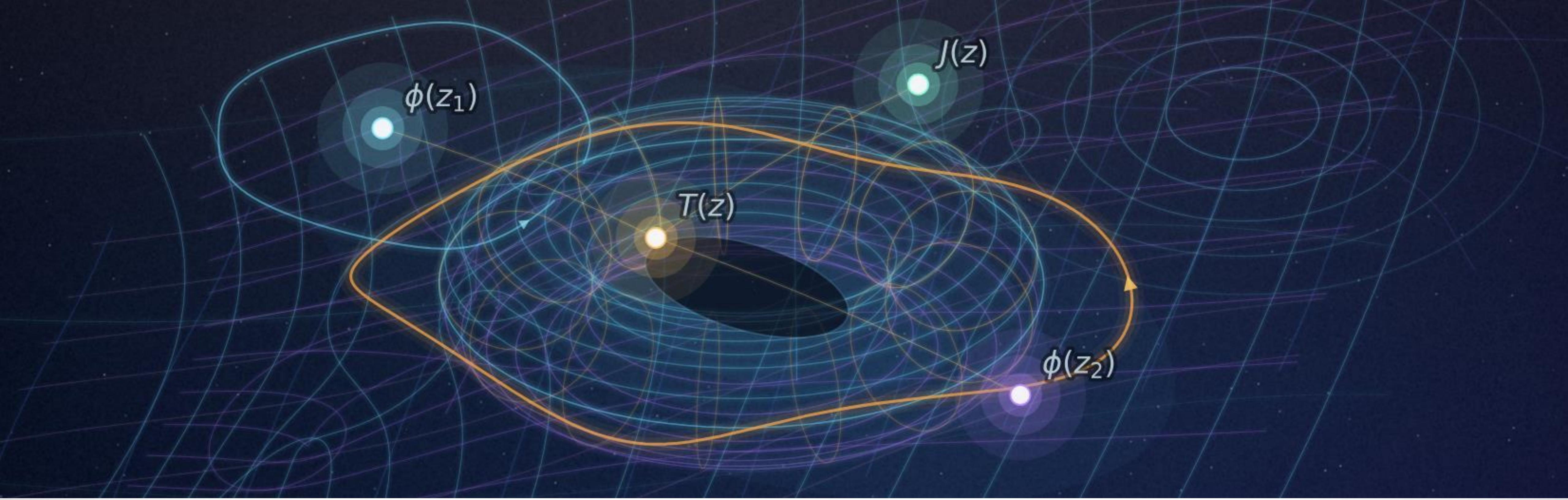

Holomorphic Conformal Field Theory & Modular Tensor Categories

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References and Collaborators

Main Paper: Generalized Level-Rank Duality, Holomorphic Conformal Field Theory, and Non-Invertible Anyon Condensation [arxiv:2512.24419]

Key recent work by others: [Rayhaun, Smith-Lin-Tachikawa-Zheng, Rayhaun-Gannon]

