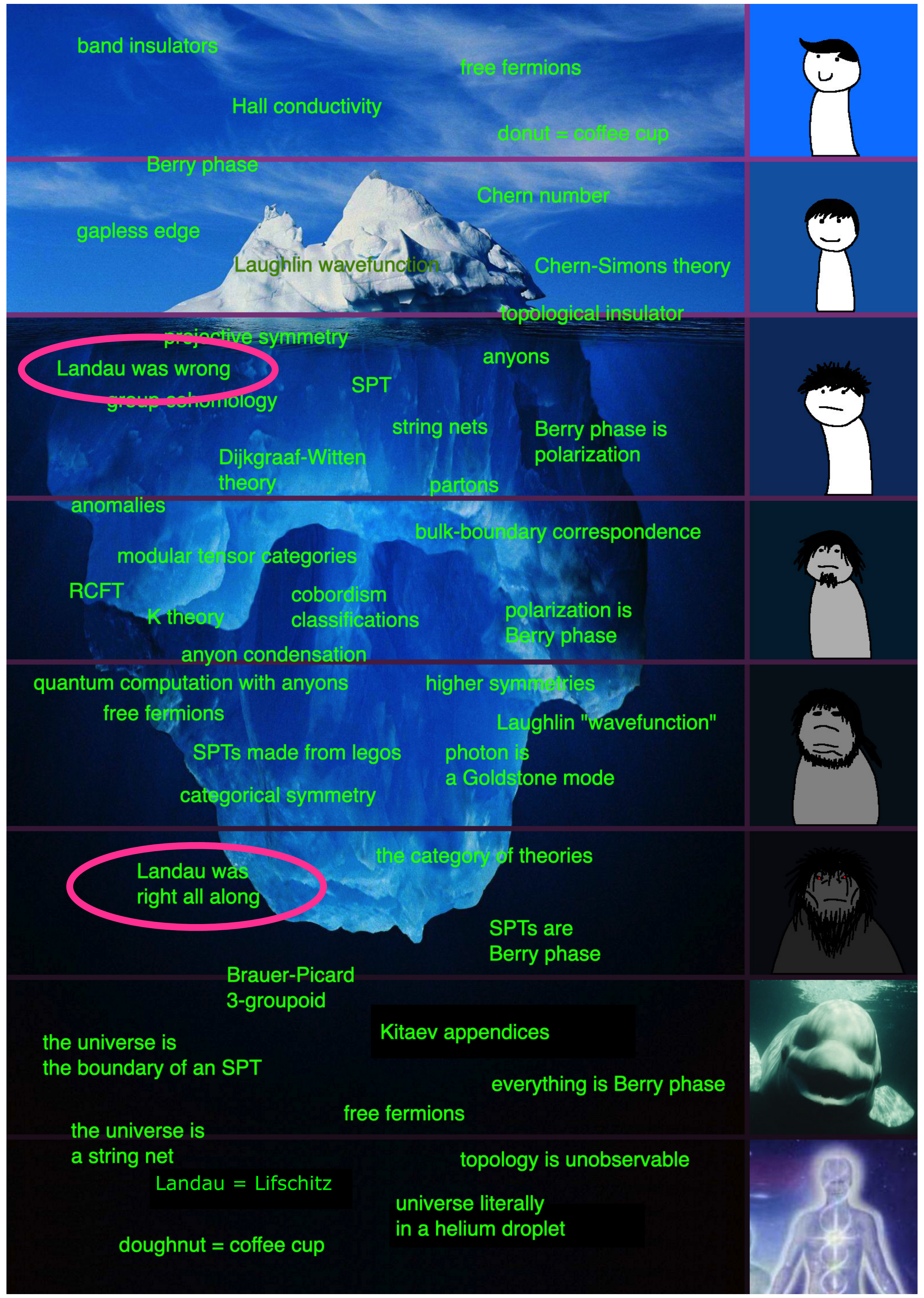
Image credit: Ryan Thorngren

PC-BARW in 1d, pair-flip Toom in 2d

Summary

Absorbing state transition?

SWSSB transition!



