Summer School IMJ-PRG 2026
Integrable Combinatorics

15 - 19 June 2026, Campus Jussieu - Sorbonne Univeristé, Paris

1 Mini-courses

"An Integrable Systems Perspective on Symmetric Function Theory", by Luigi Cantini (CY Cergy Paris University). Symmetric functions are ubiquitous objects at the crossroads of algebra, combinatorics, and representation theory: they appear as characters of classical groups, generating functions of partitions and tableaux, and observables in random matrix models. A natural way to construct and study them is via integrable vertex models, where partition functions with suitable boundary conditions realize Schur, Hall–Littlewood, Macdonald, and related families. This integrable systems perspective provides structural insights—such as Cauchy-type identities from the Yang–Baxter equation—and connects symmetric functions to diverse applications, from enumerative combinatorics, encoding tilings and plane partitions, to integrable probability, governing models in the KPZ universality class.

"Random Tilings and Integrable Structures", by Maurice Duits (KTH Stockholm). Random tiling models, such as domino tilings of the Aztec diamond and lozenge tilings of hexagons, form a meeting point between probability, combinatorics, and integrable systems. Their global behavior reveals striking geometric features like limit shapes and arctic boundaries, while their local fluctuations exhibit universal patterns connected to random matrix theory. This mini-course will present some of the ideas underlying these phenomena and explain how integrable structures emerge naturally in their analysis. The focus will be on illustrating the main mechanisms through classical examples and discussing recent progress and open questions.

"Symmetries of discrete and ultradiscrete integrable systems", by Rei Inoue (Chiba University). Some discrete and ultradiscrete integrable systems have rich symmetries related to various mathematics: algebraic/tropical geometry, combinatorics, crystal base theory and so on. In this lecture, we will discuss:

(i) the symmetries of discrete KdV equation and discrete Toda lattice. These have nice positive structures which preserves the integrability in the tropical limit (which sends rational maps to piecewise-linear maps).

(ii) the symmetry of the box-ball system (BBS). This is an integrable cellular automaton living on a cross-point of classical and quantum integrable systems, where tropical geometry and crystal base theory meet. In particular, the BBS is related to the discrete KdV/Toda via 'ultradiscretization (a tropical limit restricted on integers)'.

"Crystals and symmetric functions", by Anne Schilling (UC Davis). Crystal graphs, which originated in integrable models, provide combinatorial tools to study the representation theory of Lie algebras. For instance, crystals are well-behaved with respect to taking tensor products and hence can be used to give combinatorial interpretations for Littlewood–Richardson coefficients. The character of an irreducible crystal $B(\lambda)$ of highest weight λ in type A is the Schur function s_λ. We introduce crystal bases, explore their relation to symmetric functions and some open problems in combinatorics.

2 Introductury Research Talks

"Interacting particle systems and Macdonald polynomials", by Arvind Ayyer (IISc Bangalore). *Macdonald polynomials* are a family of symmetric polynomials of great importance in many areas of mathematics. I will present two stochastic processes and explain their connection to the Macdonald polynomials, namely the *Asymmetric Simple Exclusion Process* (ASEP) and the *t-PushTASEP*. Time permitting, I will also introduce another family called the *modified Macdonald polynomials* and explain the connection to the *Totally Asymmetric Zero Range Process* (TAZRP).

"Fock's dimer model on the Aztec diamond", by Béatrice de Tilière (Université Paris-Dauphine PSL). We consider the dimer model, or equivalently domino tilings, on the Aztec diamond, and suppose that edges are assigned Fock's weights. The main goal of this talk is to give a compact, explicit formula for the inverse Kasteleyn matrix, thus extending in this very general context previous results of the same kind; in particular, this gives an explicit expression for Boltzmann probabilities. Then, we will prove that the partition function admits a product form, and show how to recover Stanley's celebrated formula as a specific case. Finally, we will show how our expression for the inverse Kasteleyn matrix allows to recover results about limit shapes. This is based on joint work with Cédric Boutillier.

"Multiplicative statistics of Poissonized Plancherel random partitions", by Giulio Ruzza (University of Lisbon). After reviewing the Plancherel measure on partitions and its relevance in combinatorics and in the (asymptotic) representation theory of symmetric groups, I will introduce a class of multiplicative statistics of Poissonized Plancherel random partitions. Their study is motivated by connections to integrable systems (Toda equations) and to important stochastic growth models (polynuclear growth models). In particular, with Mattia Cafasso and Matteo Mucciconi we tackled the asymptotic study of these statistics. Building on the log-gas structure of the Poissonized Plancherel measure we derived optimal shapes for Poissonized Plancherel random partitions (which generalize the celebrated Vershik-Kerov-Logan-Shepp density and exhibit new behaviors naturally described in terms of elliptic functions) as well as refined asymptotic expansions for the statistics themselves (obtained, building on the study of the aforementioned optimal shapes, via the Deift-Zhou nonlinear steepest descent asymptotic analysis of Riemann-Hilbert problems). We provide applications to the q-deformed polynuclear growth model and to cylindrical Toda solutions with step-like shock initial data.

"Integrability in Schubert Calculus", by Anna Weigandt (University of Minnesota). Schubert calculus has its origins in enumerative questions asked by the geometers of the 19th century, such as "how many lines meet four fixed lines in three-space?" These problems can be recast as questions about the structure of cohomology rings of geometric spaces such as flag varieties. Borel's isomorphism identifies the cohomology of the complete flag variety with a simple quotient of a polynomial ring. Lascoux and Schützenberger (1982) defined Schubert polynomials, which are coset representatives for the Schubert basis of this ring. We will discuss Schubert polynomials from the perspective of vertex models.

3 Contributed Talks

4 Posters