

HYPERBOLIC PROBLEMS: A COMPREHENSIVE APPROACH

Conference on the occasion of
Christian Klingenberg's 70th birthday

Würzburg, 23–27 March 2026

Book of Abstracts

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1 List of all talks in alphabetical order

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1.1 Invited talks

- Rémi Abgrall: *Recent development on Active flux methodology*
- Christophe Berthon: *High-order finite volume schemes and entropy inequalities*
- Walter Boscheri: *Structure preserving semi-implicit schemes for ideal MHD*
- Praveen Chandrashekarappa: *Continuous Galerkin method for compressible flows*
- Gui-Qiang G. Chen: *Some Inverse Problems for Shock Wave Control in Compressible Fluid Flows*
- Bruno Despres: *On a notion of differentiability for Neural Networks*
- Michael Dumbser: *A simple and general framework for the construction of exactly div-curl-grad compatible discontinuous Galerkin finite element schemes on unstructured simplex meshes*
- Lukas Einkemmer: *SN-like dynamic low-rank schemes for thermal radiative transfer*
- Eduard Feireisl: *Well posedness of the Euler system of gas dynamics*
- Elena Gaburro: *High order direct ALE schemes on 1D, 2D and 3D Voronoi-like tessellations with topology changes*
- Philippe Helluy: *Random ALE scheme for compressible two-fluid flows. Application to breaking wave simulation.*
- Christiane Helzel: *Fully discrete Active Flux methods for the Euler equations*
- Shi Jin: *Quantum Computation of Hamilton-Jacobi equations: multivalued and viscosity solutions, and Quantum Scientific Computing Platform “UnitaryLab”*
- Qin Li: *Multiscale Inverse Problems: From Helmholtz to Boltzmann to Calderón*
- Maria Lukacova: *Dafermos entropy criterion and dissipative solutions for the Euler equations*
- Carlos Parés: *Local-Flux High-order well-balanced WENO finite-difference methods for hyperbolic systems with source terms*
- Gabriella Puppo: *Models for mixtures of gases, from the kinetic to the macroscopic setting*
- Nils Henrik Risebro: *Filtered scalar conservation laws*
- Wolfram Schmidt-Brückner: *Small-Scale Turbulence in Astrophysics: From Neutron Stars to Galaxies*
- Chi-Wang Shu: *Oscillation-free discontinuous Galerkin method for conservation laws*

- Eric Sonnendrücker: *Geometric Particle In Cell method with AMR*
- Volker Springel: *Exascale computing and magnetic fields – Still a challenge for cosmological simulations of structure formation*
- Min Tang: *Random ordinate method for kinetic models*
- Yinhua Xia: *Well-balanced nodal finite volume WENO scheme with flux localization for hyperbolic balance laws*

1.2 Contributed talks

- Jan Bartsch: *A Monte Carlo framework for optimal control problems governed by multi-species plasma models*
- Philipp Birken: *A well-balanced low Mach solver for atmospheric flows*
- Peter Frolkovič: *Compact implicit numerical schemes for hyperbolic balance laws*
- D. Chloe Griffin: *A Robust, Conservative, High-Order Positivity-Preserving Sweeping Procedure for Compressible Flows*
- Kathrin Hellmuth: *Kinetic PDEs and Inverse Problems: recovering the chemotaxis kernel*
- Giovanni Leidi: *A finite volume method for low-Mach MHD flows in stellar interiors*
- Mengni Li: *Estimating lifespan of solutions for MHD*
- Qianfeng Li: *Global-in-time piecewise smooth solutions for a hypersonic piston model with large velocity variations*
- Nikhil Manoj: *A convergent MUSCL–Hancock scheme for nonlocal conservation laws*
- Atul Kumar Verma: *Hyperbolic Transport Equations in the Totally Asymmetric Simple Exclusion Process*
- Yi Wang: *The time-asymptotic stability of Riemann profiles*
- Shayan Zahedi: *Discrete wave turbulence of a coupled system of quintic Schrödinger equations*
- Enrico Zampa: *An Asymptotic-Preserving and Exactly Mass-Conservative Semi-Implicit Scheme for Weakly Compressible Flows Based on Compatible Finite Elements*

2 Abstracts

2.1 Monday, 23 March 2026, morning

Nils Henrik Risebro (University of Oslo)

Title: Filtered scalar conservation laws

Abstract: We study a class of multi-dimensional non-local conservation laws of the form $\partial_t u = \operatorname{div}^\Phi \mathbf{F}(u)$, where the standard local divergence (div) of the flux vector $\mathbf{F}(u)$ is replaced by an average upwind divergence operator div^Φ acting on the flux along a continuum of directions given by a reference measure and a filter Φ . The non-local operator div^Φ applies to a general non-monotone flux \mathbf{F} , and is constructed by decomposing the flux into monotone components according to wave speeds determined by \mathbf{F}' . Each monotone component is then consistently subjected to a non-local derivative operator that utilizes an anisotropic kernel supported on the “correct” half of the real axis. We establish well-posedness, derive a priori and entropy estimates, and provide an explicit continuous dependence result on the kernel. This stability result is robust with respect to the “size” of the kernel, allowing us to specify Φ as a Dirac delta δ_0 to recover entropy solutions of the local conservation law $\partial_t u = \operatorname{div} \mathbf{F}(u)$ (with an error estimate). Other choices of Φ (and the reference measure) recover known numerical methods for (local) conservation laws. This work distinguishes itself from many others in the field by developing a consistent non-local approach capable of handling non-monotone fluxes. (joint work with G.M. Coclite and K. Karlsen)

Christophe Berthon (Nantes Université)

Title: High-order finite volume schemes and entropy inequalities

Abstract: This work concerns the numerical approximations of the weak solutions of scalar hyperbolic conservation laws. After showing how to bypass the discrete entropy inequality barrier theorems for the linear advection, the derivation of a second-order entropy-satisfying scheme is presented for non-linear equations. The fully discrete stability result is established for regular strictly convex entropy and under a parabolic CFL-like condition. Some numerical experiments are done to assess the accuracy and the stability of the proposed scheme.

