# Magnetized by Rotation: Spin and Chiral Condensates in the NJL Model

Ashutosh Dash

Institute for Theoretical Physics, Goethe University, Frankfurt

Dec 4, 2025

Institut Denis Poisson, University de Tours

Based on: arxiv: 2509.18881, Lutz Kiefer, AD and Dirk Rischke

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

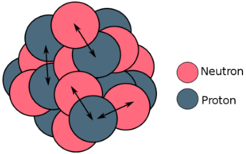
CRC–TR 211 Strong-interaction matter under extreme conditions

DFG

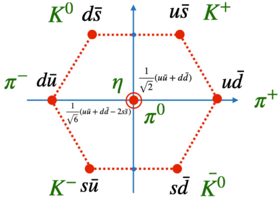
# Outline

- Brief overview of heavy-ion collision
- Rotation and heavy-ion collision
- Thermodynamics of Spin-hydrodynamics
- A simple model
- Conclusion & Outlook

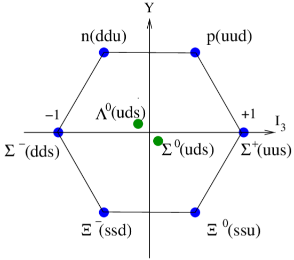
# Historical remarks




**Nucleus (1932)**



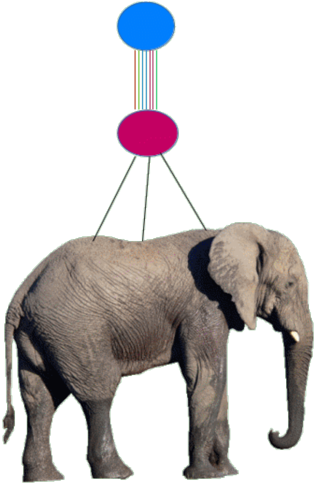

**Mesons (1940's - 1960's)**

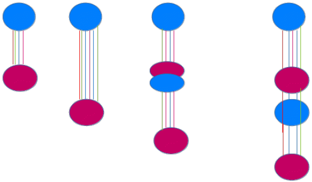



**Baryons (1940's - 1960's)**

- **1932**: nucleus of protons, neutrons bound by a strong force.
- **1940's - 1960's**: more strongly-interacting particles found - hadrons.
- **1970's**: All originate from same sub-structure - quarks and gluons.

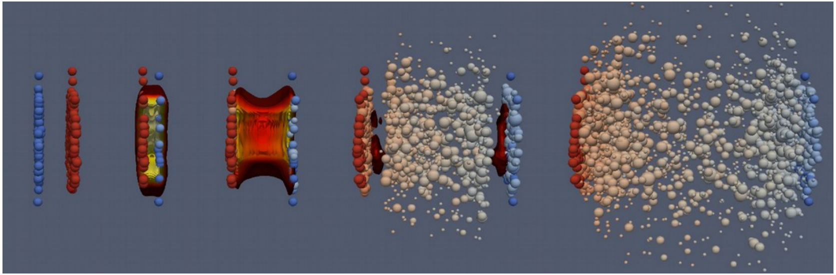
# Nature of strong force





- Electromagnetic force: force falls with distance $F_{\text{EM}} \propto 1/r^2$
- Strong force: $> 10^{-15}$ m, independent of distance between quark and antiquark. This is called Confinement.
- Strong force: $< 10^{-15}$ m, quarks and anti-quark pairs are "free". This is called Asymptotic freedom.

# How to get freedom ?

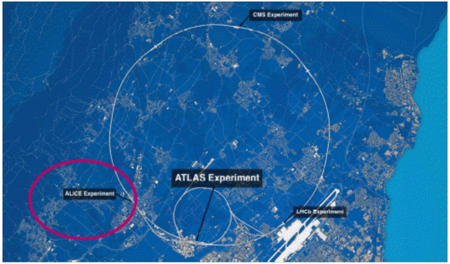


[MADAI collaboration, H. Petersen and J. Bernhard]

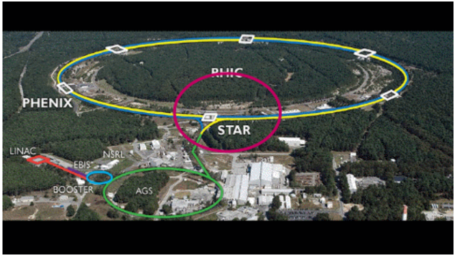
- Heat nuclear matter hot enough. Accelerate nuclei to very high-energy and collide them: Heavy Ion Collisions.
- Relevant temperature: $\hbar c/(10^{-15}\ \mathrm{m}) = 200\ \mathrm{MeV} = 2.4 \times 10^{12}\ \mathrm{K}$. Temperature at interior of sun $\sim 10^{7}\ \mathrm{K}$.

