

TOPOLOGY DEPENDENT BACKGROUND CONNECTION IN GRAVITATION

MOTIVATIONS AND CONSEQUENCES FOR COSMOLOGY

$$S_{\text{grav}} = \int_{\mathcal{M}} g^{\mu\nu} R_{\mu\nu}(\mathbf{g}) dx^4$$



$$S_{\text{grav}} = \int_{\mathcal{M}} g^{\mu\nu} \left(R_{\mu\nu}(\mathbf{g}) - \bar{R}_{\mu\nu}(\bar{\nabla}^{\text{topo}}) \right) dx^4$$



NATIONAL SCIENCE CENTRE
POLAND

Quentin Vigneron^{a,b}

^a Nicolaus Copernicus University, Toruń, Poland

^b CRAL, École Normale Supérieure of Lyon, France

26/06/2025

Mathematical Physics of Gravity and Symmetry 2025

ENS Lyon, FRANCE

$$S_{\text{grav}} = \int_{\mathcal{M}} g^{\mu\nu} \left(R_{\mu\nu}(\mathbf{g}) - \bar{R}_{\mu\nu}(\bar{\nabla}^{\text{topo}}) \right) dx^4$$

Motivations:

- * Existence of a Newtonian limit in any topology.
 - *Q.V. (PRD 2021; PRD 2022; CQG 2022; FoP 2024); Q.V. & B. Roukema (PRD 2023)*
- * From the Carrollian limit? (In prog.)
 - *Q.V. & H. Barzegar & J. Read (2025, preprint)*
- * Well-posed variational principle in any topology? (In prog.)

Cosmology: a universal dynamics

- * Expansion does not depend on Ω_K .
 - *Q.V. & V. Poulin (PRD 2023)*
- * Simple model for curved inflation.
 - *Q.V. & J. Larena (CQG 2024)*
- * Homogeneous but anisotropic solutions. (in prog.)
 - *Q.V. & H. Barzegar (in prep.)*

Cosmic topology : Study of the **shape** and **global curvature** of the Universe

Expansion in the standard model of cosmology

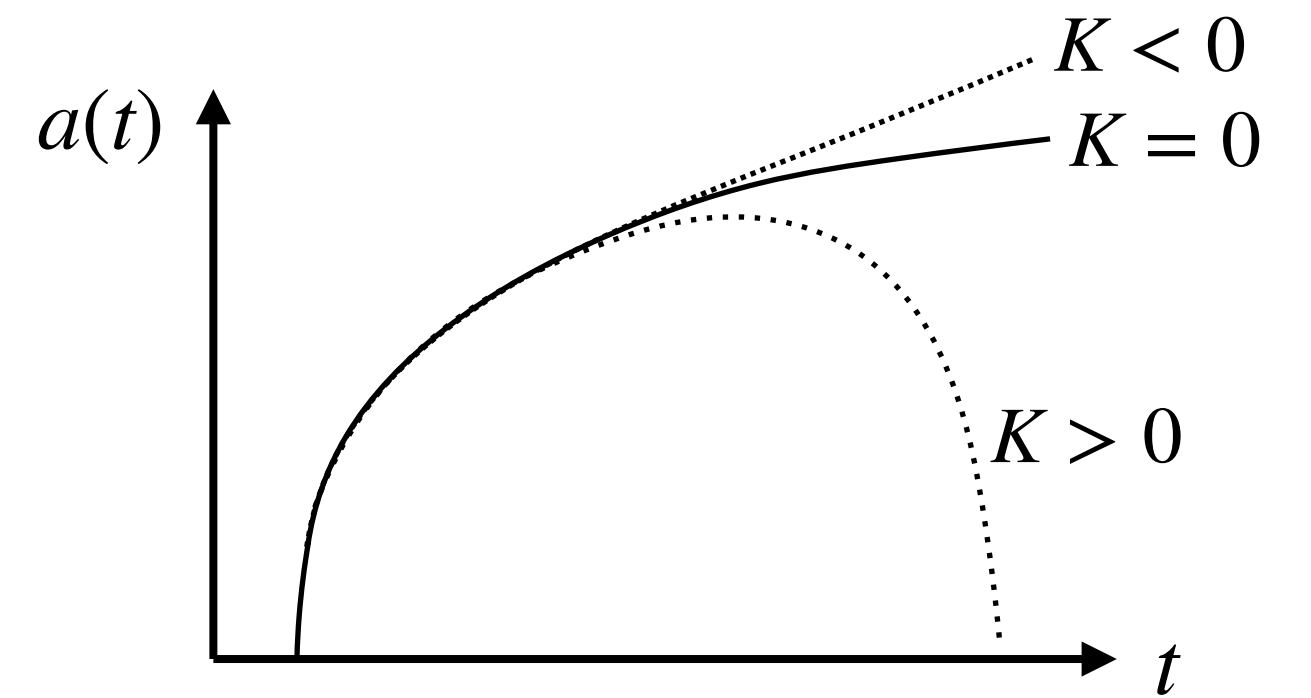
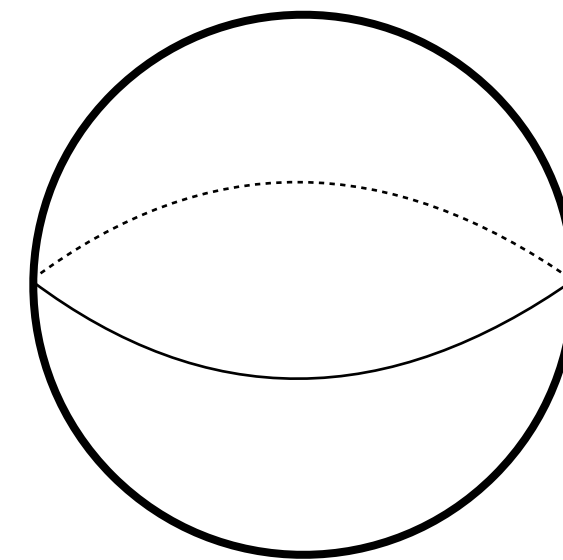
$$\Omega_{\text{matter}} + \Omega_{\text{rad}} + \Omega_{\Lambda} + \Omega_K = 1 \quad (\Omega_K := -K/(a^2 H^2))$$