Peter Frolkovič (Slovak University of Technology)

Title: Compact implicit numerical schemes for hyperbolic balance laws

Abstract: We present well-balanced implicit numerical schemes for solutions of some hyperbolic balance laws. The schemes have compact stencils that enable applications of fast iterative algebraic solvers. They are provably unconditional stable for some problems and non-oscillatory with applications of tailored limiters. This is a joint work with Michal Žeravý (Slovak University of Technology) and Carlos Parés (Universidad de Málaga).

Bruno Despres (Sorbonne University)

Title: On a notion of differentiability for Neural Networks

Abstract: Neural Networks (ML, AI, ...) currently have tremendous success, even if rigorous mathematical formalisation is not reached yet. I will focus on a notion of differentiability which is convenient for the description of functions encoded in such structures. The coupling with discrete hyperbolic equations is one example for which such a notion is needed.

Shi Jin (Shanghai Jiao Tong University)

Title: Quantum Computation of Hamilton-Jacobi equations: multivalued and viscosity solutions, and Quantum Scientific Computing Platform “UnitaryLab”

Abstract: Quantum computers are designed based on quantum mechanics principle, they are most suitable to solve the Schrodinger equation, and linear PDEs (and ODEs) evolved by unitary operators. Nonlinear PDEs are difficult due to the no-cloning theorem. In this talk we will show how to construct quantum algorithms for (nonlinear) Hamilton-Jacobi equations. Such equations develop singularities (caustics) in finite time even if the initial data are smooth. Beyond the caustics two possible solutions could be introduced: multivalued solutions which arise in geometric optics, high frequency or semiclassical limits of linear waves, etc, and viscosity solutions which arise in optimal control, level set methods for front propagations, etc. For multivalued solution our quantum algorithm is based on the level set method, while for viscosity solution it is based on an entropy penalization method. We will also introduce a quantum scientific computing platform—“UnitaryLab”, which is a AI-powered software for quantum algorithms for scientific computing problems.

2.2 Monday, 23 March 2026, afternoon

Volker Springel (Max Planck Institute for Astrophysics)

Title: Exascale computing and magnetic fields – Still a challenge for cosmological simulations of structure formation

Abstract: Hydrodynamical simulations of galaxy formation have become a powerful tool in astrophysics. Starting right after the Big Bang, they predict the dark matter backbone of the cosmic web far into the non-linear regime and follow complex galaxy formation physics with constantly improving fidelity. In my talk, I will review some of the joint efforts Christian Klingenberg and I have started to undertake a decade ago in our ‘Examag’ project, an effort to advance numerical algorithms for magnetohydrodynamics on highly adaptive moving meshes, and to make them suitable for exaflop supercomputers. In particular, we constructed interesting new schemes based on Powell cleaning and Discontinuous Galerkin methods in our joint work, helping to create the basis for successful simulation projects such as IllustrisTNG. I will also relate this work to current developments and challenges in the field.

Giovanni Leidi (Heidelberg Institute for Theoretical Studies)

Title: A finite volume method for low-Mach MHD flows in stellar interiors

Abstract: I present a finite volume method optimized for low-Mach ideal magnetohydrodynamic flows in stellar interiors. The scheme employs well-balancing techniques to preserve magnetohydrostatic equilibria, allowing small-amplitude dynamics superimposed on steep background stratifications to be captured accurately. Specialized Low-Mach Riemann solvers are used to reduce the excessive amount of numerical dissipation typical of shock-capturing schemes for highly subsonic flows. The solenoidal constraint on the magnetic field is enforced through a staggered constrained-transport formulation. To account for the relevant physical processes in stellar interiors, the proposed method also accommodates a general equation of state, incorporates nuclear reaction networks as source terms, and includes radiative diffusion treated by super time stepping.

Qianfeng Li (Friedrich-Alexander-Universität Erlangen-Nürnberg)

Title: Global-in-time piecewise smooth solutions for a hypersonic piston model with large velocity variations

Abstract: We study a piston moving into a rarefied gas. As the initial density tends to zero, the piston approaches the leading shock front, while both strict hyperbolicity and shock dissipation degenerate. Using asymptotic analysis, we derive quantitative estimates for these effects and clarify a stabilization mechanism in the model.

2.3 Tuesday, 24 March 2026, morning

Eduard Feireisl (Czech Academy of Sciences)

Title: Well posedness of the Euler system of gas dynamics

Abstract: We discuss the well posedness of the Euler system of gas dynamics from two different perspectives:

1. Admissible solutions are inviscid limits of the Navier-Stokes-Fourier system.
2. Admissible solutions comply with the maximum entropy production principle.

We show that these two criteria are in general not compatible and give rise to different classes of solutions.

Then we propose an admissibility criterion that gives rise to a unique solution semigroup of the Euler system in the class of generalized dissipative solutions. The solutions are Borel measurable meaning “almost” continuous with respect to the initial data.

