Networks in Physics and Geometry: Spectral, Exponential, and Beyond

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Laboratoire de Mathématiques d'Orsay

Programme Scientifique

Spectral and exponential networks

Spectral Networks

Spectral networks, introduced by Gaiotto, Moore, and Neitzke, are networks of trajectories on surfaces that arise in the study of 4D N=2 quantum field theories. Furthermore, they establish a geometric link between flat nonabelian connections on a Riemann surface and flat abelian connections on a branched cover (the spectral curve). Additionally, they are deeply connected to the theory of ordinary differential equations through WKB analysis and the study of Stokes phenomena.

Exponential Networks

Exponential Networks are a variant of Spectral Networks. They arise in the study of local mirror symmetry. On the gauge theory side, they encode information about 5D N=1 quantum field theories. These networks are conjectured to provide local coordinates on cluster varieties arising from cluster integrable systems introduced by A. Goncharov and R. Kenyon. Moreover, they are conjecture to provide Stokes lines for difference equations.

Higher-rank Teichmüller theory

Higher-rank Teichmüller theory generalizes classical Teichmüller spaces—which correspond to distinguished connected components of principal PSL(2,R) bundles over smooth surfaces—to analogous components of principal G-bundles, where G is a real Lie group. Spectral networks play a fundamental role in Higher-rank Teichmüller theory through the non-abelianization maps they define. These maps provide local coordinate charts on moduli spaces of flat G-connections.

Topological Recursion

Topological recursion is a universal recursive formalism that computes invariants from a spectral curve—typically a plane algebraic curve endowed with additional data. Originating in matrix models, it applies broadly to problems in geometry, physics, and knot theory. It produces a tower of multidifferentials w_g,n encoding enumerative and quantum information.
Topological recursion captures the semiclassical and quantized structure of the associated curves. It offers a bridge between the geometry of networks, the behavior of BPS states, and the difference or differential equations arising from quantization.

Knot Theory

M. Aganagic and C. Vafa showed that the quantized augmentation polynomial of a knot annihilates the colored HOMFLY-PT polynomial. Exponential networks offer a powerful tool for studying the resulting difference equations, shedding new light on the structure of knot invariants and their quantization.

Resurgence

Resurgence provides a framework for understanding the full nonperturbative content of asymptotic expansions arising in quantum field theory and geometry. In the context of spectral and exponential networks, and their associated difference or differential equations, resurgence links the perturbative topological recursion data to nonperturbative effects, such as Stokes phenomena and BPS jumps. It reveals how different sectors of the theory—perturbative, nonperturbative, and resummed—are interconnected, offering a unified view of quantum curves, and wall-crossing structures.