Three types of universes (if global isotropy)

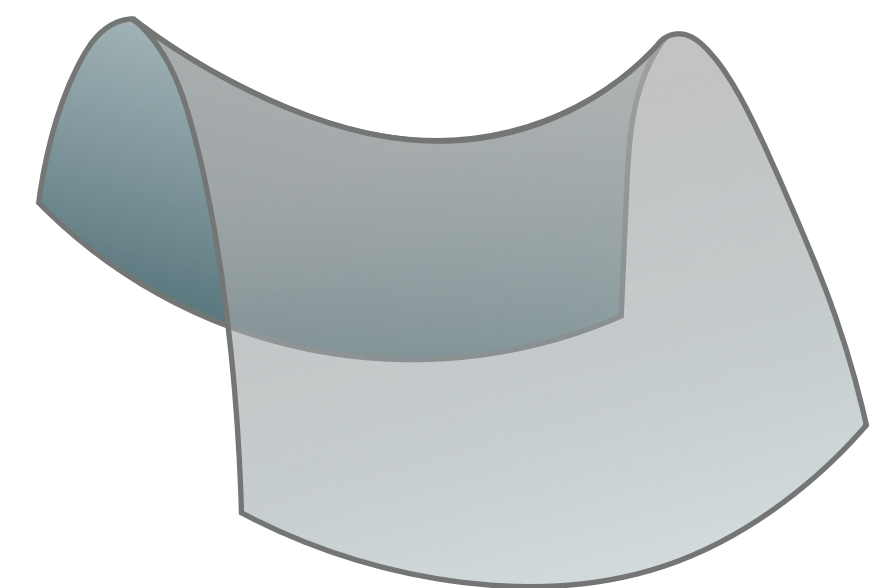
Euclidian ($K=0$)



Spherical ($K > 0$)



Hyperbolic ($K < 0$)