Maria Lukacova (Johannes Gutenberg University Mainz)

Title: Dafermos entropy criterion and dissipative solutions for the Euler equations

Abstract: The dissipative solutions of the Euler equations can be obtained as a limit of suitable structure-preserving, consistent and stable numerical schemes. If the strong solution to the above equations exists, the dissipative solutions coincide with the strong solution on its life span. Otherwise, we apply the concept of K-convergence and prove the strong convergence of the empirical means of numerical solutions to a dissipative weak solution. The latter is the expected value of the dissipative measure-valued solutions and satisfies a weak formulation of the Euler equations, modulo the Reynolds turbulent stress. We will discuss the relation between numerical approximations of oscillatory solutions and selection criteria, such as the Dafermos criterion based on maximising the entropy production or the optimisation of the energy defect. A series of numerical simulations will illustrate theoretical results.

Yi Wang (Institute of Applied Mathematics, AMSS, Chinese Academy of Sciences)

Title: The time-asymptotic stability of Riemann profiles

Abstract: I will talk about our recent developments on the time-asymptotic of generic Riemann profiles, which are viscous version of generic Riemann solutions to inviscid system, including the shock profile, rarefaction wave and viscous contact discontinuity, to the barotropic/full compressible Navier-Stokes equations, Boltzmann equation and non-convex viscous conservation laws. The talk is based on the several joint works with Moon-Jin Kang, Alexis Vasseur and Feimin Huang, Jian Zhang and Qiuyang Yu.

Eric Sonnendrücker (Max Planck Institute for Plasma Physics)

Title: Geometric Particle In Cell method with AMR

Abstract: In order to deal with local mesh refinement in the geometric PIC code GEMPICX, we developed a field solver for Maxwell's equations based on patchwise discontinuous Finite Element spaces forming a discrete de Rham Complex. These broken Finite Element spaces are constructed from a conforming de Rham sequence using the Galerkin Difference concept, so that the degrees of freedom are the natural geometric degrees of freedom, i.e. points values, edge integrals, face integrals and volume integrals respectively for the four spaces of the complex.

Wolfram Schmidt-Brückner (Hamburg Observatory, University of Hamburg)

Title: Small-Scale Turbulence in Astrophysics: From Neutron Stars to Galaxies

Abstract: Turbulence in astrophysical environments naturally arises from the nonlinear, hyperbolic structure of the governing conservation laws, leading to complex, multiscale dynamics that span many orders of magnitude in space and time. Direct numerical simulation of all relevant scales are computationally intractable. Large eddy simulations (LES) offer a solution by explicitly resolving large-scale dynamics while modeling subgrid-scale (SGS) turbulent processes. In this talk, I will briefly introduce the LES methodology and discuss advances in structural SGS models, which aim to represent unresolved turbulent stresses and magnetic field dynamics in a physically consistent way. I will highlight how these models can enhance the treatment of complex sub-resolution physics, such as turbulence-regulated star formation in simulations of isolated and interacting galaxies and magnetic field amplification in binary neutron star mergers.

2.4 Tuesday, 24 March 2026, afternoon

Chi-Wang Shu (Brown University)

Title: Oscillation-free discontinuous Galerkin method for conservation laws

Abstract: We discuss a recently developed class of oscillation-free discontinuous Galerkin (OFDG) method for solving hyperbolic conservation laws. Discontinuous Galerkin methods are well-known for their nice properties for solving hyperbolic conservation laws, such as entropy stability, high order convergence, parallel efficiency and easiness for h-p adaptivity. However, for strong shocks, unmodulated discontinuous Galerkin methods still suffer from spurious oscillations. Traditional strategies to control such oscillations include artificial viscosity and various limiters. Recently, a class of oscillation-free discontinuous Galerkin method has been developed, which contains a carefully designed local damping term and ensures essentially oscillation-free performance. The OFDG method maintains the nice theoretical properties of discontinuous Galerkin methods such as entropy stability, optimal order of convergence, and superconvergence. The method has been developed for scalar equations and systems, including Euler equations and MHD equations. Numerical results demonstrate the nice performance of the method. This is a joint work with Yong Liu and Jianfang Lu.

D. Chloe Griffin (Brown University)

Title: A Robust, Conservative, High-Order Positivity-Preserving Sweeping Procedure for Compressible Flows

Abstract: High-order numerical methods for hyperbolic conservation laws lack inherent mechanisms to preserve positivity, often leading to computation failure near strong shocks or in near-vacuum states. In this talk, we present a novel, conservative sweeping procedure for density and the nonlinear pressure in a finite difference WENO numerical solution of the compressible Euler equations. Formulated as a mass-redistribution technique, this approach generalizes the Zhang-Shu scaling limiter within a sweeping framework. A key feature is modularity: the method functions as a standalone post-processing step applicable to finite difference, finite volume, and discontinuous Galerkin methods without modifying the spatial discretization. Beyond pressure in the Euler equations, the procedure extends to any concave function of conserved variables. We demonstrate its robustness using fifth-order WENO with extreme flow conditions and high Courant-Friedrichs-Lewy numbers. Finally, we discuss extensions to continuous Galerkin schemes and implicit time integration, where standard flux limiters are difficult to apply. This presentation is based on joint work with Chi-Wang Shu.

Mengni Li (Southeast University; University of Würzburg)

Title: Estimating lifespan of solutions for MHD

Abstract: This talk concerns the lifespan estimate for three-dimensional ideal incompressible MHD systems. By introducing $L^{3/2}$ -type energy, we lower the regularity assumption for the initial data as well as the order of the highest derivatives involved in the total weighted energy estimate. The new weights are symmetric, unified, and compatible with both L^2 -type energy and $L^{3/2}$ -type energy. This is based on joint work with Hongwei Yuan.