Jeffrey Harvey

Diego García-Sepúlveda

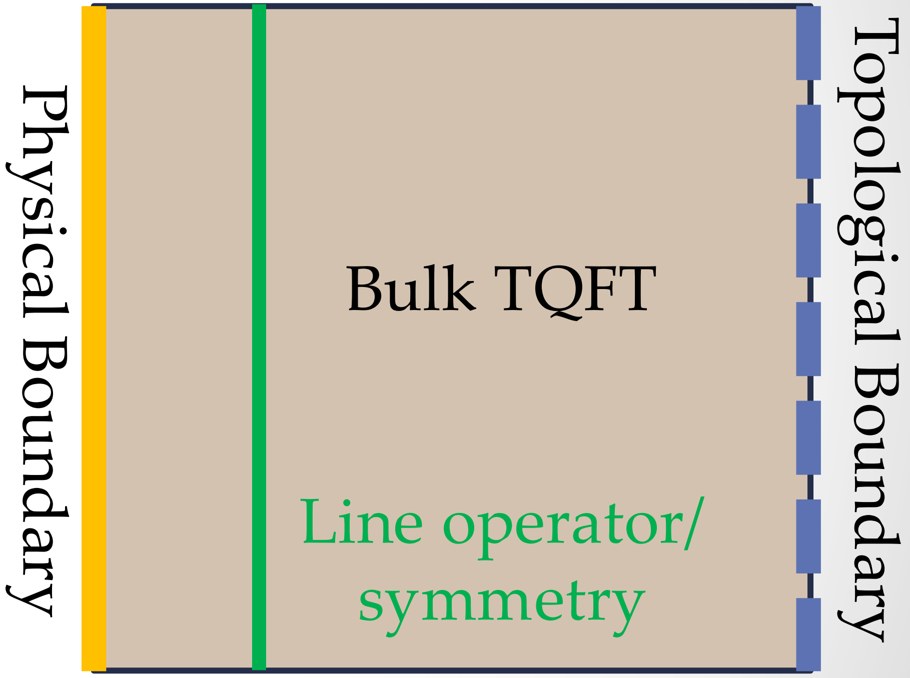
Introduction & Motivation

Symmetry TQFT

Interplay between a quantum field theory and its generalized symmetry:

[Fuchs-Runkel-Schweigert, Gaiotto-Kulp, Kong-Zheng, Freed-Moore-Teleman, Kaidi-Ohmori-Zheng, Apruzzi-Bonetti-Garcia-Etxebarria-Hosseni-Shafer-Nameki]

QFT of interest is boundary condition of bulk TQFT



The bulk TQFT is locally trivial, but supports extended operators characterizing symmetries

The topological boundary condition selects which bulk operators reach the physical boundary as genuine symmetries

Paradigmatic Example in 2D/3D

Most developed example: Boundary = 2D Bulk TQFT =3D

Symmetry is fusion category C, with a finite # of simple lines/objects

3D TQFT is Z[C], the Drinfeld center of C. (Admits a topological boundary)

Maximal interest is when the symmetry C is not spontaneously broken, (unique vacuum)

Implies that the physical boundary is Neumann-like relative to the topological boundary

Special Case of 2D CFTs

Typical QFT is an RG flow. Limits have enhanced conformal symmetry:

UV CFT → IR CFT

A special case is when there is no RG flow, a conformal fixed point

Such theories have a left and right central charge: $c_L, c_R \geq 0$

The restriction to positive values is for unitarity. Strict positivity implies gapless.

The simplest class of possible examples and our focus below are purely chiral

$$c_L \equiv c > 0, \qquad c_R = 0.$$

A Motivating Example

Well known examples of chiral CFTs often show up in duality of TQFT

For instance consider the theory of NK massless chiral complex 2D fermions

$$c_L \equiv c = NK\,, \qquad c_R = 0$$

Bilinears in these fermions form a Kac-Moody algebra: $U(NK)_1$

There is conformal embedding of algebras: $U(K)_N \times SU(N)_K \subset U(NK)_1$

In turn can be used to derive the level-rank TQFT duality:

[Naculich-Schnitzer, Hsin-Seiberg]

$$U(K)_{-N} \cong SU(N)_K$$

Questions and Answers

Motivated by this example we would like to understand:

What is the general relationship between chiral CFT and duality of TQFT?

Methods:

Use symmetry TQFT paradigm to realize holomorphic CFTs in a slab of Chern-Simons gauge theories

Results:

Show chiral CFTs imply new dualities of such Chern-Simons TQFTs (level-rank)

Holomorphic Conformal Field Theory

Basic Structure of Holomorphic CFT

Purely chiral CFT also known as **holomorphic CFT**, or **vertex operator algebra**

The is a holomorphic operator product algebra

$$V_i(z)V_j(0) \sim \sum_k \frac{\lambda_{ijk}V_k(0)}{z^{h_i+h_j-h_k}}$$

Allowed values of the central charge are dictated by modular invariance:

- $c \in \frac{1}{2}\mathbb{Z} \Rightarrow$ spin: invariant under modular fixing preserving spin structure
- $c \in 8\,\mathbb{Z} \Rightarrow$ bosonic: invariant under full modular $SL(2,\mathbb{Z})$