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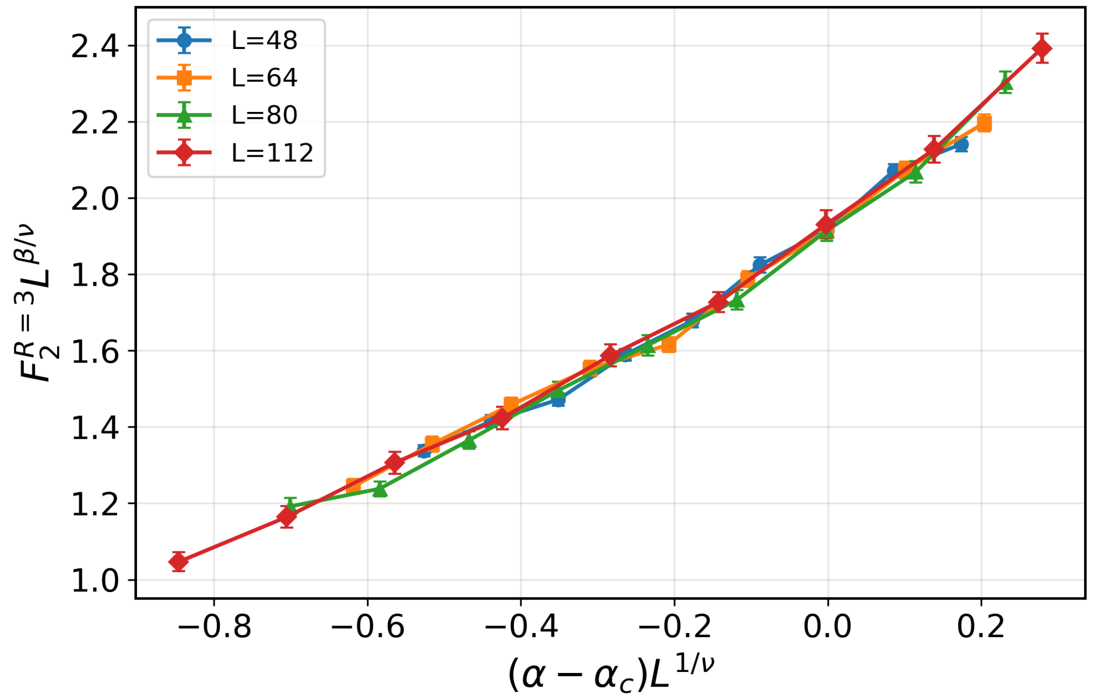
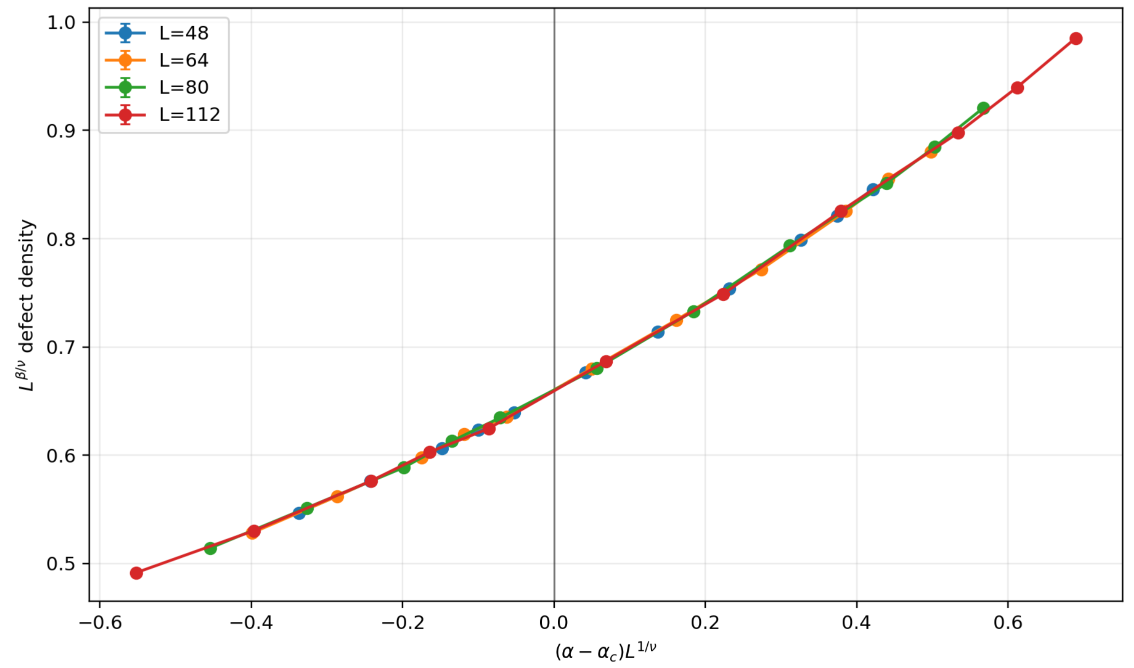
PC-BARW in 1d, pair-flip Toom in 2d