Lukas Einkemmer (University of Innsbruck)

Title: SN-like dynamic low-rank schemes for thermal radiative transfer

Abstract: Dynamic Low Rank (DLR) methods are a promising way to reduce the computational cost and memory footprint of the high-dimensional thermal radiative transfer (TRT) equations. The TRT equations are a system of nonlinear PDEs that model the energy exchange between the material temperature and the radiation energy density. Due to their high dimensionality, solving the TRT equations is often a bottleneck in multi-physics simulations. DLR methods represent the solution in terms of time-evolving SVD-like factors of angle and space. Here we present our recently developed method that uses a discrete set of time-evolving angles. This DLR formulation enables us to use the highly optimized SN transport sweep as our main computational kernel, and thus results in a practical way of leveraging low-rank methods in production TRT codes. We demonstrate the methods on several challenging, highly heterogeneous problems in two spatial dimensions (4D), where these DLR schemes can give a significant reduction in angular artifacts (i.e., ray effects) with the same cost as gold-standard SN methods.

2.5 Wednesday, 25 March 2026, morning

Qin Li (University of Wisconsin-Madison)

Title: Multiscale Inverse Problems: From Helmholtz to Boltzmann to Calderón

Abstract: Inverse problems for PDEs arise across a wide range of physical scales. At the microscopic and mesoscopic levels, one encounters inverse Helmholtz and inverse Boltzmann problems, modeling wave propagation and kinetic particle interactions. At the macroscopic level, inverse diffusion problems of Calderón type seek to recover material parameters from boundary measurements. While these problems originate in different physical regimes and involve distinct analytic structures, they exhibit a striking structural commonality. Partly motivated by the unifying vision behind Hilbert's sixth problem that examines the systematic mathematical treatment of physical theories across scales, this talk proposes a unified perspective on these multiscale inverse problems. I will argue that, despite differences in equation type, they share the same underlying mechanism, with one regime emerging asymptotically from another through appropriate scaling limits. This viewpoint reveals parallels in stability, ill-posedness, and identifiability across models, and suggests common principles underlying computational reconstruction methods.

Min Tang (Shanghai Jiao Tong University)

Title: Random ordinate method for kinetic models

Abstract: The Discrete Ordinates Method (DOM) is the most widely used velocity discretization method for simulating kinetic models. However, the ray effect is a long-standing drawback of DOM. In benchmark tests that exhibit the ray effect, we observe low regularity in the velocity variable of the solution. To address this issue, we propose a Random Ordinate Method (ROM) to mitigate the ray effect. Compared to other strategies proposed in the literature for mitigating the ray effect, ROM offers several advantages: 1) For benchmark tests that exhibit the ray effect, the computational cost is lower than that of DOM; 2) it is simple and requires minimal changes to existing DOM-based code; 3) it is easily parallelizable and independent of the problem setup. We apply the ROM to the steady-state radiative transport equation and the BGK model. Numerical tests demonstrate the reduction in computational cost compared to DOM, as well as its effectiveness in mitigating the ray effect. Moreover, the convergence orders of the error and bias are provided both numerically and theoretically.

Kathrin Hellmuth (California Institute of Technology)

Title: Kinetic PDEs and Inverse Problems: recovering the chemotaxis kernel

Abstract: Inverse problems related to kinetic PDEs seek to reconstruct their parameters, such as the absorption cross section or an interaction kernel, from experimental data. They appear in a plethora of applications, such as optical tomography or model fitting, and yield insight into microscopic behaviour of various systems. In biology, for instance, we seek to recover the parameter that drives bacterial motion in response to an external stimulus, termed chemotaxis. In this talk, we demonstrate how an analytic tool from kinetic theory not only grants insight into theoretical existence and uniqueness of the parameter reconstruction, but also informs data collection strategies that facilitate recovery in a discretized setting. This is based on joint work with Christian Klingenberg, Qin Li and Min Tang.

Gabriella Puppo (Università La Sapienza Roma)

Title: Models for mixtures of gases, from the kinetic to the macroscopic setting

Abstract: Macroscopic models for mixtures of gases are often derived from heuristic considerations. This is the case in particular of two phase flows. This talk will describe how to derive macroscopic models for mixtures from kinetic equations. The MHD equations for plasmas can be derived from a BGK mixture model, under suitable scalings, as has already been shown, from different perspectives, in several works. But an analogous approach can be used to derive macroscopic two phase flow models of Baer Nunziato type. This instead is, as far as we know, a novel approach.

Jan Bartsch (University of Würzburg)

Title: A Monte Carlo framework for optimal control problems governed by multi-species plasma models

Abstract: The Vlasov-Poisson equation describes the evolution of plasma in the so-called collisionless regime. The investigation of a high-temperature plasma that is influenced by an exterior magnetic field is one of the most significant aspects of the research on thermonuclear fusion. However, there are only a few works on optimal control problems governed by the Vlasov-Poisson. We formulate and analyse a Vlasov-Poisson kinetic optimal control problem where the control is assumed to be the external magnetic field. The purpose of this control is to drive an ensemble of particles to acquire a desired mean and variance in phase space. To characterize the optimal control, we use the widely known Lagrange multiplier framework to derive an optimality system. We build a Monte Carlo framework to solve all arising equations numerically and apply gradient-based optimization schemes to solve the optimal control problem. We perform numerical experiments that successfully validate our framework.

