

Maximal Hilbert functions of Artinian quotients of a product ring

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Given a field k and a graded k -algebra A , let $|F\Psi^h|_A$ and $|H\Psi^h|_A$, be the schemes parameterizing filtered quotients and graded quotients of A with Hilbert function h . Let $|F\Psi^{\{h,t\}}|_A$ and $|H\Psi^{\{h,t\}}|_A$ be their subschemes of Artinian quotients of socle type t .

In 1984, Iarrobino proved that, if k is infinite, if A is a polynomial ring, if t is permissible in a certain sense, and

if $h = h^I$ where

$h^I(p) := \min\{a(p), \sum\{q>0\} t(q)a(q-p)\}$ and $a(i) := \dim A_{-i}$,

then $|F\Psi^{\{h,t\}}|_A$ is an affine space bundle over $|H\Psi^{\{h,t\}}|_A$, and

$|H\Psi^{\{h,t\}}|_A$ is nonempty, irreducible and covered by open subschemes, each isomorphic to $/A^N$ with N explicit. For any A , there's a similar maximal h , but it's not necessarily equal to h^I .

In this talk, we analyze the case where $A := S \times T$ and $h \neq h^I$. When $S := k[x]$, a polynomial ring in one variable, we prove that $|F\Psi^{\{h,t\}}|_A$ and $|H\Psi^{\{h,t\}}|_A$ are close to be as nice as when $h = h^I$. In 2001, Cho and Iarrobino gave such examples with $T := k[y,z]/(z^5)$ in the graded case. The new work described here is joint work with Steve Kleiman.

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