Holomorphic CFTs with Small c

- $c = 8$: Unique possible theory given by compact bosons defined on the root lattice of the Lie-Algebra E_8. This is the vertex algebra $E_{8,1}$

$$Z(q) = \mathrm{Tr}(q^{L_0 - c/24}) = \frac{E_4(\tau)}{\eta(\tau)^8} = q^{-\frac{1}{3}}(1 + 248\, q \ + 4124\, q^2 + \cdots)$$

- $c = 16$: Two distinct theories, both related to Kac-Moody algebras

$$E_{8,1} \times E_{8,1}. \quad \text{and} \quad Spin(32)_1/\mathbb{Z}_2$$

These theories are isospectral but have different OPEs

$$Z(q) = \mathrm{Tr}(q^{L_0 - c/24}) = \frac{E_4(\tau)^2}{\eta(\tau)^{16}} = q^{-\frac{2}{3}}(1 + 496\, q \ + 69752\, q^2 + \cdots)$$

Schellekens Classification at $c = 24$

The last value where we have complete results is c=24. **Our focus case**

Schellekens showed that there are exactly 71 distinct possibilities for the Kac-Moody subalgebra of a holomorphic c=24 theory (including no currents)

Schellekens also conjectured that there is a unique holomorphic CFT at c=24 given the Kac-Moody subalgebra. Now essentially proven:

- Examples with Kac-Moody symmetry are obtained by orbifolds from the Leech lattice theory and the 23 Niemeier lattice theories

- The special case with no currents is famous VOA with monster symmetry

Holomorphic CFTs as Boundaries

Gravitational Anomaly Inflow

Modular invariance implies that holomorphic CFTs are absolute QFTs

They are boundaries of invertible TQFTs. Relevant bulk theory gravitational:

$$I_{inflow} = \frac{c}{192\pi} \int \mathrm{Tr}\left(\omega \wedge d\omega + \frac{2}{3}\, \omega \wedge \omega \wedge \omega \right) = 2c\; CS_{grav}$$

Often the minimal consistent values identified with Chern-Simons theories:

At $c = 8$, we have an equality of partition functions 16 CS_{grav} = $E_{8,1}$

Enriching with Kac-Moody

Every holomorphic CFT is boundary of gravitational SPT

Does not manifest any boundary internal symmetries

Aside from the monster, each $c = 24$ theory has a Kac-Moody sub-algebra with the same central charge

$$24 = c = \sum_i \frac{k_i d_i}{k_i + h_i^\vee}, \qquad k_i = \text{level}, \; d_i = \text{dimension}, \; h_i^\vee = \text{dual Coxeter}$$

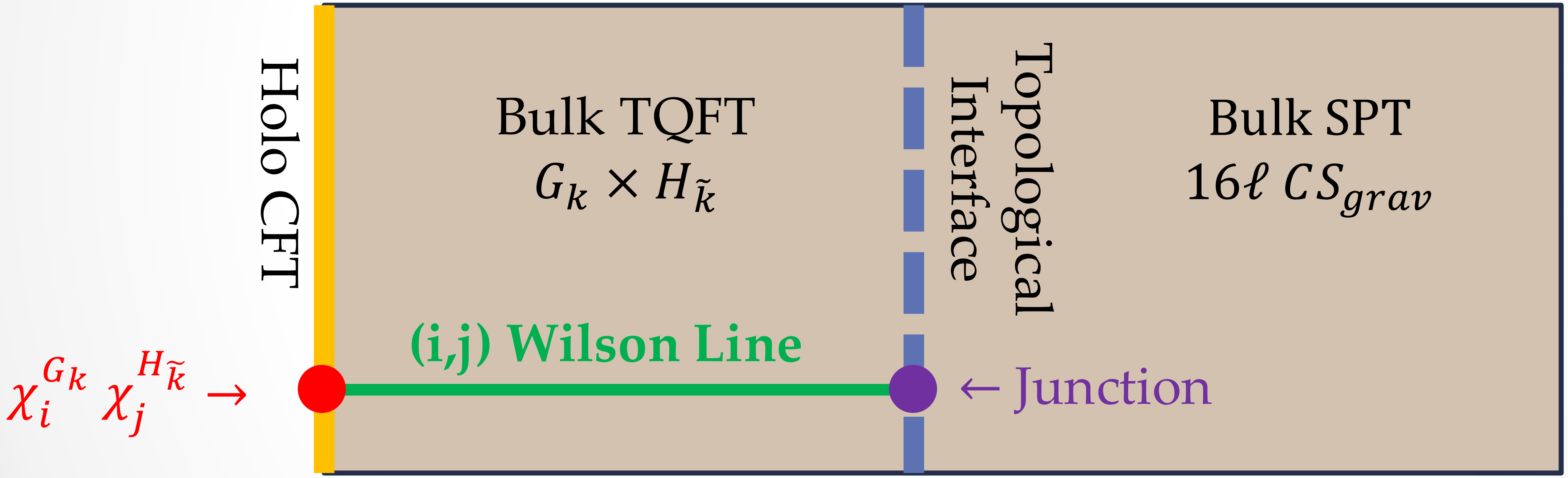
Full VOA viewed as a conformal extension of Kac-Moody sub-algebra:

$$Z(q) = \sum_\lambda N_\lambda \, \chi_\lambda(q), \qquad \chi_\lambda(q) = \text{Kac} - \text{Moody Charaters}, \;\; N_\lambda \in \mathbb{N}_{\geq 0}$$

Enriching with Kac-Moody

Boundary Kac-Moody symmetry implies that bulk is a Chern-Simons theory
[Witten, Elitzur-Moore-Seiberg-Schwimmer]

For instance in a theory with two Kac-Moody factors $G_k \times H_{\tilde{k}}$ we have a slab:



Topological junction is pairing data $N_{i,j}$ specifying which characters occur

Makes manifest fusion category symmetries commuting with Kac-Moody

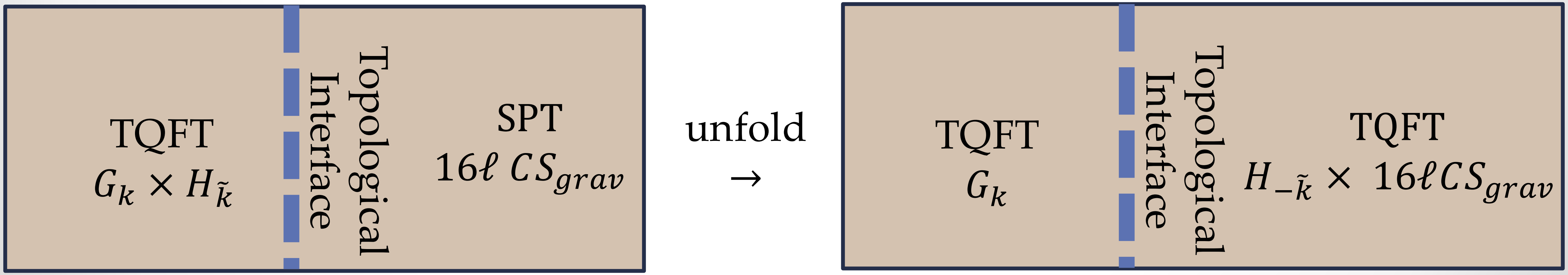
TQFT Duality From Holomorphic CFT

TQFT Duality from Unfolding

Key observation:

Given holomorphic CFT and Kac-Moody subalgebra with equal c there is a topological interface of associated Chern-Simons theory to gravitational SPT [Davidov-Müger-Nikshych-Ostrik , Kaidi-Komargodski-Ohmori-Seifnashri-Shao]

We can unfold this to an interface between the Chern-Simons factors:



Note that unfolding time reverses levels in the Chern-Simons theories

TQFT Duality from Interfaces

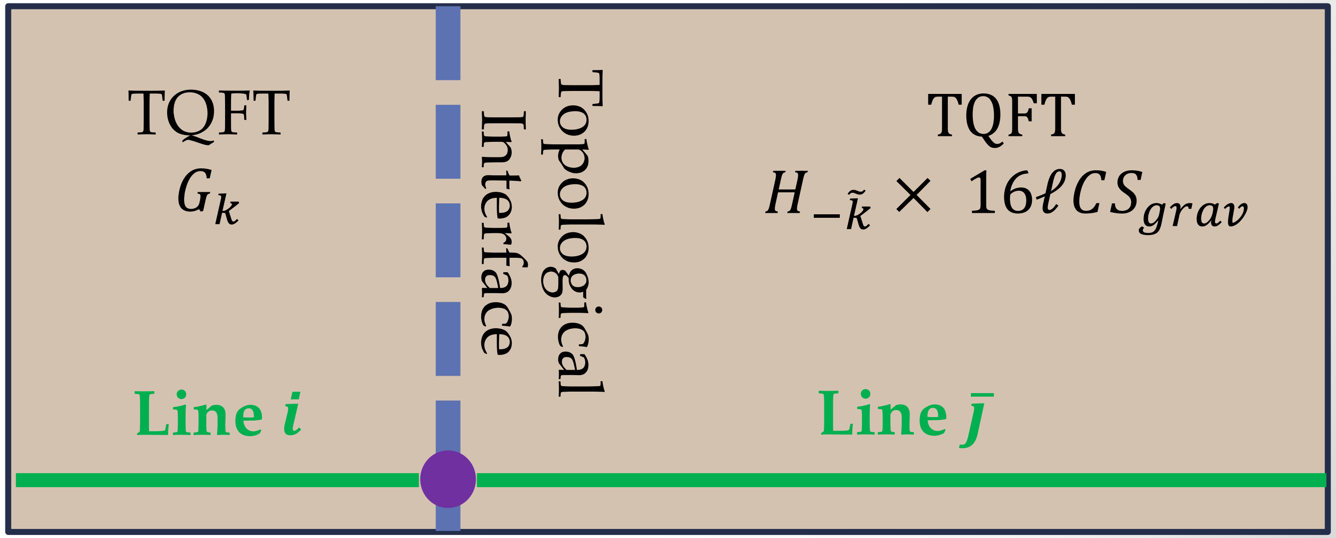
Topological interface between distinct TQFTs is almost a duality of TQFTs

For instance, this setup gives rise to level-rank duality $U(K)_{-N} \cong SU(N)_K$
(here the symbol $\cong$ means that TQFTs are equal up to an invertible theory)

Key data of such a duality is a map between the distinct lines of the TQFTs

Map specified by junction data

This is a Lagrangian characterizing the boundary of Chern-Simons



Duality Requires Condensation

The reason a topological interface is only almost a duality is that the implied map on simple lines is not one-to-one. Instead a variety of junctions exist

Mathematically this means that both $(i,\bar{j})$ and $(i,\bar{j}\,')$ occur in the Lagrangian

We can restore uniqueness at the cost of condensing anyons (ie algebras)

Theorem: If two MTCs $\mathcal{T}_1$ and $\mathcal{T}_2$ are Witt equivalent (topological interface)
[Davidov-Müger-Nikshych-Ostrik]

Then there exist condensable algebras $\mathcal{A}_1 \subset \mathcal{T}_1$ and $\mathcal{A}_2 \subset \mathcal{T}_2$ with: $\frac{\mathcal{T}_1}{\mathcal{A}_1} \cong \frac{\mathcal{T}_1}{\mathcal{A}_2}$