2.6 Thursday, 26 March 2026, morning

Gui-Qiang G. Chen (University of Oxford)

Title: Some Inverse Problems for Shock Wave Control in Compressible Fluid Flows

Abstract: Controlling shock waves is of fundamental importance in a wide range of scientific and engineering applications, including aerodynamics, aerospace engineering, explosion mitigation, and shock tube experiments. In this talk, we present several inverse problems and related stability issues arising in the control of multidimensional steady shock waves. In particular, we discuss recent progress on the analysis of inverse problems for controlling both the leading shock waves generated by wedges or conical bodies and the associated fluid flows. The objective is to design boundary geometries of wedges or cones that achieve prescribed pressure distributions and/or specified leading shock locations. We will also outline further perspectives and highlight several open problems in this direction.

Michael Dumbser (University of Trento)

Title: A simple and general framework for the construction of exactly div-curl-grad compatible discontinuous Galerkin finite element schemes on unstructured simplex meshes

Abstract: We introduce a new family of discontinuous Galerkin (DG) finite element schemes for the discretization of first order systems of hyperbolic partial differential equations (PDE) on unstructured simplex meshes in two and three space dimensions that respect the two basic

vector calculus identities exactly also at the discrete level, namely that the curl of the gradient is zero and that the divergence of the curl is zero. The key ingredient here is the construction of two compatible discrete nabla operators, a primary one and a dual one, both defined on general unstructured simplex meshes in multiple space dimensions. Our new schemes extend existing cell-centered finite volume methods based on corner fluxes to arbitrary high order of accuracy in space. An important feature of our new method is the fact that only two different discrete function spaces are needed to represent the numerical solution, and the choice of the appropriate function space for each variable is related to the origin and nature of the underlying PDE. The first class of variables is discretized at the aid of a discontinuous Galerkin approach, where the numerical solution is represented via piecewise polynomials of degree N and which are allowed to jump across element interfaces. This set of variables is related to those PDE which are mere consequences of the definitions, derived from some abstract scalar and vector potentials, and for which involutions like the divergence-free or the curl-free property must hold if satisfied by the initial data. The second class of variables is discretized via classical continuous Lagrange finite elements of approximation degree $M=N+1$ and is related to those PDE which can be derived as the Euler-Lagrange equations of an underlying variational principle. The primary nabla operator takes as input the data from the FEM space and returns data in the DG space, while the dual nabla operator takes as input the data from the DG space and produces output in the FEM space. The two discrete nabla operators satisfy a discrete Schwarz theorem on the symmetry of discrete second derivatives. From there, both discrete vector calculus identities follow automatically. We apply our new family of schemes to three hyperbolic systems with involutions: the system of linear acoustics, in which the velocity field must remain curl-free and the vacuum Maxwell equations, in which the divergence of the magnetic field and of the electric field must remain zero. In our approach, only the magnetic field will remain exactly divergence free. As a third model we study the Maxwell-GLM system of Munz et al. [1], which contains a unique mixture of curl-curl and div-grad operators and in which the magnetic field may be either curl-free or divergence-free, depending on the choice of the initial data. In all cases we prove that the proposed schemes are exactly total energy conservative and thus nonlinearly stable in the L^2 norm.

Shayan Zahedi (Ecole Polytechnique)

Title: Discrete wave turbulence of a coupled system of quintic Schrödinger equations

Abstract: We derive rigorously the macroscopic system associated to a microscopic system of coupled quintic Schrödinger equations in the framework of discrete wave turbulence under a particular scaling law that describes the limiting process. Our system evolves from a pair of well-prepared random initial data. More precisely, in dimensions $d \geq 2$, we set up our microscopic system on a large box of size L with weak non-linearity of strength ϵ . In the limit $L \rightarrow \infty$ and $\epsilon \rightarrow 0$, under the scaling law $\epsilon L^{\frac{1}{\beta}} = 1$ with $\beta \in (1, \infty)$, we prove that the long-time behaviour of our microscopic system is statistically described by a resonant system whose dynamics are driven by exact resonances. The difference between systems that are approximated by wave kinetic equations and our microscopic system is that our system does not display standard symmetries such as mass conservation. In such systems with fewer invariances, exact resonances contribute significantly compared to quasi-resonances and are essentially responsible for the effective dynamics in the large-box limit. We justify the emergence of discrete wave turbulence for our microscopic system.

Christiane Helzel (Heinrich-Heine-University Düsseldorf)

Title: Fully discrete Active Flux methods for the Euler equations

Abstract: The Active Flux method is a finite volume method for hyperbolic conservation laws which employs cell averages and point values as degrees of freedom. Several versions of this method are currently being developed, building on the pioneering work of Eymann and Roe.

We present recent results on the construction of third-order-accurate, fully discrete Active Flux methods. A crucial component of these methods is the evolution operator, which is used to update the point values. We will discuss several multi-dimensional operators and present numerical results for the two-dimensional Euler equations.

This is joint work with Erik Chudzik and Amelie Porfetye (Düsseldorf) and Maria Lukacova (Mainz).

