Celestial Symmetries of Black Hole Horizons

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Based on [2504.08027], [2506.x] with R. Ruzziconi

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Introduction: Celestial Symmetries

- $Lw_{1+\infty}$ algebra $\sim$ "Higher spin BMS"

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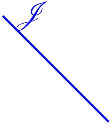
- $Lw_{1+\infty}$ algebra $\sim$ "Higher spin BMS"
- Infinity of conserved charges in absence of radiation

$$\frac{\mathrm{d}H_s}{\mathrm{d}v} = 0 \Leftrightarrow \Psi_4^0 = 0 \qquad\qquad s = -1, 0, 1, 2, ...$$

$Lw_{1+\infty}$ algebra at $\mathscr{I}$



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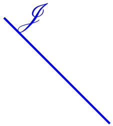

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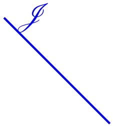

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- canonical generators in "Noether-type prescription" realizations
  - Noether charges associated to gauge symmetries of the self-dual twistor action [Kmec, Mason, Ruzziconi, Yelleshpur Srikant '24]
  - Ashtekar-Streubel phase space of radiative data (spacetime realisation) [Freidel, Pranzetti, Raclariu '21, Geiller '24]

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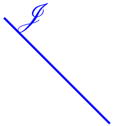

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This sector is interesting to consider within full gravity

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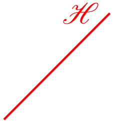
Arbitrary null hypersurface at finite distance $\mathscr{H}$.
Example: black hole horizon, cosmological horizon



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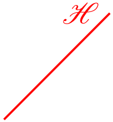

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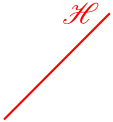

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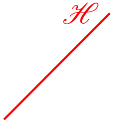

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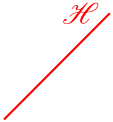

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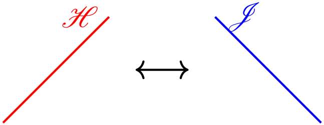
- Carroll physics at finite distance [Sachs '61, Damour '78, ...]
- Black hole entropy [Bekenstein '73, Hawking '75]
- Asymptotic symmetries: BMS-like symmetries on $\mathscr{H}$ [Penna '15, Donnay, Giribet, Gonzalez, Pino '15 '16; Hawking, Perry, Strominger '16; Haco, Hawking, Perry, Strominger '16; Chandrasekaran, Flanagan, Prabhu '18; Grumiller, Sheikh-Jabbari, Troncoso, CZ '19; Adami, Grumiller, Sheikh-Jabbari, Taghiloo, Yavartanoo, CZ '21, ...]

Where are the celestial symmetries and their canonical generators at $\mathscr{H}$*?*

Today's program

- Map $\mathscr{I} \leftrightarrow \mathscr{H}$



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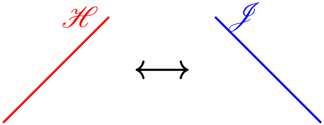

- Import celestial symmetries, a notion of radiation and new observables for an observer closed to $\mathscr{H}$

Method

- Newman-Penrose formalism [Newman, Penrose '62] and Geroch-Held-Penrose operators [Geroch, Held, Penrose '73]
- Conformal compactification [Penrose '63]

Subtle point

Einstein equations are not Weyl invariant, hence off-shell mapping

Contents

Mapping $\mathscr{I}$ to $\mathscr{H}$

Newman-Penrose at $\mathscr{I}$

- Newman-Unti (NU) null tetrad [Newman, Unti '62]

$$\ell = -\partial_{r_{\mathscr{I}}} \qquad n = -\partial_v + \mathcal{O}(r_{\mathscr{I}}^{-1})$$
$$m = \frac{1}{r_{\mathscr{I}}} m_0^A \partial_A + \mathcal{O}(r_{\mathscr{I}}^{-2}) \qquad \bar{m} = \frac{1}{r_{\mathscr{I}}} \bar{m}_0^A \partial_A + \mathcal{O}(r_{\mathscr{I}}^{-2})$$

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- Peeling property of Weyl scalars

$$\Psi_n = \frac{\Psi_n^0}{r_\mathscr{I}^{5-n}} + \mathcal{O}(r_\mathscr{I}^{-(6-n)})$$

  - $\Psi_4^0, \Psi_3^0 \sim$ radiation , $\mathrm{Re}\Psi_2^0 \sim$ mass , $\Psi_1^0 \sim$ angular momentum , $\Psi_0 \sim$ higher spins
  - Weyl scalars are related to celestial symmetries