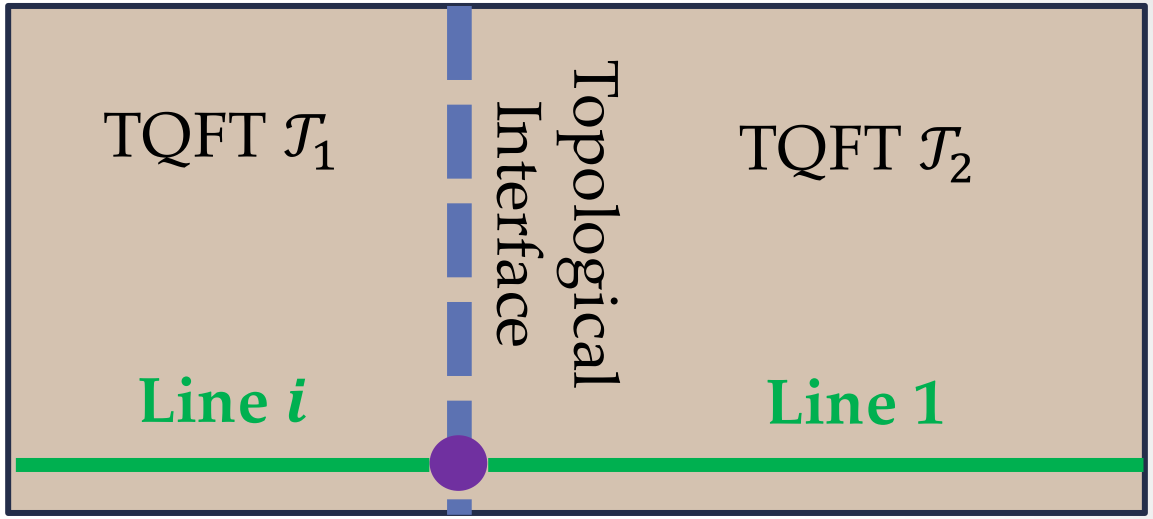
Quotient is MTC defined by local modules. Equivalence is as MTCs

Identification of the Algebras

Identify algebras that must condense by inspecting junctions with identity

After condensation the identity lines must map to each other in duality

Thus all lines with a junction with the identity must condense



$$\mathcal{A}_1 = \left\{ \sum i \mid (i,1) \text{ allowed junction} \right\}, \quad \mathcal{A}_2 = \left\{ \sum j \mid (1,j) \text{ allowed junction} \right\}$$

In our examples these algebras can be deduced from the decomposition of the CFT character into Kac-Moody characters isolating the trivial rep

Duality Engine

We now have a machine to produce new dualities of Chern-Simons TQFTs

- Find a holomorphic CFT and Kac-Moody subalgebra with equal c

- This implies a topological interface between the factors (e.g. $G_k \times H_{\tilde{k}}$) in the associated Chern-Simons gauge theories

- Invoking the theorem we then deduce a duality e.g.: $\frac{G_k}{\mathcal{A}_1} \cong \frac{H_{-\tilde{k}}}{\mathcal{A}_2}$

Notably all level-rank dualities are instances of this construction

We will now harvest Shellekens c=24 list of examples to deduce new dualities

Example Dualities of TQFT

Complexity Menu

Natural to organize the new dualities by the complexity of the quotient:

No condensation: topological interface gives one-to-one map directly. For instance this occurred in our simple level-rank pair $U(K)_{-N} \cong SU(N)_K$

Abelian anyon condensation: allowed for anyons which are mutual bosons [Moore-Seiberg]

1. Remove lines that braid nontrivially with condensing algebra
2. Identify remaining lines that differ by fusion with condensing algebra
3. A line invariant under fusion with s condensing anyons splits into s new lines

Non-Abelian anyon condensation: full local module calculation required [Kong, Bais-Slingerland, Fuchs-Runkel-Schweigert]

Sporadic Examples

As warmup, inspect Schellekens list to find an instance without condensation:

$$E_{8,2} \cong Spin(17)_{-1}$$

How this arises: There is a Schellekens theory with Kac-Moody $E_{8,2} \times Spin(17)_1$

What it means: $Spin(17)_{-1}$ is the Ising fusion category with particular spin
Duality implies that $E_{8,2}$ is also that same MTC

Another case: $E_{7,2} \cong Spin(11)_{-1} \times F_{4,-1}$

Here in addition to the Ising fusion category we have $F_{4,-1} \cong$ Fibonacci

Examples with Abelian Condensation

Abelian condensation in Chern-Simons theory can often be understood as removing some of the lines arising from the center of the gauge group

This corresponds to Chern-Simons theory with the quotient group.