Rémi Abgrall (University of Zurich)

Title: Recent development on Active flux methodology

Abstract: Considering an hyperbolic problem defined on a domain Ω of \mathbb{R}^d , $d = 1, 2, 3$ covered by polygons, we are interested in methods that use globally continuous approximation of the data, while being able to handle discontinuities. One possible method uses as degrees of freedom the Gauss-Lobatto points on the boundary of the polygons K , and moments in K of the solution. From this, it is possible to use a globally continuous approximation where

- when the polygon is a triangle, for an approximation of order $k + 1$, one uses an approximation space which contains \mathbb{P}^k and is contained in \mathbb{P}^{k+1} , see [1]
- when a general polygon is used by using the approximation tools defined for Virtual Finite Elements, [2], see [3].

To illustrate, let us take the example of third order of accuracy, i.e. $k = 2$. On each face of the polygon, we have 3 Gauss-Lobatto points (defining a quadratic polynomial on this face), and we need one moment, the average value. The average is updated by

$$|K| \frac{d\bar{\mathbf{u}}_K}{dt} + \int_{\partial K} \mathbf{f}(u) \mathbf{n} \, d\gamma = 0$$

which is doable (without Riemann solver) because we know \mathbf{u} on the boundary, and the point values are updated by

$$\frac{d\mathbf{u}_\sigma}{dt} + \sum_{K, \sigma \in K} \Phi_\sigma^K(u) = 0$$

The residuals are approximations of $\text{div} \mathbf{f}(\mathbf{u})$ on the polygon K which are consistent with this operator, in the sense that if the solution were linear, and the flux linear with constant Jacobian, the sum $\sum_{K, \sigma \in K} \Phi_\sigma^K(u)$ would exactly give $\text{div} \mathbf{f}(\mathbf{u})$. We have shown that this condition is essentially sufficient, together with the classical stability bounds, to get a Lax-Wendroff theorem.

This kind of schemes is within the family of active flux, initially designed by Roe and his co-worker, see [5] for an early contribution, and has since then received a lot of attention: one may mention the work of Barsukow, Klingenberg, Heltzel in Europe, Wu, Kurganov and others in China.

It turns out that this kind of scheme has a lot of connection with discontinuous Galerkin schemes. Using a Runge Kutta time discretisation, one can reinterpret the above scheme, for a linear flux with constant Jacobian as, for each Runge Kutta cycle, one step of DG followed by

a projection of the discontinuous solution onto the solution space. The projector operator can be a simple arithmetic average (at least in 1D), or a more complex one with upwinding flavor, see [6]. We also have gone further by showing how to properly describe the implementation of boundary conditions in this kind of scheme in [7].

It is well known that solutions of hyperbolic problems are generically discontinuous. This implies that some non linear stabilisation mechanism must be added. We have proposed a very simple algorithm to guaranty that the solution stays in the invariant domain in [3], and where in addition a non linear mechanism is proposed to remove spurious oscillations [7].

In the talk, we will make a coherent summary of all these findings.

REFERENCES:

- [1] Rémi Abgrall, Jianfang Lin, and Yongle Liu. Active flux for triangular meshes for compressible flows problems. *Beijing J. of Pure and Appl. Math.*, 2:1–33, 2025.
- [2] L. Beirão da Veiga, Franco Brezzi, L. D. Marini, and A. Russo. The Hitchhiker’s guide to the virtual element method. *Math. Models Methods Appl. Sci.*, 24(8):1541–1573, 2014.
- [3] Rémi Abgrall, Walter Boscheri, and Yongle Liu. Virtual finite element and hyperbolic problems: The PAMPA algorithm. *Journal of Computational Physics*, 546:114521, 2026. ArXiv 2502.10069.
- [4] R. Abgrall. A combination of residual distribution and the active flux formulations or a new class of schemes that can combine several writings of the same hyperbolic problem: application to the 1d Euler equations. *Commun. Appl. Math. Comput.*, 5(1):370–402, 2023.
- [5] T.A. Eyman and P.L. Roe. Active flux schemes. In *49th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition*, 2011. doi:10.2514/6.2011-382.
- [6] Rémi Abgrall, Philipp Öffner, and Yongle Liu. Some new properties of the PAMPA scheme, 2026. in revision for the *Journal of Scientific Computing*.
- [7] Rémi Abgrall and Yongle Liu. Robust PAMPA scheme in the dg formulation on unstructured triangular meshes: bound preservation, oscillation elimination, and boundary conditions, 2025. in revision for the *Journal of Computational Physics*.

2.7 Thursday, 26 March 2026, afternoon

Philippe Helluy (Université de Strasbourg)

Title: Random ALE scheme for compressible two-fluid flows. Application to breaking wave simulation.

Abstract: The compressible two-fluid Euler model presents no additional theoretical difficulties compared to the single-fluid case. However, its numerical resolution is notoriously more difficult due to the phenomenon of pressure oscillations at the interface between fluids. We present an approach based on a "Glimm-like" random sampling at the interface, which overcomes this drawback. The resulting scheme is applicable to unstructured meshes and exhibits excellent robustness and convergence properties. We apply it to breaking wave cases.

Nikhil Manoj (Institute of Mathematics, University of Würzburg)

Title: A convergent MUSCL–Hancock scheme for nonlocal conservation laws

Abstract: The MUSCL-Hancock scheme is a single-stage, fully discrete second-order method for conservation laws, known for its simplicity and computational efficiency compared to standard Runge-Kutta schemes. In this talk, we develop a second-order MUSCL-Hancock (MH) scheme for conservation laws with nonlocal flux functions and analyze its convergence. The main difficulty in proposing such a scheme in the nonlocal setting arises from convolution operators in the flux, which complicate the construction and analysis of higher-order schemes. We propose a novel discretization strategy for the convolution terms that preserves second-order accuracy and supports rigorous analysis. This yields uniform L^∞ bounds, bounded variation (BV) estimates, and L^1 -Lipschitz continuity in time. Together with Kolmogorov’s compactness theorem, these estimates yield convergence of a subsequence of approximate solutions to a weak solution. As discrete entropy inequalities are generally unavailable for second-order schemes, entropy convergence is nontrivial. We overcome this by introducing a mesh-dependent modification of the slope limiter and prove convergence to the entropy solution. Numerical experiments support the analysis and demonstrate improved accuracy over the corresponding first-order scheme, as well as improved efficiency compared to standard second-order Runge–Kutta methods.

