

Small-N expansion in the $O(N)$ model: constructive field theory and transseries

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Transseries are an improved version of asymptotic series that include both perturbative and nonperturbative contributions to a given problem. They play an important role in the theory of resurgence, that was developed by Jean Ecalle in the context of dynamical systems. It has been conjectured that also perturbative series in quantum field theory (QFT) can be upgraded to resurgent transseries, but proving it in a nontrivial QFT remains an open challenge. On the other hand, the study of the Borel summability of perturbative series in QFT is the subject of constructive field theory, the branch of theoretical physics devoted to rigorously proving that QFTs exist and satisfy the Osterwalder-Schrader axioms. It is somewhat surprising that resurgence in QFT and constructive field theory have so far remained two essentially separated fields of research. In this talk, I will present a first small step in trying to bridge these two areas of research. In order to do that, we have considered a trivial and extensively studied toy model: a zero-dimensional QFT with quartic interaction and $O(N)$ global symmetry. Although most of the results are not new, the main point is to show that it is possible to obtain them by means of a constructive expansion due to Vincent Rivasseau, the so-called loop vertex expansion (LVE), that is at least in principle applicable also in genuine higher-dimensional QFT. The LVE is a repackaging of the perturbative series into a convergent expansion, and in the case of the $O(N)$ model it can be interpreted as a small-N expansion. Using the Brydges-Kennedy-Abdesselam-Rivasseau forest formula, one can then prove rigorous results about the convergence domain of the free energy of the model in the LVE representation. Lastly, the latter allows to derive its transseries expansion. [Based on work with R. Gurau, H. Kepler, D. Lettera]

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