# Heavy-ion collisions

CRC–TR 211

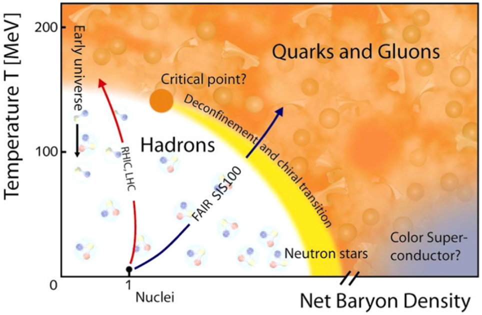



**ALICE** @ **CERN**, Switzerland




**STAR** @ **BNL**, USA

- Huge collaborations: $40+$ countries, $150+$ institutions with $2000+$ members.
- Upcoming experiment: **CBM** @ **FAIR**, GSI, Germany

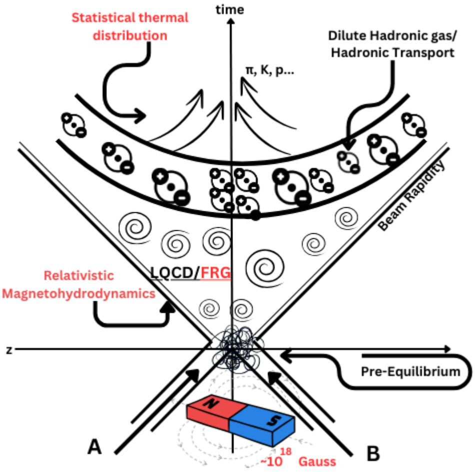



[NuPECC Long Range Plan 2017]

**Emergent properties of QCD using relativistic heavy-ion collision**:

- QCD transitions: De-confinement and chiral symmetry restoration.
- Deconfined state of quarks and gluons : Quark Gluon Plasma (QGP).
- Properties of QGP: viscosity, conductivity, opacity, polarization and vorticity.
- Phase diagram of QCD: Thermalization, crossover, first order, critical point ?

# Theoretical Frameworks




**Canonical picture of heavy-ion with various theoretical tools used.**

- Equation of state, thermodynamics and transport.
  1. LQCD (Lattice QCD): First principle Monte-Carlo calculation of QCD on a lattice.
  2. FRG (Functional Renormalization Group): First-principle functional method of QCD in continuum.
- Bulk evolution: Relativistic hydrodynamics/magnetohydrodynamics (RHD/RMHD).
- Statistical thermal models: Hadron resonance gas (with interactions - S-matrix formalism)