Enrico Zampa (University of Vienna)

Title: An Asymptotic-Preserving and Exactly Mass-Conservative Semi-Implicit Scheme for Weakly Compressible Flows Based on Compatible Finite Elements

Abstract: Compatible finite elements—also known as Finite Element Exterior Calculus [1]—have demonstrated significant advantages in structure-preserving discretizations of incompressible flows, see e.g. [2,3]. However, their potential for simulating compressible flows remains largely unexplored, with most current approaches relying on Finite Volume (FV), Discontinuous Galerkin (DG), or Continuous Galerkin (CG) methods. In this talk, we address this gap by presenting a semi-implicit compatible finite element method for weakly compressible flows, see [4]. Our scheme has the following key properties:

- Asymptotic preservation: In the low Mach number limit with constant density, the method consistently recovers a discretization of the incompressible Navier–Stokes equations.
- Exact mass conservation: In the incompressible regime, the discrete velocity field is exactly divergence-free, owing to the use of an $H(\text{div}, \Omega)$ -conforming momentum discretization.
- Shock capturing: A MOOD-style a posteriori artificial viscosity mechanism ensures robust shock resolution.
- Computational efficiency: The method requires solving only symmetric positive definite (SPD) systems at each time step, and the CFL condition remains independent of the Mach number.

We conclude with implementation insights and present numerical results that demonstrate the accuracy, robustness, and efficiency of the proposed method.

REFERENCES:

- [1] D. N. Arnold, R. S. Falk, R. Winther Finite element exterior calculus, homological techniques, and applications. *Acta Numerica*. 2006;15:1–155

- [2] Christoph Lehrenfeld, Joachim Schoeberl, High order exactly divergence-free Hybrid Discontinuous Galerkin Methods for unsteady incompressible flows, *Computer Methods in Applied Mechanics and Engineering*, Volume 307, 2016, Pages 339-361,
- [3] A. Palha, M. Gerritsma, A mass, energy, enstrophy and vorticity conserving (MEEVC) mimetic spectral element discretization for the 2D incompressible Navier–Stokes equations, *Journal of Computational Physics*, Volume 328, 2017, Pages 200–220,
- [4] E. Zampa and M. Dumbser. An asymptotic-preserving and exactly mass-conservative semi-implicit scheme for weakly compressible flows based on compatible finite elements. *Journal of Computational Physics*, Volume: 521, 113551, 2025.

Walter Boscheri (CNRS)

Title: Structure preserving semi-implicit schemes for ideal MHD

Abstract: We present a second-order accurate semi-implicit, well-balanced finite volume scheme for the equations of ideal magnetohydrodynamics with gravitational source terms. The method treats acoustic pressure-related contributions implicitly while retaining the remaining terms in an explicit sub-system, making it particularly effective in low-Mach-number regimes. By combining the semi-implicit formulation with a deviation-based well-balancing strategy, we show that magnetohydrostatic equilibrium solutions are preserved up to machine precision. To enforce the solenoidal constraint on the magnetic field, a constrained transport technique is embedded within the semi-implicit framework. To address flows across all Mach and Alfvén numbers, we further introduce an operator-splitting approach in which convective terms are advanced explicitly, whereas hydrodynamic pressure and magnetic field effects are integrated implicitly, resulting in two decoupled linear implicit systems. Linearity of the implicit operators is achieved via semi-implicit time linearization. Finally, we design implicit cell-centered finite difference operators that discretely maintain the divergence-free condition of the magnetic field on three-dimensional Cartesian meshes, ensuring consistency and robustness of the overall scheme across a wide range of flow regimes.

Atul Kumar Verma (Indian Institute of Technology (ISM) Dhanbad, India)

Title: Hyperbolic Transport Equations in the Totally Asymmetric Simple Exclusion Process

Abstract: The totally asymmetric simple exclusion process (TASEP) is a prototypical stochastic model for one-dimensional transport phenomena. In this talk, we will present the one-dimensional TASEP and its macroscopic description in terms of hyperbolic transport equations. We present the derivation of the resulting hyperbolic conservation laws and discuss analytical properties of their solutions, including characteristic behaviour and shock formation. The hyperbolic equations are solved using suitable numerical schemes, and the results are compared with Monte Carlo simulations of the underlying microscopic TASEP. This dual numerical verification provides a consistent validation of the macroscopic model and illustrates the effectiveness of hyperbolic PDEs in describing transport dynamics in interacting particle systems.

2.8 Friday, 27 March 2026, morning

Carlos Parés (Universidad de Málaga)

Title: Local-Flux High-order well-balanced WENO finite-difference methods for hyperbolic systems with source terms

Abstract: Different strategies have been discussed in the literature to extend high-order WENO finite-difference methods to hyperbolic systems with source terms and/or non-conservative products. In "High-order WENO finite-difference methods for hyperbolic non conservative systems of PDEs" (B. Ren, C. Parés, JCP 2025), a new strategy based on the so-called Local-Flux approach has been introduced. In this framework, source terms and non-conservative products are represented as the derivative of a generalized flux function that, unlike the Global Flux approach [1], is defined locally based on the choice of a family of paths. The goal of the talk is to present this strategy together with two different techniques to design well-balanced methods. The approach is tested to different hyperbolic systems like the Burger's equations, the nonlinear shallow water equations and the two layer shallow water equations.

