Journée d'équipe Mathématiques Physiques

Institut de Mathematiques de Bourgogne

May, 14, 2025

Contents

Abstracts

Mathematical Study of a Quasi-linear Schrödinger-Type Equation

Meriem Bahhi

In this talk, I will present recent results on a quasilinear Schrödinger equation. After establishing the existence and uniqueness of the positive radial solution, which depends on the model parameter, we will discuss its behavior near the critical values of this parameter .

Quasi-normal mode expansion of black hole perturbations: a hyperboloidal Keldysh approach

Jérémy Besson

Evolution systems driven by non-selfadjoint operators present certain genuinely dissipative dynamical features, absent in conservative systems. In a gravitational setting, the choice of a hyperboloidal foliation for the description of wave dynamics on a background spacetime with outgoing boundary conditions (e.g. black holes), casts the (linear) dynamics in terms of an evolution problem whose infinitesimal time generator is a non-selfadjoint operator. A general framework for non-conservative systems is given in terms of i) the spectral theory of non-selfadjoint operators and ii) the non-modal analysis associated with non-normal dynamics. They provide, respectively, complementary spectral and time-domain descriptions of non-conservative dynamics, sharing the key feature of being defined for normalisable states in a Hilbert (Banach) space. We focus here on two genuinely non-selfadjoint problems, illustrating this spectral/time-domain complementarity, namely: a) the construction of asymptotic resonant expansions and b) the assessment of dynamical linear transients. Regarding a), the so-called Keldysh expansion of the resolvent permits to express time-evolved fields as quasinormal mode expansions, with quasinormal frequencies characterised as eigenvalues of the non-selfadjoint operator. As a result, we provide general closed expressions for the spectral decomposition of the field that generalise (in an asymptotic sense) the standard (spectral-theorem) projection scheme in self-adjoint problems. Regarding b), and making use of Sobolev norms weigthing high-derivatives to assess the regularity/small-scale structure of solutions, we demonstrate the presence of an initial growth in the Sobolev-norm of evolving (linear) fields, indicating an initial transitory enhancement of the "small scale/regularity”, akin to the Aretakis instability but not restrained to extremality.

On the Distinction of Quantum States through Local Unitary Invariants

Johann Chevrier

In this presentation, we explore the structure of multipartite quantum entanglement through the lens of local unitary (LU) invariants. We begin with the bipartite case, where entanglement entropy admits a clear LU-invariant characterization via the Schmidt decomposition. We then extend the discussion to multipartite systems, where no canonical notion of spectrum exists, and the classification problem becomes significantly more intricate. Motivated by the problem of distinguishing quantum states belonging to different LU orbits, we highlight how LU-invariant polynomials, represented by colored graphs, provide a rich algebraic and combinatorial framework to define entropy-like quantities. Building on recent geometrical quantities, we introduce a tree-based construction of these invariants, and analyze their ability to differentiate between inequivalent states. We conclude by discussing potential connections with holography, where multipartite entropies might admit dual interpretations as geometric cuts in bulk space.

Simulation of Serre–Green–Naghdi and Singularities

Théo Gaudry

The Serre–Green–Naghdi (SGN) equations model the propagation of nonlinear and dispersive shallow water waves. This talk will present numerical methods designed to improve the accuracy and stability of SGN simulations, with a particular focus on capturing the formation of singularities

---

TBA

Ouneïs Gloton

Bihamiltonian Formulation of the Whitham Hierarchy

Dimitrios Makris

The genus-zero Whitham hierarchy, originally introduced by Krichever in 1994, is a family of dispersionless evolutionary PDEs for an infinite number of dependent variables, which arises from the Whitham averaging method. It is an extension of the dispersionless KP hierarchy and is related, by reduction, to the dispersionless limit of several important integrable systems, such as the KdV, Gelfand–Dickey and constrained KP hierarchies. Using the R-matrix techniques originally developed by Semenov-Tian-Shansky et al., we construct a family of bihamiltonian structures on the algebra of (m+1)-tuples of formal Laurent series, with an R-matrix associated with a splitting of this algebra in two subalgebras. By Dirac reduction, we obtain an infinite family, indexed by m+1 positive integers, of bihamiltonian structures for the Whitham hierarchy. Finally, we show how these bihamiltonian structures correspond to the flat metric and intersection form of a recently introduced family of infinite-dimensional Frobenius manifolds, thereby providing an explicit bihamiltonian formulation of their principal hierarchies.