# Establishing the formation of a thermal system-I

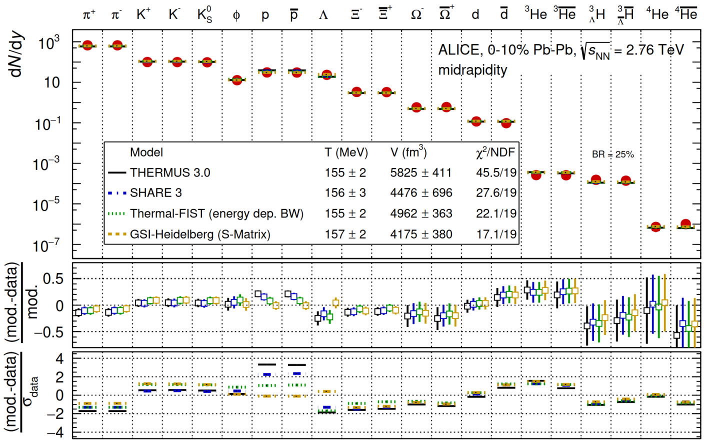



Figure: ALICE: Eur.Phys.J.C 84 (2024) 8, 813

$$n = \frac{1}{V}\frac{\partial(T\ln Z)}{\partial\mu}$$

$Z$ is the partition function of an ideal gas of hadrons

$$\mathbf{T \sim 156\ MeV}$$

# Establishing the formation of a thermal system-II

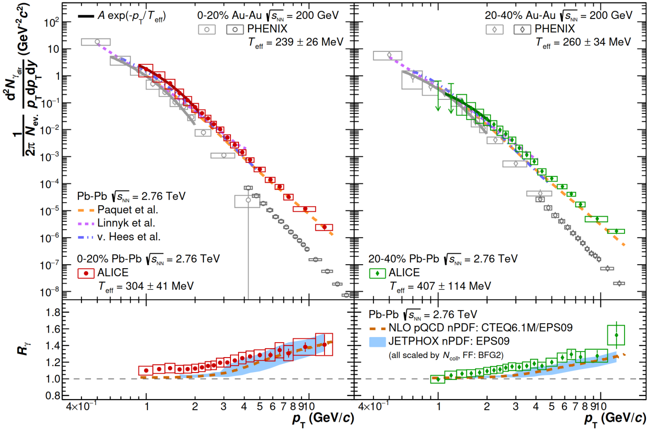



Figure: ALICE: Eur.Phys.J.C 84 (2024) 8, 813

$$p_T \lesssim 3\ \text{GeV};\quad A\exp(-p_T/T_{\rm eff})$$

$$p_T \gtrsim 5\ \text{GeV};\quad B(p_T)^{-\alpha}$$

$$\mathbf{T_{eff} \sim (304 - 407)\ MeV}$$

Radial expansion of the system, which causes a blue-shift of the emitted photons

# Hydrodynamical evolution

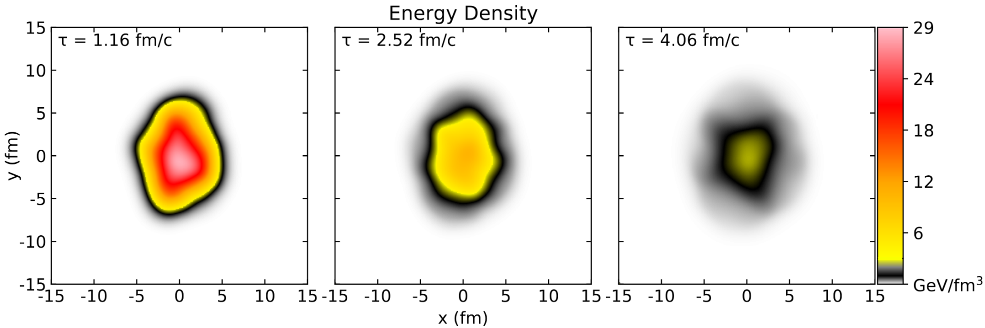
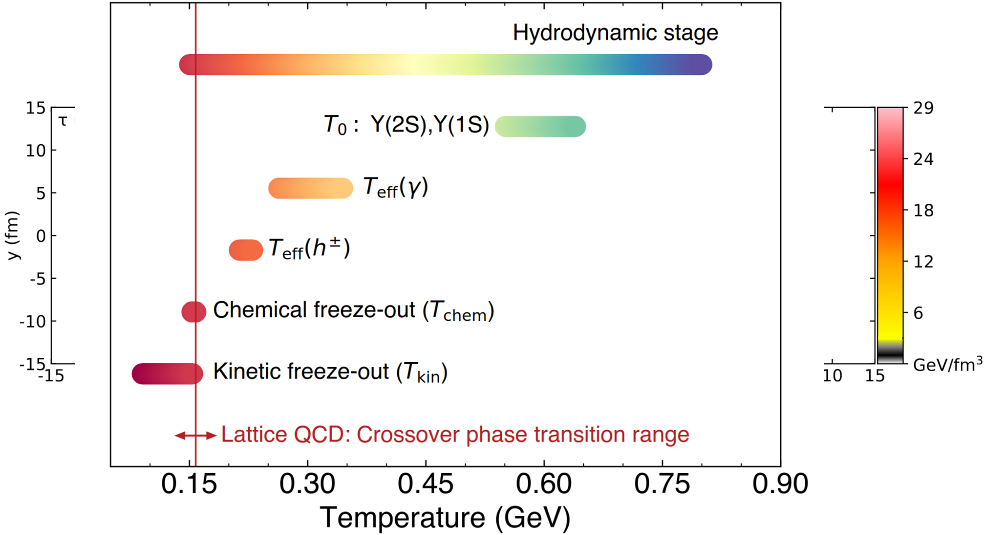



Figure: A mapping of the energy density in the QGP phases vs time and space for a mid-central collision

# Hydrodynamical evolution



# Rotation and heavy-ion collision

# Weak decay of Lambda hyperon

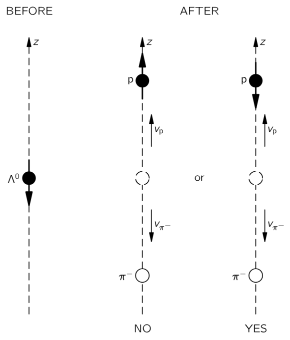



Figure: Lambda decay in the rest frame

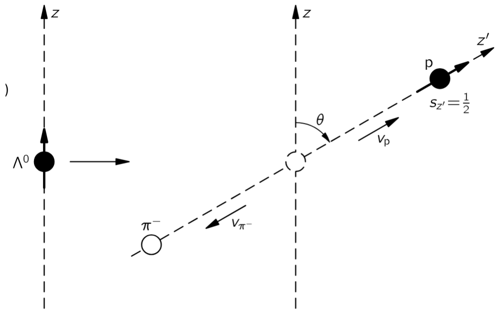
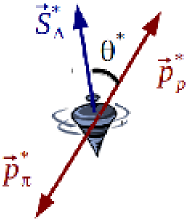



Figure: Rotated view showing decay–proton direction

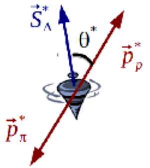
- If the $\Lambda^0$ has spin up along the $z$-axis, what is the probability that the decay proton will go off at the angle $\theta$ ?
- $\frac{dN}{d\cos\theta} = |a|^2\cos^2(\theta/2) + |b|^2\sin^2(\theta/2) = \frac{1}{2}(1 + \alpha\cos\theta)$




$$\Lambda \to p + \pi^+$$

- Now suppose that the $\Lambda^0$ is not completely polarized but only partially given by strength $\mathcal{P}$
- Measurement of $\frac{dN}{d\cos\theta^*} = \frac{1}{2}\left(1 + \alpha\mathcal{P}\cos\theta^*\right)$

# Measurement of spin-polarization in a medium




$$\Lambda \to p + \pi^+$$

- Now suppose that the $\Lambda^0$ is not completely polarized but only partially given by strength $\mathcal{P}$
- Measurement of
  $$\frac{dN}{d\cos\theta^*} = \frac{1}{2}\left(1 + \alpha\mathcal{P}\cos\theta^*\right)$$
- Why is the medium polarized ?

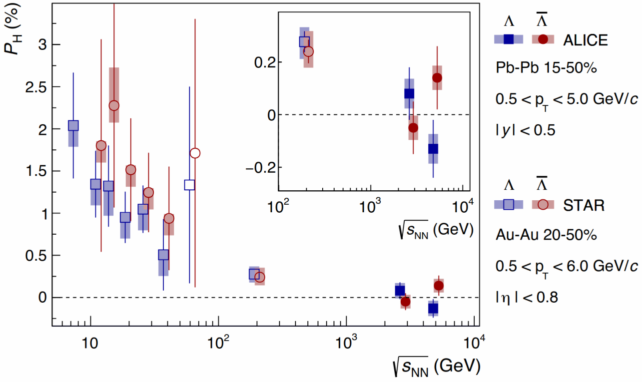
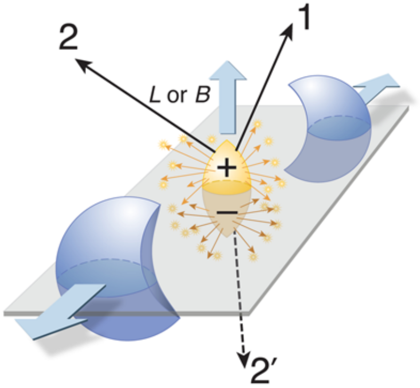