Yinhua Xia (University of Science and Technology of China)

Title: Well-balanced nodal finite volume WENO scheme with flux localization for hyperbolic balance laws

Abstract: In this talk, we present a general framework for constructing arbitrarily high-order well-balanced "nodal" finite volume weighted essentially non-oscillatory (WENO) schemes, specifically tailored for hyperbolic balance laws. To handle complex systems involving non-conservative products and steady-state solutions with global integral quantities, we introduce a flux localization technique. This technique defines the numerical fluxes, incorporating the entire source term and nonconservative products, locally within each computational cell. By exactly preserving the nodal equilibrium state, the method avoids both reference-state reconstruction and the use of ODE solvers. The well-balanced property is achieved by performing high-order WENO reconstruction and employing a linear segment path in terms of equilibrium variables, rather than conservative ones. A rigorous theoretical analysis confirms that the scheme exactly preserves all stationary solutions. The effectiveness and versatility of the proposed method are demonstrated through a series of numerical experiments on various systems of hyperbolic balance laws.

Philipp Birken (Lund University)

Title: A well-balanced low Mach solver for atmospheric flows

Abstract: We consider numerical methods for the dynamical core in numerical weather prediction (NWP). Thus, we are interested in schemes for the Euler equations in 3D at low Mach numbers with a gravity source term. Currently, the trend in NWP is to increase the resolution. Thus, high order Discontinuous Galerkin (DG) methods are competitive, and also well suited for High Performance Computing due to their high arithmetic intensity. Due to stiffness, implicit discretizations are needed. The design goal is thus a suitable highly parallel stable implicit DG implementation. To deal with the gravitational source term, a well-balanced method is needed, suitable for large domains. Additionally, one wants a numerical flux suitable for low Mach numbers, and some form of nonlinear stability such as entropy stability, suitable for large simulation times. Currently, Environment and Climate Change Canada (ECCC) is doing a redesign of their dynamical core along these lines in the code WxFactory. We present

the implementation of a well-balanced DG method suitable for low Mach numbers and provide numerical results showing the performance on atmospheric test cases in 2D and 3D.

Praveen Chandrashekarappa (TIFR Centre for Applicable Mathematics)

Title: Continuous Galerkin method for compressible flows

Abstract: Continuous Galerkin methods use a continuous representation of the solution using piecewise polynomials. They can suffer from too much dispersion errors when applied to wave-type problems especially when the solution has steep gradients or is not very well resolved. Some form of stabilization like SUPG/GLS/VMS etc. is usually required to make these methods less prone to such errors. However many of these methods can introduce too much artificial dissipation and have tunable parameters. For the same reasons, convection dominated problems like Navier-Stokes equations at high Reynolds numbers are difficult to solve with central-type or continuous Galerkin methods and require some stabilization mechanism. In recent years, several ideas for stabilizing such methods have emerged which include summation-by-parts, split form schemes and gradient penalty. They help to achieve consistent evolution of other quantities like kinetic energy, internal and entropy, which are found to lead to robust schemes for complex flows. SBP property allows to prove stability of the method even under inexact integration which is usually the case in practice. Gradient penalty is another idea to add extra dissipation which is found to reduce dispersion errors by acting on the high wave-number part of the solution. It is also useful as a subgrid scale model in the context of Large Eddy Simulation (LES). Here we will explore these ideas for the computation of compressible flows and try to identify useful schemes for DNS and LES of complex, turbulent flows. The eventual goal of this work is to develop high order, low-dissipation, robust schemes that can be used for compressible flows on unstructured, curvilinear grids.

Elena Gaburro (University of Verona)

Title: High order direct ALE schemes on 1D, 2D and 3D Voronoi-like tessellations with topology changes

Abstract: The framework of this work is that of direct Arbitrary-Lagrangian-Eulerian (ALE) methods of high order of accuracy for solving hyperbolic equations on moving Voronoi-like tessellation. What has become clear over the past years is that in order to guarantee a good quality of the moving mesh, we cannot just relocate, stretch and deform the elements, but also so-called topology changes (change of the elements shape and connectivity) should be allowed. At this point, the crucial step of a direct method consists in being able to build, even around a topology change, the space-time control volumes connecting elements with potentially different shapes and neighborhood at successive time-steps. Unfortunately, just linking the existing elements, by collapsing or expanding their edges, leaves a hole in the space-time. Thus, we have decided to fill it by considering an additional space-time degenerate hole-like sliver element. In this talk, I will first describe the shape and the numerical treatment of a sliver element in the 2D+time framework with a direct ALE ADER Discontinuous Galerkin (DG) method. Then, I will justify the stability of this ALE ADER-DG approach applied to sliver elements by presenting a von Neumann stability analysis performed in the 1D+time setting at the aid of a fictitious 1D+time sliver element. Finally, I will show preliminary novel results about sliver elements in 3D+time: their degenerate 4D hole-like shape will be fully characterized and a visualization strategy will be proposed. E. Gaburro gratefully acknowledges the support received from the European Union with the ERC Starting Grant ALcHyMiA (grant agreement No. 101114995).