Fidelity correlator



Defect density



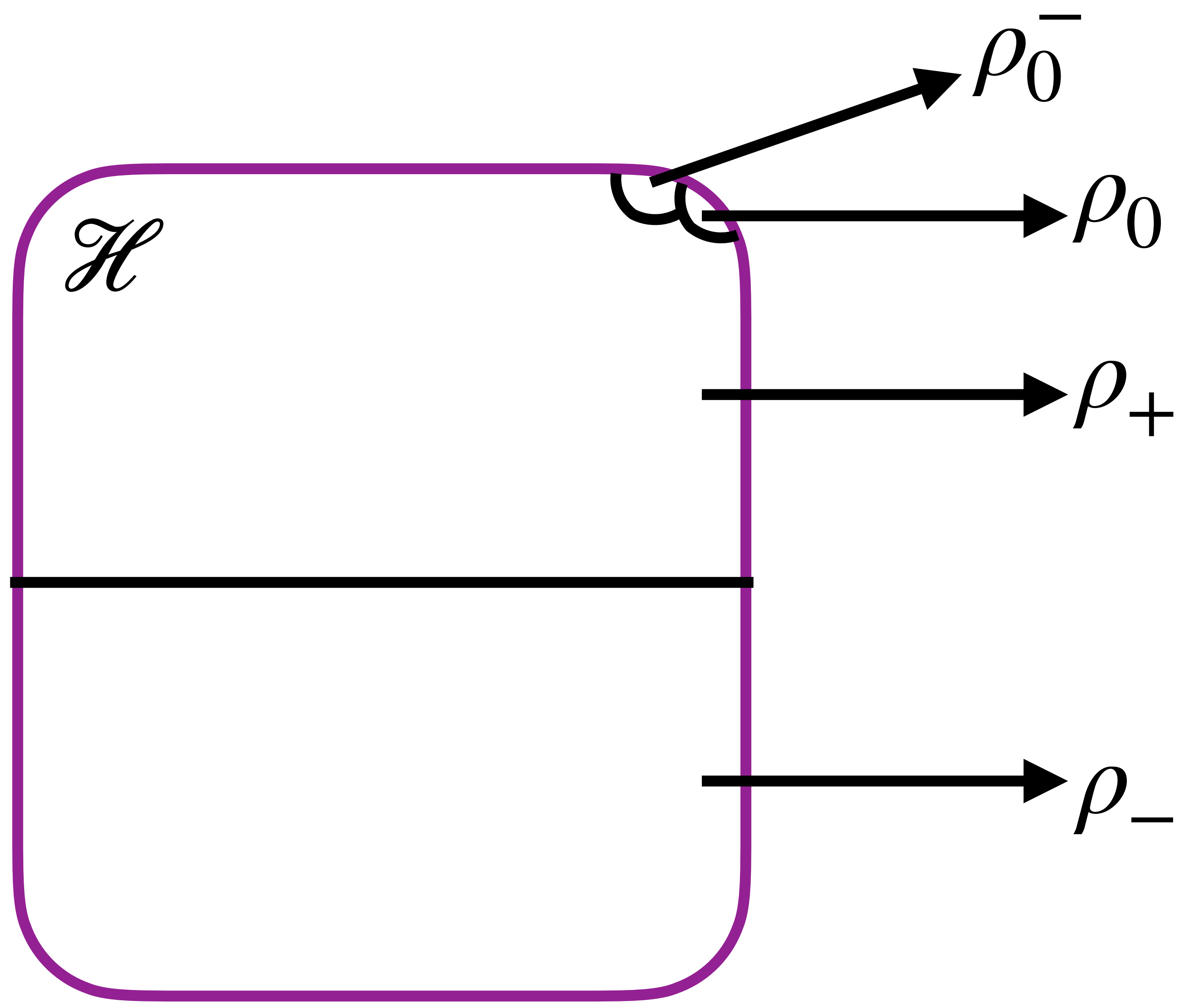
Marginal fidelity

- Monotonic
- Stable
- Complexity implications
- Can lower bound in certain cases and prove convergence
- Universal

Comments/Outlook

- 2+1d model can be thought of as toric code robust to branching error.
- Rigorous proofs of stability/instability of certain steady state spaces?
- Thinking of dynamical constraints (i.e. forbidding certain branchings) in terms of the steady states they add or symmetries?
- Other kinds of phase transitions in highly non-equilibrium settings?
- More direct relationship between marginal fidelity and traditional order parameters?

Comments/Outlook



We have a decent understanding of states. But in what ways are we allowed to put them together into interesting steady state spaces? And how do we use this understanding to construct (quantum) dynamical phase transitions?