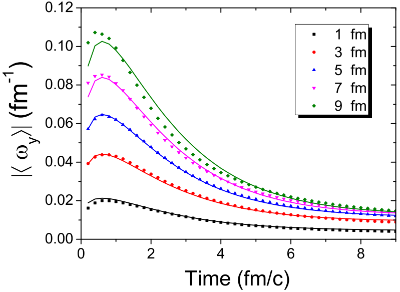

Figure: $\Lambda$ polarization **[Adamczyk et al. (2017)]**

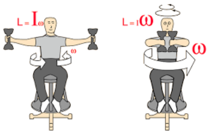
# Heavy-ion collisions

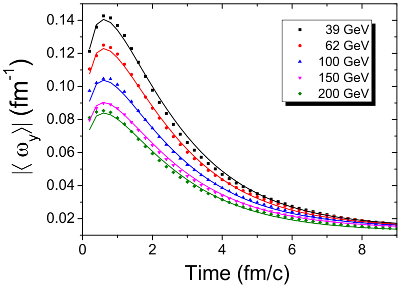



- Reaction plane: Impact parameter and beam axis.
- L and B perpendicular to reaction plane.
- No rigid rotation, but local fluid **vorticity.**

# How large is the vorticity ?



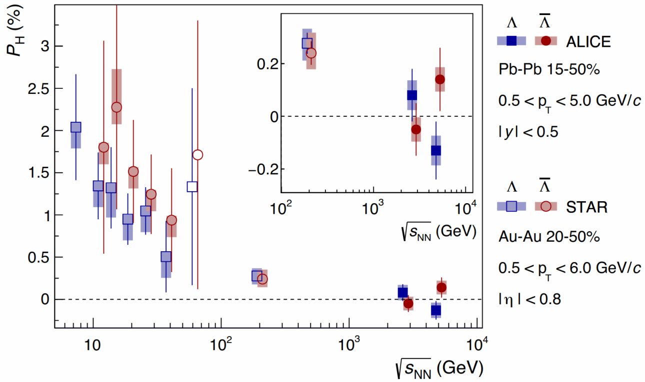



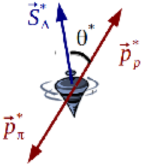




[Jiang et al. (2016)]

- **Angular momentum conservation**: Vorticity decreases with time $\rightarrow$ moment of inertia increases due to expansion.
- For $\sqrt{s} = 200$ GeV and $b = 5$ fm, $\omega \sim 10$ MeV $\rightarrow v \sim c$. System is relativistic.

# Measurement of vorticity from spin-polarization



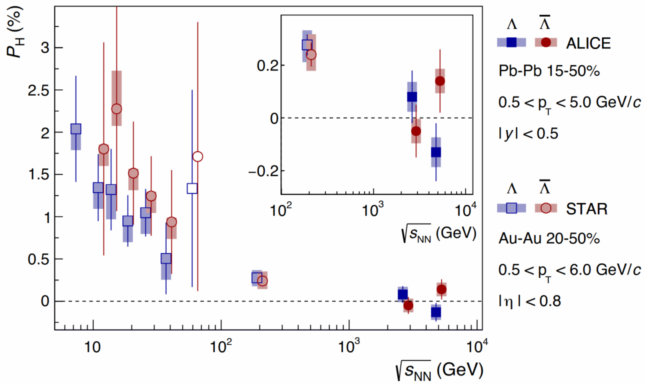

$$\Lambda \to p + \pi^+$$

- Measurement of $\frac{dN}{d\cos\theta^*} = \frac{1}{2}\left(1 + \alpha\mathcal{P}\cos\theta^*\right)$
- At **global equilibrium** $\frac{dN_s}{d\mathbf{p}} = e^{-(H_0 - \boldsymbol{\omega}.\boldsymbol{S})/T}$

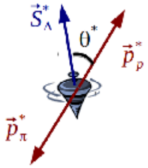
  $\mathcal{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\omega}{2T}$
- $\omega \sim (5 - 20)$ MeV

[Adamczyk et al. (2017)] [Liang and Wang (2005); Becattini et al. (2013)]

# Measurement of vorticity from spin-polarization




[Adamczyk et al. (2017)] [Becattini et al. (2013)]




$$\Lambda \to p + \pi^+$$

- Measurement of $\frac{dN}{d\cos\theta^*} = \frac{1}{2}\left(1 + \alpha\mathcal{P}\cos\theta^*\right)$
- At **global equilibrium** $\frac{dN_s}{d\mathbf{p}} = e^{-(H_0 - \boldsymbol{\omega}.\boldsymbol{S})/T}$
  $\mathcal{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\omega}{2T}$
- $\omega \sim (5 - 20)$ MeV

But what about local equillibrium ? → Spin Hydrodynamics

# Thermodynamics of Spin-hydrodynamics

# Spin hydrodynamics and spin chemical potential

Equations of motion for hydrodynamics consist of local conservation laws

- **Energy-momentum conservation**

$$\nabla_\mu T^{\mu\nu} = 0$$

- **Spin-(non) conservation**

$$\partial_\lambda S^\lambda_{\mu\nu} = 2T_{[\mu\nu]} \leftarrow \text{(Anti-symmetric part of stress-energy tensor)}$$

- Non-conservation equation for the spin current implies the existence of a **spin potential** $\mu_{ab}$, i.e. the spin analog of electric chemical potential $\mu$.