Classification and stability of black hole event horizon births

Oscar Meneses-Rojas

A classical result by Penrose establishes that null geodesics generating a black hole event horizon can only intersect at their entrance to the horizon in "crossover" points forming the "crease set". Such "crease sets" finish at points where light rays focus on "caustics". This can be paraphrased by saying that black hole horizons "are born" at their crease set. A natural question in this context refers to the classification and stability of crease sets, that is, of the structural possibilities in which black holes can be born. In this talk I will revisit some basic concepts of the so-called black hole event horizon and the "crease set" but in the setting of singularity theory in contact geometry. Specifically, in such contact geometry framework, the black hole event horizon is identified as a component (not connected to null infinity) of a so-called "bigfront". The characterisation of bigfronts as Legendrian projections of Legendrian submanifolds of appropriate contact manifolds permits to classify the possible crease sets and "cuspidal sets" (the latter corresponding, in the contact geometry terminology, to Penrose's caustics) by applying classical results established by the school of V.I. Arnol'd. This analysis permits to establish the causal connection with null infinity of the other (connected) components of the crease set and (most of) the cuspidal set of the entire bigfront. Such visibility of "caustics" by distant observers invites to consider potential observational signatures of black hole births.

New systems of log-canonical coordinates on the moduli space of Riemann surfaces M_g

Jordi Pillet

In this presentation I would like to introduce new systems of log-canonical coordinates on the moduli space of unpunctured Riemann surfaces M_g. These systems of coordinates correspond to mixtures of Fenchel-Nielsen twist-length coordinates with Thurston-Fock-Goncharov shear coordinates. The relationship with classical Fenchel-Nielsen coordinates will be also discussed as well as potential extension to higher Teichmüller theory.

Perturbed Schwarzschild black holes: towards background integrable structures in an hyperboloidal approach

Corentin Vitel

Integrability structures are known to play a key role in one- dimensional scattering. In the context of perturbed Schwarzschild black holes (BHs), Lenzi & Sopuerta recently showed that Einstein equations decouple into wave-like master equations which split into two branches connected via so-called Darboux transformations. These master equations exhibit an infinite number of isospectral symmetries governed by the flow of the infinite hierarchy of Korteweg-de Vries (KdV) equations, thus paving the way for identifying inte- grable structures within BH perturbation theory. At this stage, the discussion holds on Cauchy slices. In a second step, we move on to hyperboloidal folia- tions, where the master equations become neatly separated between bulk and asymptotic contributions. The KdV structures in the bulk are well conserved, although the complete picture should now also include asymptotic integrable structures.

A construction of Reshetikhin-Turaev invariants via factorization homology and microcosm principle

Hidir-Deniz Yeral

Reshetikhin-Turaev invariants form an important class of closed oriented three-manifold invariants that is part of a three-dimensional topological field theory. Factorization homology is a homology theory for topological manifolds in the sense that it satisfies an analogue of Eilenberg-Steenrod axioms for homology theories. In this talk, I will give a description of Reshetikhin-Turaev invariants in terms of factorization homology and a principle from categorical algebra called the microcosm principle.

Spacetime with a helical Killing vector modeling binary black holes in bispherical coordinates

El Mehdi Zejly

The concept of black holes has evolved significantly, from early theoretical ideas to their formal description within Einstein's theory of General Relativity. Schwarzschild's solution was the first exact solution describing a non-rotating, spherically symmetric black hole, marking a pivotal advancement in their understanding. A central tool in analyzing symmetries in spacetime is the Killing vector. Of particular interest is the helical Killing vector, which combines time-translation and rotational symmetries. This type of symmetry is especially relevant when describing stationary binary systems in general relativity. In the presence of a Killing vector, the Ernst formalism allows a simplification of Einstein's field equations, reducing them to a more tractable form. This formalism is instrumental in exploring spa- cetimes with such symmetries, including those that may represent binary black holes. A spacetime admitting a helical Killing vector can serve as an approximation for a binary black hole system where the incoming gravitational radiation from infinity precisely balances the outgoing radiation. This idealized configuration represents a stationary spacetime, and while it does not capture the full dynamical evolution, it can be a valid approximation over a single orbital period when the black holes are widely separated. To describe such systems, bispherical coordinates are introduced. These coordinates offer a natural global framework for covering both black hole horizons. Their geometric properties make them parti- cularly suited to modeling binary configurations. Existing solutions, such as the double Kerr solution in Weyl coordinates, offer some insight but come with limitations. These solutions often neglect rotational effects and rely on non-physical constructs such as massless struts to maintain equilibrium. Current work involves analyzing such solutions in bispherical coordinates, both analytically and numerically, to better understand gauge choices and construct more realistic models in the presence of a helical Killing vector.
