

Mini-course 2: orbital stability of periodic waves in Hamiltonian systems under localized perturbations

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In Hamiltonian systems, periodic waves often correspond to coherent structures: recurrent, robust patterns that persist over time. Notable examples include water waves, periodic sequences of light pulses in nonlinear optical fibers, and soliton trains in Bose-Einstein condensates. To date, nonlinear stability results for periodic standing or traveling waves in Hamiltonian systems have primarily addressed co-periodic perturbations. A longstanding open problem concerns their stability with respect to localized perturbations: a natural setting in many physical applications. We begin this minicourse by reviewing classical stability methods for Hamiltonian systems with symmetry. These approaches characterize stable solutions as constrained minimizers of an appropriate Lagrangian functional, which is built from conserved quantities of the system and is positive definite on a finite-codimensional constraint space. We then explain why this framework breaks down for periodic waves under localized perturbations and introduce a novel approach that combines variational methods, Floquet-Bloch theory, and Duhamel-based estimates with a modulational ansatz. This alternative approach yields orbital stability results for periodic waves in key Hamiltonian models, such as the Korteweg-de Vries, Klein-Gordon, and nonlinear Schrödinger equations, with respect to L^2 -localized perturbations.

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