[Jensen et al. (2014); Becattini et al. (2019); Bhadury et al. (2021); Gallegos et al. (2021); Hongo et al. (2021); Weickgenannt et al. (2022)]...

# Generating functional

In the hydrostatic limit, the generating functional is given as

$$\ln Z_{\text{id}} = W_{\text{id}} = \int d^4x\, P(T, M^2, m\cdot\tilde{M}, m^2) \leftarrow \text{(Equation of state for spin-hydro)}$$

[Gallegos et al. (2021)]

where we have decomposed spin potential $\mu_{ab}$ into transverse components,

$$\mu_{ab} = 2u_{[a}m_{b]} + M_{ab}$$

analogous to the generating functional of electrically polarised matter

$$\ln Z_{\text{id}} = W_{\text{id}} = \int d^4x\, P(T, B^2, E\cdot B, E^2)$$

[Kovtun (2016)]

# Polarization ambiguities

For polarizable media, the current $J^\alpha$ is given as

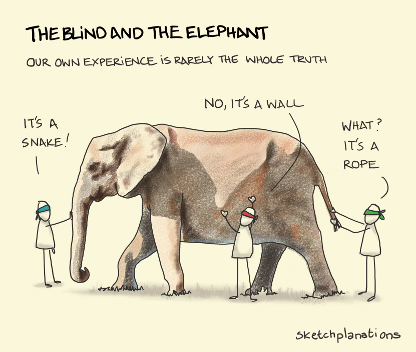
$$\mathrm{J}^\alpha = \rho u^\alpha \text{ (free charge)} - \nabla_\lambda M^{\lambda\alpha} \text{ (bound charge)}$$

[Jensen et al. (2014); Kovtun (2016)]

The second term in the right-hand side is a derivative of an anti-symmetric tensor.

$$\nabla_\alpha J^\alpha = \nabla_\alpha(\rho u^\alpha)$$

Similar terms can be added to the spin current and energy momentum tensor.




Tittha Sutta (Buddhist text), Udāna 6.4, Khuddaka Nikaya

# Generating functional

The stress tensor and current associated are given by

$$\mathsf{T}^{\mathsf{id}}_{\alpha\beta} = \epsilon u_\alpha u_\beta + P\Delta_{\alpha\beta} - 2\left(\frac{\partial P}{\partial m^2} + 4\frac{\partial P}{\partial M^2}\right) u_\alpha M_\beta{}^\gamma m_\gamma$$

$$\mathsf{S}^{\mathsf{id}}_{\lambda\,\alpha\beta} = u_\lambda \rho_{\alpha\beta} \leftarrow \text{(Spin density)}$$

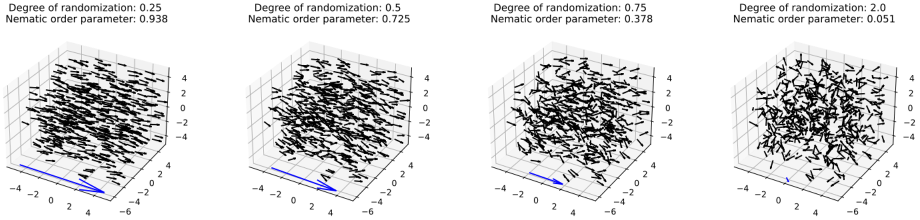
$$\epsilon = -P + \frac{\partial P}{\partial T}T + \frac{1}{2}\rho_{ab}\mu^{ab}$$

$$\rho_{\alpha\beta} = 8\frac{\partial P}{\partial M^2}M_{\alpha\beta} - 2\frac{\partial P}{\partial m^2}\left(u_\alpha m_\beta - u_\beta m_\alpha\right) + \mathcal{O}(m\tilde{M})$$

**[Gallegos et al. (2021)]**

# Analogy with other system: liquid crystals

# Liquid crystals & order parameter tensor




- Liquid crystals are compound objects, typically non-spherical, with an orientational degree of freedom.
- Prototype for uni-axial particles being rod or disc like shape.
- At high temperature, the orientation of the molecules is completely random, and the system is an isotropic liquid.
- At low temperature, the molecules start to orient in a common direction and form the nematic phase.

# Landau–de Gennes theory

CRC-TR 211

The Landau-de Gennes theory for the phase transition from isotropic $\leftrightarrow$ nematic is based on minimizing the free enrgy $\mathcal{F}$ associated with alignment.

$$\mathcal{F} = k_B T \Phi = T k_B \left( \frac{1}{2} A I_2 - \frac{1}{3} B I_3 + \frac{1}{4} C I_2^2 \right)$$

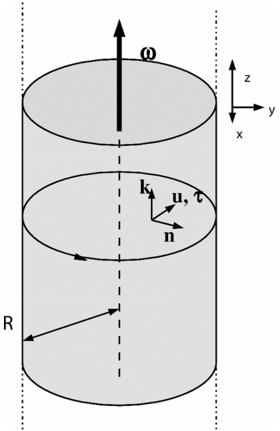
[De Gennes (1969)]

where $I_2$ and $I_3$ are scalar invariants (norm and determinant) formed from the second-rank alignment tensor $a_{\mu\nu}$.

- $a_{\mu\nu}$ is bi-axial and the order parameter of the theory.
- $A, B, C$ are phenomenological coefficients.
- One of the outcome of this theory is that equilibrium state is uni-axial and not bi-axial.

