SUMMER SCHOOL "OPERATOR ALGEBRAS : APPROXIMATION, RIGIDITY AND DYNAMIC"

CARGESE, SEPTEMBER 14 – SEPTEMBER 18

1. Shirly Geffen: C*-Algebras Arising from Dynamical Systems

This minicourse explores key dynamical properties that play a central role in current research on crossed product C*-algebras. We begin by examining how dynamical comparison can be established for minimal actions of the integers on compact metrizable spaces, following work of Petr Naryshkin. We then turn to amenable actions of free groups on compact metrizable spaces, highlighting the fundamentally different techniques required in this setting. In particular, while actions of amenable groups (such as $\mathbb{Z}$) always admit invariant measures, amenable actions of non-amenable groups (such as free groups) do not admit invariant measures.

We will discuss how dynamical comparison informs structural properties in C*-algebra theory. This includes an overview of the anticipated dichotomy between stable rank one (i.e., density of invertible elements) and pure infiniteness in crossed product C*-algebras, depending on the presence or absence of invariant measures.

We will conclude with results establishing stable rank one in the amenable setting, as shown by Li and Niu, and more recent generic results for certain classes of free group actions on the Cantor set, due to Bell, Geffen, and Kerr. We will also highlight several open questions in this area.

2. Yves Benoist: Arithmeticity of discrete groups

The topic of this course is the discrete subgroups of higher rank semisimple Lie groups. We will discuss a criterion that ensures that such a subgroup is arithmetic. This criterion, obtained with Sébastien Miquel in 2020, extends previous works of Raghunathan, Venkataramana, Selberg and Hee Oh and solves a conjecture of Margulis. Most of the lectures will focus on concrete examples like the special linear groups in small dimension and their products.

3. Amine Marrakchi: Ultrapower methods in von Neumann algebras

The notions of central sequences and ultrapowers play a central role in the study of operator algebras and their automorphisms. This mini-course will be an introduction to ultrapower techniques for tracial von Neumann algebras. By combining this techniques with an averaging argument due to Haagerup, we will give simple and unified proofs of various classical results (such as Connes' theorems on property Gamma and approximately inner automorphisms) as well as some recent new applications. If time allows, I will also discuss how this methods can be extended to type III von Neumann algebras.

4. William Slofstra: Nonlocal games and finite-dimensional approximations

Nonlocal games are simple cooperative game used in quantum information to study the power of entanglement. They've attracted attention in operator algebras due to their use in the resolution of the Connes embedding problem, via the MIP*=RE theorem. In this course, I'll give an overview of nonlocal games, covering the following topics:

(1) The origins of nonlocal games in physics, and applications in quantum information like entanglement certification and device independent testing.
(2) Connections with proof systems in computer science, including the PCP theorem and the parallel repetition theorem, and complexity classes like MIP*.
(3) Efforts to characterize the optimal strategies for nonlocal games, e.g. synchronous games and algebras, as well as special cases like linear system games.
(4) Connections with finite-dimensional approximations of algebras and groups.
(5) An overview of the MIP* = RE theorem, including a key concept, compression.