- Spin coefficients

$$\sigma = \frac{\sigma_0}{r_{\mathscr{I}}^2} + \mathcal{O}(r_{\mathscr{I}}^{-3})$$

σ_0 is the (normal) shear

$$\lambda = \lambda_0 + \frac{\lambda_1}{r_{\mathscr{I}}} + \mathcal{O}(r_{\mathscr{I}}^{-2})$$

λ_0 is the (intrinsic) shear , λ_1 is the news

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On-shell

- Boundary condition: $m_0^A, \bar{m}_0^A$ are the dyad of a fixed 2-sphere
- $\lambda_0 = 0$
- $\Psi_4^0 = \partial_v \lambda_1 = \partial_v^2 \bar{\sigma}_0\,, \qquad \lambda_1 = \partial_v \bar{\sigma}_0$

Covariantisation

Advantage of NP formalism: trade tensors for weighted scalars under spin and boost transformations of the tetrad

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Advantage of NP formalism: trade tensors for weighted scalars under spin and boost transformations of the tetrad

Definition

Under the transformation of the tetrad

$$\ell^\mu \to \mathcal{V}\ell^\mu, \quad n^\mu \to \mathcal{V}^{-1}n^\mu, \quad m^\mu \to e^{ib}m^\mu, \quad \bar{m}^\mu \to e^{-ib}\bar{m}^\mu,$$

a scalar η has weight (p,q) if it transforms

$$\eta \to \mathcal{V}^{\frac{p+q}{2}} e^{i\frac{p-q}{2}b}\eta$$

Example:

$$\sigma : (3,-1)\,, \lambda : (-3,1)\,, \Psi_n : (2(2-n),0)$$

Derivatives

$$\ell^\mu \partial_\mu, n^\mu \partial_\mu, m^\mu \partial_\mu, \bar{m}^\mu \partial_\mu$$

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GHP operators (in NU tetrad)

$$þ = \ell^\mu \partial_\mu \,, \quad þ' = n^\mu \partial_\mu + p\,\gamma + q\,\bar{\gamma} \,, \quad ð = m^\mu \partial_\mu + p\,\beta + q\,\bar{\alpha} \,, \quad \bar{ð} = \bar{m}^\mu \partial_\mu + p\,\alpha + q\,\bar{\beta}$$

"The symbol þ is pronounced "thorn" and ð is pronounced "e(d)th". þ and ð are the phonetic symbols for the soft and hard "th", respectively" [Geroch, Held, Penrose '73]

If η has weight (p, q), then $þ'\eta$ has weight $(p-1, q-1)$.

Weyl rescalling

Definition

Under Weyl rescaling of the null tetrad

$$\ell^\mu \to \Omega^{-2}\ell^\mu, \quad n^\mu \to n^\mu, \quad m^\mu \to \Omega^{-1}m^\mu, \quad \bar{m}^\mu \to \Omega^{-1}\bar{m}^\mu$$

a scalar a Weyl weight W transforms

$$\eta \to \Omega^W \eta$$

Example:

$$W(\Psi_n) = n - 5\,, \quad W(\sigma) = 2\,, \quad W(\lambda) = 0$$

Rem: Most spin coefficients do not have a well-defined Weyl weight.

Partial derivatives do not have a well-defined Weyl weight.

GHP operators (in NU tetrad)

$$\text{þ}'_{\mathscr{C}} = \text{þ}' + (W + p + q)\mu\,, \quad \eth_{\mathscr{C}} = \eth - (W + q)\,\tau$$

[Penrose, Rindler '84]

- At $\mathscr{I}$, $\lim_{r_{\mathscr{C}}\to\infty} \text{þ}'_{\mathscr{C}} = -\partial_v\,, \lim_{r_{\mathscr{C}}\to\infty} \eth_{\mathscr{C}} = \eth$
- $\text{þ}'_{\mathscr{C}}\eta$ has a well defined weight of W

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Off-shell Weyl rescalling

To go from $\mathscr{I}(r_{\mathscr{I}} \to \infty)$ to $\mathscr{H}(r \to 0)$, we perform a Weyl rescalling $\Omega \sim \frac{1}{r_{\mathscr{I}}} \sim r$

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$\mathscr{I}$	$\Omega \sim \frac{1}{r_{\mathscr{I}}} \sim r$	$\mathscr{H}$
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$\Psi_n = \frac{\Psi_n^0}{r_{\mathscr{I}}^{5-n}} + \mathcal{O}(r_{\mathscr{I}}^{-(6-n)})$		$\Psi_n = \Psi_n^0 + \mathcal{O}(r)$