# A simple model

# Rotating fermions




*[Gibbons and Hawking (1977)]

The metric tensor is given as

$$ds^2 = dt^2 - dr^2 - r^2\left(d\theta + \Omega dt\right)^2 - dz^2$$

The Euclidean metric is given as $t \to -i\tau$

$$ds^2 = -d\tau^2 - dr^2 - r^2\left(d\theta - i\Omega d\tau\right)^2 - dz^2$$

- Space-time is stationary but not static.
- Outcome: Complex Riemannian* section instead of real Riemannian when Wick rotation is done.
- Analytic continuation $\Omega \to i\Omega$.
- We will stay with real rotation.

# NJL(Nambu–Jona-Lasino) model

The Lagrangian of the NJL-model is given as

$$\mathcal{L}_{NJL} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi + G(\bar{\psi}\psi)^2 + G_A(\bar{\psi}\gamma_{\mu}\gamma^5\psi)^2$$

and in a non-trivial background:

$$\partial_{\mu} \to D_a \equiv e_a^{\mu}\left(\partial_{\mu} - \Gamma_{\mu}\right) \qquad \Gamma_{\mu} \equiv -\frac{1}{4}\gamma^a\gamma^b e_a^{\nu}\partial_{\mu}e_{b\nu}$$

and the connection coefficients are

$$\Gamma_t = \Omega\Gamma_{\theta}\ , \quad \Gamma_{\theta} = -\frac{1}{2}\gamma^1\gamma^2\ , \quad \Gamma_r = \Gamma_z = 0$$

[Vilenkin (1980); Iyer (1982); Ambrus and Winstanley (2016); Ebihara et al. (2017); Chernodub and Gongyo (2017); Buzzegoli and Palermo (2024)]

CRC-TR 211

We rewrite the Lagrangian in terms of a scalar and axial-vector condensate and then perform the Hubbard -Stratonovich transformation leading to the effective action:

$$S_E\left[\sigma, \mathbf{s}^\mu\right] = \int d^3x \int_0^\beta d\tau \sqrt{g_E}\, \bar{\psi}(x) \left(i\gamma_E^\mu D_{E,\mu} - \sigma - \gamma_E^3 \gamma_E^5 \mathbf{s}^\mu\right) \psi(x) - \frac{V_4}{2G}(\sigma - \nu)^2 - \frac{V_4}{2G_A}\mathbf{s}^2$$

$$\langle \bar{\psi}\psi \rangle = -\frac{\sigma - m}{G}, \quad \langle \bar{\psi}\gamma^\mu\gamma^5\psi \rangle = -\frac{\mathbf{s}^\mu}{G_A}$$

# Mean-field effective potential

Resulting in the effective potential:

$$V_{eff}\left(\sigma, \mathbf{s}\right) = \frac{1}{2G}(\sigma-\nu)^2 + \frac{1}{2G_A}\mathbf{s}^2 + \frac{i}{V}\,\mathrm{Tr}\Big\{\ln\left(i\gamma_E^\mu D_{E,\mu} - \sigma - \gamma_E^3\gamma_E^5\mathbf{s}\right)\Big\}$$

where the $\mathrm{Tr}$ acts in color, flavor, Dirac and coordinate space. But how to calculate the inverse thermal propagator?

$\left(i\gamma_E^\mu D_{E,\mu} - \sigma - \gamma_E^3\gamma_E^5\mathbf{s}\right) \;= ?$ use Ritus method [Ritus (1972)]

# Ritus method

Fourier-like method that uses eigenfunction matrices $E_p^l(x)$ to diagonalize the propagator

$$E_p^l(x) = e^{i(p_z z + l\theta + \tau\omega_n)} \times \mathrm{diag}\left(J_-(\tilde{x}), J_+(\tilde{x}), J_-(\tilde{x}), J_+(\tilde{x})\right)$$

where $J_\pm(\tilde{x}) \equiv J_{l\pm\frac{1}{2}}(p_\perp r)$ are the cylindrical Bessel functions of first kind .

with this the inverse thermal propagator becomes:

$$\begin{aligned} G_l^{-1}(p,p') &= \int d^4x d^4x' \, \bar{E}_p^{\,l}(i\gamma_E^\mu D_\mu - \sigma - \gamma_E^3\gamma_E^5 \mathbf{s})\delta^{(4)}(x-x')E_{p'}^{l'} \\ &= (2\pi)^4\delta^{(4)}(p-p')\delta_{l,l'}\tilde{G}_l^{-1}(\bar{p}) \end{aligned}$$

# Effective potential

CRC-TR 211

After performing the trace and the Matsubara sum we have:

$$V_{\text{eff}} = \frac{(\sigma - m)^2}{2G} + \frac{\mathbf{s}^2}{2G_A} - \\ - \frac{N_f N_c}{(2\pi)^3} \sum_{l=0}^{\infty} \sum_{e=\pm} \sum_{s=\pm} \int p_\perp dp_\perp dp_z \left[ T \ln\left(1 + e^{-(E_s - e\Omega l)/T}\right) + E_s \right]$$

where $e = \pm$ denotes the contribution of particles/antiparticles, $s = \pm$ denotes spin up/down, and the dispersion relation is

$$E_s = \sqrt{p_\perp^2 + \left(\sqrt{p_z^2 + \sigma^2} + s|\mathbf{s}|\right)^2}$$

# Divergences and regularization

- The vacuum part is divergent and needs regularization.
- Pauli-Villars regularization has been used.