Example dualities from Schellekens list manifesting this principle are:

$$\frac{USP(16)_1}{\mathbb{Z}_2} \cong F_{4,-1} \times F_{4,-1} \cong \text{Fibonacci}^2, \qquad \frac{E_{6,3}}{\mathbb{Z}_3} \cong G_{2,-1} \times G_{2,-1} \times G_{2,-1} \cong \text{Fibonacci}^3$$

We also find a simple abelian infinite series: $\frac{SU(2n^2)_1}{\mathbb{Z}_n} \cong E_{7,(-1)^n}$

The $c = 24$ example is the case $n = 3$. Gives a clue to the general pattern

Examples Implying Time-Reversal

A particularly interesting kind of duality is one between $\mathcal{T}$ and $\bar{\mathcal{T}}$

In that case the theory is said to possess time-reversal symmetry T

e.g. the level-rank duality $SO(N)_k \cong SO(k)_{-N}$ implies T symmetry if $N = k$

From the Schellekens $c = 24$ list we find several new T symmetric theories

$$SU(7)_1 \times SU(7)_1, \qquad \frac{Spin(7)_2 \times Spin(7)_2}{\mathbb{Z}_2}$$

Easy to verify a posteriori. Holomorphic CFT provides a principle

Non-Abelian Condensation Examples

Novel examples occur when we condense bosons with non-abelian fusion

For instance from Schellekens list of $c = 24$ theories we deduce equivalence

$$\frac{Spin(18)_2}{\mathcal{A}} \cong SU(8)_{-1}$$

Algebra object $\mathcal{A} = \mathbf{1} + \Lambda^6(\mathbf{18})$, (label by Wilson line irrep), $d_{\Lambda^6(\mathbf{18})} = 2$. Fusion:

$$\Lambda^6(\mathbf{18}) \times \Lambda^6(\mathbf{18}) = \mathbf{1} + 2(\mathbf{18}) + \Lambda^6(\mathbf{18}),$$

Note as consistency check: $\dim\left(\frac{Spin(18)_2}{\mathcal{A}}\right) = \frac{\dim(Spin(18)_2)}{\dim(\mathcal{A})^2} = \frac{72}{9} = \dim(SU(8)_{-1})$

This is the first in an infinite series of dualities for odd n: $\frac{Spin(2n^2)_2}{\mathcal{A}_n} \cong SU(8)_{-1}$

$Z[\mathbb{Z}_{2n+1}$ Tambara-Yamagami$]$

Another infinite series construction can be generalized from $c = 24$

Schellekens implies that $\frac{Spin(13)_2 \times SU(13)_1}{\mathcal{A}}$ is the bulk of a holmorphic CFT with fusion category symmetry that includes $\mathbb{Z}_{13}$ Tambara-Yamagami

$$D \times D = \sum_{i=0}^{12} \eta^i$$

More generally this corresponds to the general pattern that for a particular FS indicator and bicharacter Z[TY] is a Chern-Simons gauge theory:

$$Z\left[TY[\mathbb{Z}_{2n+1}]\right] \cong Spin\,(2n+1)_2 \times SU(2n+1)_{-1}$$

[Ardonne-Cheng-Rowell-Wang, Ardonne-Finch-Titsworth, Evans-Gannon]

Example with Only One Gauge Group

To engineer dualities we look at theories with multiple Kac-Moody factors

What about holomorphic CFTs with a single irreducible factor?

These examples also yield a TQFT surprise: associated CS theory is a center

For example $c = 24$ yields an example with Kac-Moody $Spin(25)_2$

After tensoring with an invertible theory to cancel c, we deduce:

$$Spin(25)_2 \cong \text{DW twisted } \mathrm{D}_5 \text{ gauge theory}$$

This is the first in an infinite series for odd n: $Spin(n^2)_2 \cong \text{twisted } \mathrm{D_n} \text{ gauge theory}$

Conclusions

VOAs that are conformal extensions of Kac-Moody algebras imply topological boundaries of Chern-Simons gauge theories

Boundaries can be converted into dualities after anyon condensation

Applying this to Schellekens list of $c = 24$ theories yields **novel dualities**

Future topics:

What novel dualities can be extracted from holomorphic spin CFTs?

Can we realize these dualities as the IR of Chern-Simons matter theories?