Peeling to Taylor expansion

Solution space at $\mathcal{H}$

- The tetrad we obtained yields a metric in Gaussian null coordinates (with $g_{vr} = +1$) and fall-offs used in near finite surface literature.
- For Kerr black hole, $\Psi_4^0 = 0 = \Psi_3^0$ and $\Psi_2^0 \sim$ mass and $\Psi_1^0 \sim$ rotation parameter

On-shell at $\mathscr{H}$:

- $\Psi_4^0 = -\text{þ}'_{\mathscr{C}} \lambda_0 \quad \lambda_0 = -\bar{m}_0^A \text{þ}'_{\mathscr{C}} \bar{m}_A^0$
- To have radiation (Ψ_4^0) the induced geometry on $\mathscr{H}$ has to vary, in particular time dependence
- $\lim_{r\to 0} \text{þ}'_{\mathscr{C}} \neq -\partial_v$ because for instance for a black hole $\gamma_0 + \bar{\gamma}_0$ is related to the surface gravity.

Recall at $\mathscr{I}$:

- $m_0^A, \bar{m}_0^A$ are the dyad of a fixed 2-sphere
- $\Psi_4^0 = \partial_v^2 \bar{\sigma}_0 = \partial_v \lambda_1 \,, \qquad \lambda_1 = \partial_v \bar{\sigma}_0 \,, \qquad \lambda_0 = 0$

Summary of Part I

- We used the power of NP and GHP formalism to map off-shell $\mathscr{I} \to \mathscr{H}$
  (see [Ashtekar, Speziale '24] for on-shell mapping/compactification where $\mathscr{I}$ is mapped to a weakly isolated horizon)
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Expert: asymptotic symmetries

Weyl scalars ReΨ_2^0 and Ψ_1^0 appear in the subleading charges associate to supertranslation and superrotations respectively.

Celestial symmetries

Celestial symmetries at $\mathscr{I}$

3 ingredients [Freidel, Pranzetti, Raclariu '21; Geiller '24; Cresto, Freidel '24; Kmec, Mason, Ruzziconi, Yelleshpur Srikant '24]

1. Self-dual gravity equations of motion/ Recursion relations
2. Construction charges and fluxes
3. Ashtekar-Streubel symplectic structure

1. Self-dual equations of motion/Recursion relations

$$-\partial_v Q_s = \eth Q_{s-1} - (s+1)\sigma_0\, Q_{s-2} \qquad s = -1, 0, 1, ..$$
$$Q_{-2} = \Psi_4^0$$

Example:

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- For lower spins (up to spin 4), SD eom = Gravity eom (Bianchi identities)

2. Construction charges and fluxes

$$H_s := \frac{1}{8\pi G}\oint \left(T_s\, Q_s + \text{corrections}\right)\,, F_s := -\frac{dH_s}{dv} \text{ such that } F_s = 0 \Leftrightarrow Q_{-2} = 0$$

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Ex:

$$H_{-1} = \frac{1}{8\pi G}\oint T_{-1}\, Q_{-1} \text{ and } F_{-1} = -\partial_v H_{-1} = \frac{1}{8\pi G}\oint T_{-1}\eth Q_{-2}$$

$$H_0 = \frac{1}{8\pi G}\oint T_0(Q_0 + v\eth Q_{-1})$$
$$F_0 = \frac{1}{8\pi G}\oint T_0(-\sigma_0 Q_{-2} - v\partial_v \eth Q_{-1}) = \frac{1}{8\pi G}\oint T_0(-\sigma_0 Q_{-2} + v\eth^2 Q_{-2})$$

3. Ashtekar-Streubel symplectic structure

$$\Omega_{r_{\mathscr{I}}} = \frac{1}{8\pi G} \oint \int dv\, \delta\lambda_1 \wedge \sigma_0 + c.c.$$

$$\{\lambda_1(v_1, x_1^A), \sigma_0(v_2, x_2^A)\} = 8\pi G\, \delta(v_1 - v_2)\delta(x_1^A - x_2^A)$$

[Ashtekar, Streubel '81]

The fluxes $\mathcal{F}_s = \int dv F_s$ satisfy $Lw_{1+\infty}$ algebra

$$\{\mathcal{F}_{T_{s_1}}, \mathcal{F}_{T_{s_2}}\} = \mathcal{F}_{T_{s_1+s_2-1}}, \qquad T_{s_1+s_2-1} = (s_2+1)T_{s_2}\eth T_{s_1} - (s_1+1)T_{s_1}\eth T_{s_2}$$

where we noted $\mathcal{F}_s \equiv \mathcal{F}_{T_s}$

Note: To be well-defined Schwartzian falloffs have to be imposed $\lim_{v\to\pm\infty} \lambda_1 \sim e^{-|v|^2}$, which implies the wedge condition $\eth^{s+2} T_s = 0$