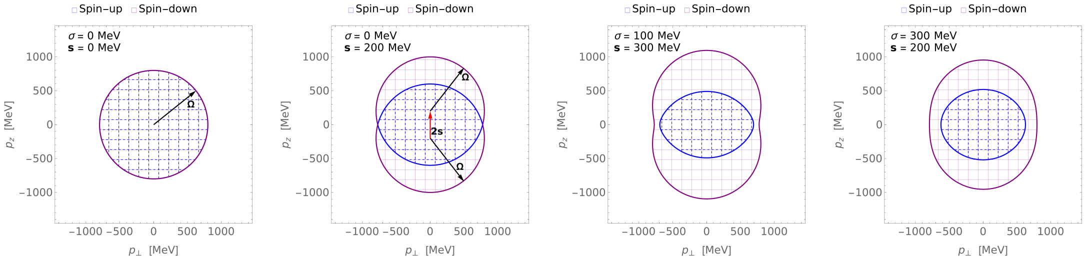
$$E_s \quad \longrightarrow \quad \sum_{j=0}^{3} c_j \sqrt{E_s^2 + j\Lambda^2}$$

- $\sum_j c_j = 0$ kills **quartic** divergences
- $\sum_j c_j m_j^2 = 0$ kills **quadratic** divergences
- $\sum_j c_j m_j^4 = 0$ kills **logarithmic** divergences
- $c_0 = 1$, others 3 coefficients are obtained by solving three equations.

# Influence of the spin condensate on the Fermi sea

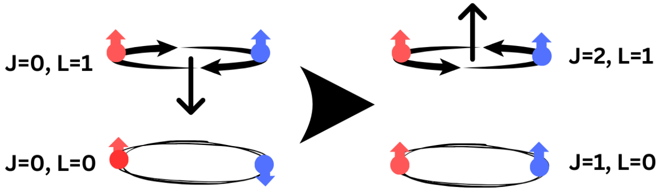
In the zero-temperature limit, the following identity is useful:

$$\lim_{T\to 0} T \ln\left(1 + e^{-(E-\mu)/T}\right) = (\mu - E)\,\theta(\mu - E)\,, \tag{1}$$




- $E_\uparrow = \frac{1}{12}\left(-s^4 + 2\Omega s^3 + \Omega^4 - 2s\Omega^3\right)$ $\quad E_\downarrow = \frac{1}{12}\left(s^4 - 2\Omega s^3 + \Omega^4 + 2s\Omega^3\right)$
- $E_\uparrow + E_\downarrow = \frac{\Omega^4}{6}$
- Thus, a nonzero spin condensate s is never energetically favored, unless the chiral condensate $\sigma$ is also non-vanishing.

# Rotational suppression



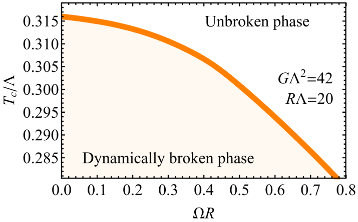
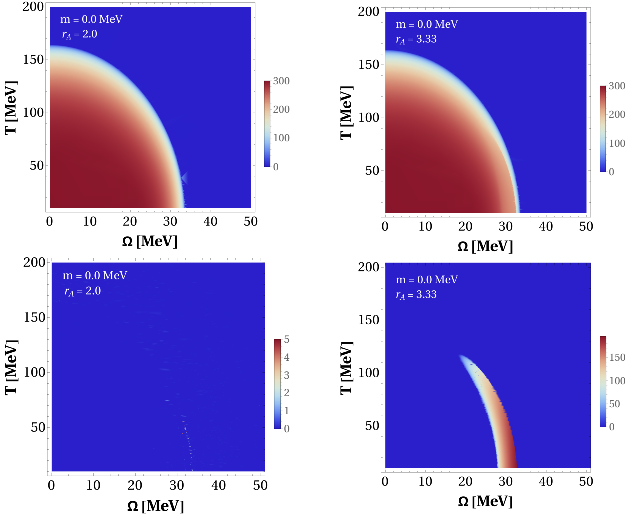




Figure: Meson supersession (left) and schematic phase diagram (right) **[Chernodub and Gongyo (2017)]**.

- Rotation typically suppresses scalar $L = 1, S = 1$ but $J = 0$ (or pseudoscalar) pair of fermions.
- However, if a spin condensate forms corresponding to quark-antiquark pairs with total angular momentum $J = 1$ , the rotation enhances the stability of such mesons.

# Phase diagram

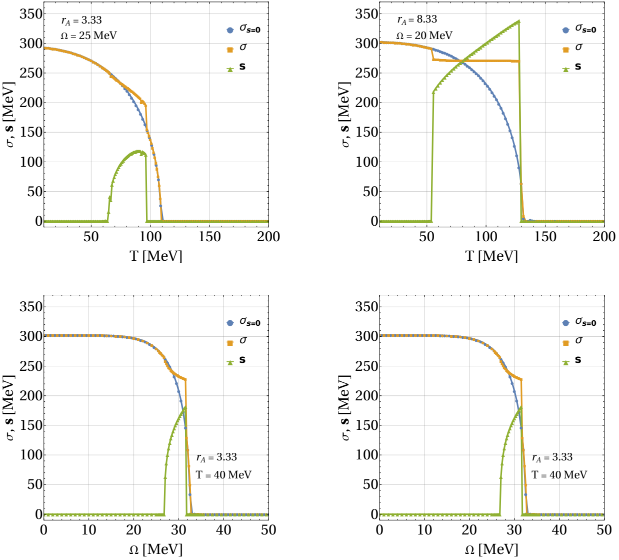



- Solve the coupled gap equations self-consistently

$$\frac{\partial V_{\text{eff}}}{\partial \sigma} \stackrel{!}{=} 0\,, \qquad \frac{\partial V_{\text{eff}}}{\partial \text{s}} \stackrel{!}{=} 0\,. \qquad (2)$$

- Define $r_A \equiv G_A/G$. $G$ is fixed by vacuum $\sigma = 300$ MeV, and $f_\pi = 88$ MeV.
- For a given $T$, $\sigma$ decreases with increasing $\Omega$.
- $T_c \simeq 165$ MeV where as $\Omega_c \simeq 33$ MeV .
- For small $r_A$, no spin condensates, but appears as we increase $r_A$.

