

## Maximal Hilbert functions of Artinian quotients of a product ring

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Given a field  $k$  and a graded  $k$ -algebra  $A$ , let  $|\mathbb{F}\Psi^h_A$  and  $|\mathbb{H}\Psi^h_A$ , be the schemes parameterizing filtered quotients and graded quotients of  $A$  with Hilbert function  $h$ . Let  $|\mathbb{F}\Psi^{\{h,t\}}_A$  and  $|\mathbb{H}\Psi^{\{h,t\}}_A$  be their subschemes of Artinian quotients of socle type  $t$ .

In 1984, Iarrobino proved that, if  $k$  is infinite, if  $A$  is a polynomial ring, if  $t$  is permissible in a certain sense, and if  $h = h^A$  where

$h^A(p) := \min\{a(p), \sum_{q>0} t(q)a(q-p)\}$  and  $a(i) := \dim A_i$ , then  $|\mathbb{F}\Psi^{\{h,t\}}_A$  is an affine space bundle over  $|\mathbb{H}\Psi^{\{h,t\}}_A$ , and  $|\mathbb{H}\Psi^{\{h,t\}}_A$  is nonempty, irreducible and covered by open subschemes, each isomorphic to  $\mathbb{A}^N$  with  $N$  explicit. For any  $A$ , there's a similar maximal  $h$ , but it's not necessarily equal to  $h^A$ .

In this talk, we analyze the case where  $A := S \times T$  and  $h \neq h^A$ . When  $S := k[x]$ , a polynomial ring in one variable, we prove that  $|\mathbb{F}\Psi^{\{h,t\}}_A$  and  $|\mathbb{H}\Psi^{\{h,t\}}_A$  are close to be as nice as when  $h = h^A$ . In 2001, Cho and Iarrobino gave such examples with  $T := k[y,z]/(z^5)$  in the graded case. The new work described here is joint work with Steve Kleiman.

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