Celestial symmetries at $\mathscr{H}$

1. Covariantize self-dual gravity equations of motion

$$\text{þ}'_{\mathscr{C}} Q_s = \eth_{\mathscr{C}} Q_{s-1} - (s+1)\sigma_0\, Q_{s-2} \qquad s = -1, 0, 1, ..$$
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- It reduces to what we know for $\mathscr{I}$
- They can be used at $\mathscr{H}$ because they transform well under Weyl rescalling

2. Construction of charges and fluxes

$$H_s := \frac{r}{8\pi G} \oint (T_s\, Q_s + \text{corrections}) \qquad F_s := -\partial_v H_s \qquad \text{þ}'_{\mathscr{C}} T_s = 0$$

Note: The factor r will be explain soon

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But already at spin 0, we face

$[\eth_{\mathscr{C}}, \text{þ}'_{\mathscr{C}}] \neq 0$

Self-duality conditions

We impose

$$\mu_0 = 0\,, \quad \bar{\lambda}_0 = 0\,, \quad \gamma_0 + \bar{\gamma}_0 = -\kappa$$

with κ constant, such that $[\eth_{\mathscr{C}}, \text{þ}'_{\mathscr{C}}] = 0$

Now we can construct the full tower as at $\mathscr{I}$

$$H_{-1} = \frac{r}{8\pi G} \oint T_{-1} Q_{-1},$$
$$H_0 = \frac{r}{8\pi G} \oint T_0 \left(Q_0 - U\, \eth_{\mathscr{C}} Q_{-1}\right)$$

where U satisfy $\text{þ}'_{\mathscr{C}} U = 1$.

$$F_{-1} = \frac{r}{8\pi G} \oint T_{-1} \eth_{\mathscr{C}} Q_{-2},$$
$$F_0 = \frac{r}{8\pi G} \oint T_0 \left[-\sigma_0\, Q_{-2} - U\, \eth^2_{\mathscr{C}} Q_{-2}\right]$$

3. Symplectic structure

But

$[\delta, \text{þ}'_{\mathscr{C}}] \neq 0, [\delta, \eth_{\mathscr{C}}] \neq 0$

More conditions (consistent with equations of motion)

$$\delta m_A^0 = 0\,, \quad \delta\kappa = 0\,, \quad \delta\sqrt{q} = 0$$

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$[\delta, \text{þ}'_{\mathscr{C}}] \neq 0, [\delta, \eth_{\mathscr{C}}] \neq 0$

More conditions (consistent with equations of motion)

$$\delta m^0_A = 0\,, \quad \delta\kappa = 0\,, \quad \delta\sqrt{q} = 0$$

The symplectic form is subleading

$$\omega^r_{EH} = 0 + \frac{r}{8\pi G}\int dv \oint \delta\sigma_0 \wedge \delta\lambda_0 + \delta\bar{p}^A \wedge \delta\bar{m}^0_A + \mathcal{O}(r^2)$$

Subleading phase space

$$\mathbf{\Omega}_{\mathcal{H}} = \frac{r}{8\pi G}\oint\int dv\,\delta\sigma_0\wedge\delta\lambda_0 + \mathcal{O}(r^2)$$

$$\{\sigma_0(v_1,x_1^A),\lambda_0(v_2,x_2^A)\} = \frac{8\pi G}{r\sqrt{q}}\delta^2(x_1^A - x_2^A)\delta(v_1 - v_2)$$

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The fluxes $\mathcal{F}_s = \int dv F_s$ satisfy $Lw_{1+\infty}$ algebra

$$\{\mathcal{F}_{T_{s_1}}, \mathcal{F}_{T_{s_2}}\} = \mathcal{F}_{T_{s_1+s_2-1}}, \qquad T_{s_1+s_2-1} = (s_2+1)T_{s_2}\eth_{\mathscr{C}}T_{s_1} - (s_1+1)T_{s_1}\eth_{\mathscr{C}}T_{s_2}$$

where we noted $\mathcal{F}_s \equiv \mathcal{F}_{T_s}$

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Expert: asymptotic symmetries

Q_0 and Q_1 are related to the action of supertranslation and superrotation respectively.

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Merci!