Figure: Phase diagram of $\sigma$ (top) and s (bottom) condensates; $r_A = 2.0$ (left) and $r_A = 3.33$ (right).




- For the case with $s = 0$, the chiral transition is second order both in $T$ and $\Omega$ direction.
- For smaller $r_A$, the presence of spin condensation slightly enhances the chiral condensate.
- For larger $r_A$, spin condensate slightly reduces the value of the chiral condensate.
- The chiral transition here is noticeably steeper, resembling a first-order transition.

Figure: $T$ (top) and $\Omega$ (bottom) depend. of $\sigma$ and s for $r_A = 3.33$ (left) and $r_A = 8.33$ (right).

# Conclusion & Outlook

# Conclusion & Outlook

CRC-TR 211

- In order to understand the hydrodynamics of polarized matter subject to rotation, one needs to understand its thermodynamics first.
- The alignment of spin, driven by rotational dynamics, manifests as anisotropies in momentum space and deformations of the Fermi surface
- Secondly, the interplay between the chiral and spin condensates under rotation results in a region of non-vanishing spin condensate in the plane of temperature and angular velocity.
- In this region, we observe "rotational catalysis", albeit small and a change of the nature of the chiral phase transition from second to first order.

Thanks for your attention

Adamczyk, L. et al. (2017). Global $\Lambda$ hyperon polarization in nuclear collisions: evidence for the most vortical fluid. *Nature*, 548:62–65.

Ambrus, V. E. and Winstanley, E. (2016). Rotating fermions inside a cylindrical boundary. *Phys. Rev. D*, 93(10):104014.

Becattini, F., Chandra, V., Del Zanna, L., and Grossi, E. (2013). Relativistic distribution function for particles with spin at local thermodynamical equilibrium. *Annals Phys.*, 338:32–49.

Becattini, F., Florkowski, W., and Speranza, E. (2019). Spin tensor and its role in non-equilibrium thermodynamics. *Phys. Lett. B*, 789:419–425.

Bhadury, S., Florkowski, W., Jaiswal, A., Kumar, A., and Ryblewski, R. (2021). Relativistic dissipative spin dynamics in the relaxation time approximation. *Phys. Lett. B*, 814:136096.

Buzzegoli, M. and Palermo, A. (2024). Emergent Canonical Spin Tensor in the Chiral-Symmetric Hot QCD. *Phys. Rev. Lett.*, 133(26):262301.

Chernodub, M. N. and Gongyo, S. (2017). Interacting fermions in rotation: chiral symmetry restoration, moment of inertia and thermodynamics. *JHEP*, 01:136.

De Gennes, P. (1969). Phenomenology of short-range-order effects in the isotropic phase of nematic materials. *Physics Letters A*, 30(8):454–455.

Ebihara, S., Fukushima, K., and Mameda, K. (2017). Boundary effects and gapped dispersion in rotating fermionic matter. *Phys. Lett. B*, 764:94–99.

Gallegos, A. D., Gürsoy, U., and Yarom, A. (2021). Hydrodynamics of spin currents. *SciPost Phys.*, 11:041.

Gibbons, G. W. and Hawking, S. W. (1977). Action Integrals and Partition Functions in Quantum Gravity. *Phys. Rev. D*, 15:2752–2756.

Hongo, M., Huang, X.-G., Kaminski, M., Stephanov, M., and Yee, H.-U. (2021). Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation. *JHEP*, 11:150.

Iyer, B. R. (1982). DIRAC FIELD THEORY IN ROTATING COORDINATES. *Phys. Rev. D*, 26:1900–1905.

Jensen, K., Loganayagam, R., and Yarom, A. (2014). Anomaly inflow and thermal equilibrium. *JHEP*, 05:134.

Jiang, Y., Lin, Z.-W., and Liao, J. (2016). Rotating quark-gluon plasma in relativistic heavy ion collisions. *Phys. Rev. C*, 94(4):044910. [Erratum: Phys.Rev.C 95, 049904 (2017)].

Kovtun, P. (2016). Thermodynamics of polarized relativistic matter. *JHEP*, 07:028.

Liang, Z.-T. and Wang, X.-N. (2005). Globally polarized quark-gluon plasma in non-central A+A collisions. *Phys. Rev. Lett.*, 94:102301. [Erratum: Phys.Rev.Lett. 96, 039901 (2006)].

Ritus, V. (1972). Radiative corrections in quantum electrodynamics with intense field and their analytical properties. *Annals of Physics*, 69(2):555–582.

Vilenkin, A. (1980). QUANTUM FIELD THEORY AT FINITE TEMPERATURE IN A ROTATING SYSTEM. *Phys. Rev. D*, 21:2260–2269.

Weickgenannt, N., Wagner, D., Speranza, E., and Rischke, D. H. (2022). Relativistic second-order dissipative spin hydrodynamics from the method of moments. *Phys. Rev. D*, 106(9):096014.