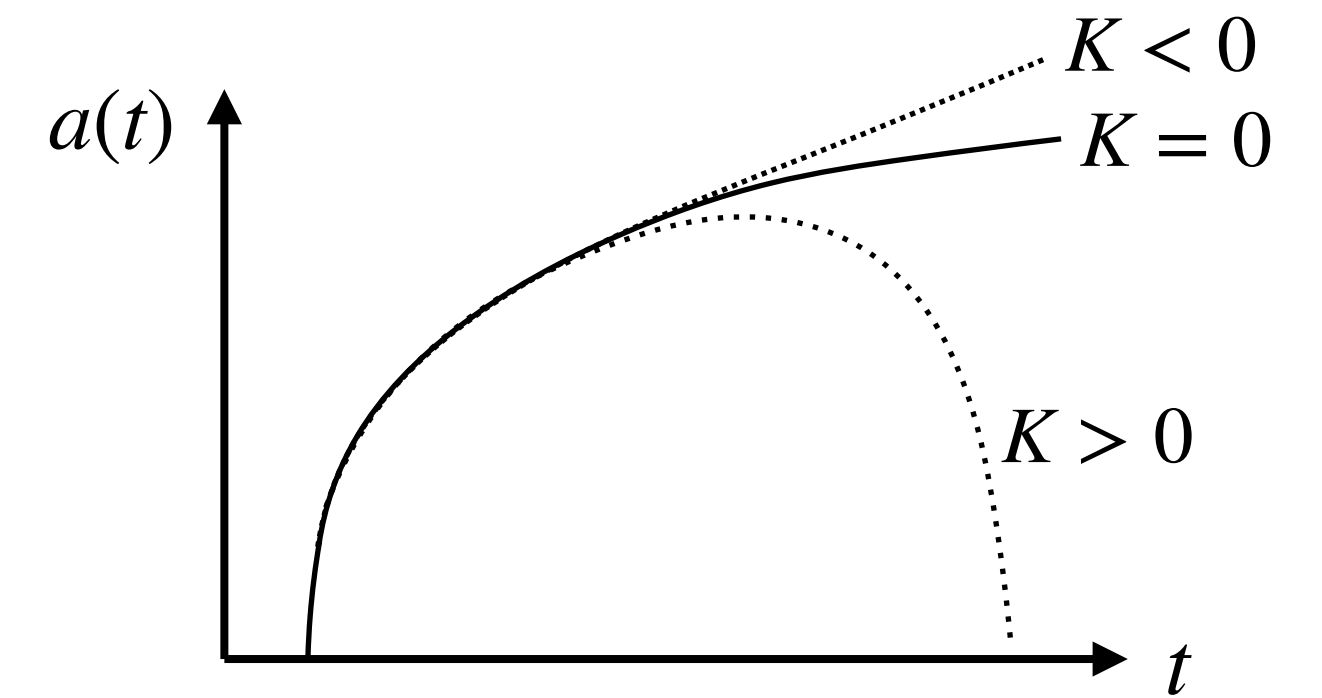


Cosmic topology : Study of the **shape** and **global curvature** of the Universe

Expansion in the standard model of cosmology

$$\Omega_{\text{matter}} + \Omega_{\text{rad}} + \Omega_{\Lambda} + \Omega_K = 1 \quad (\Omega_K := -K/(a^2 H^2))$$

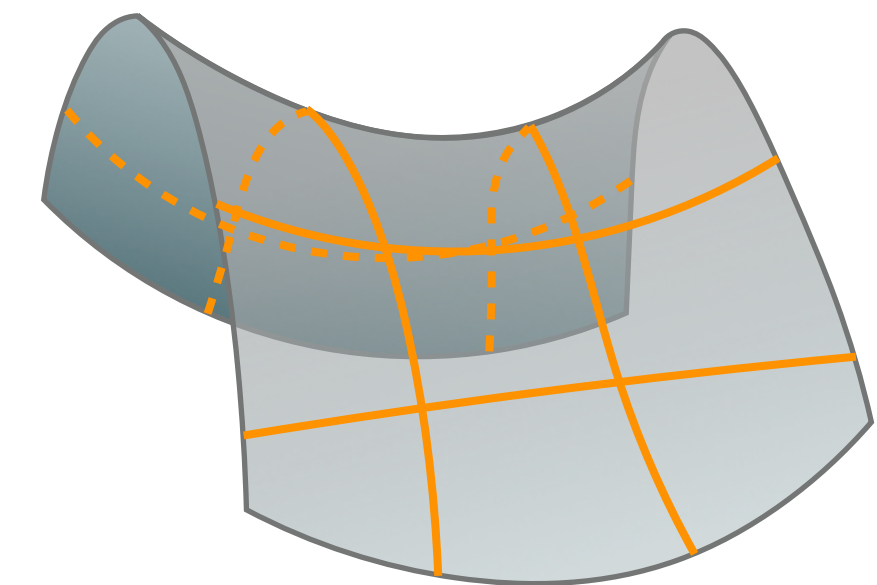
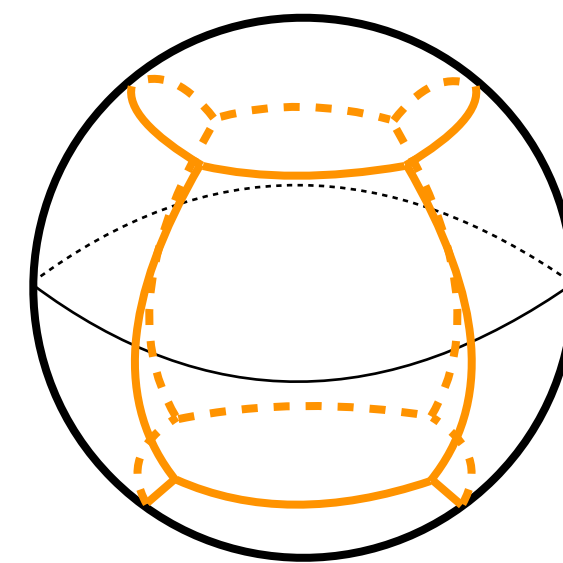
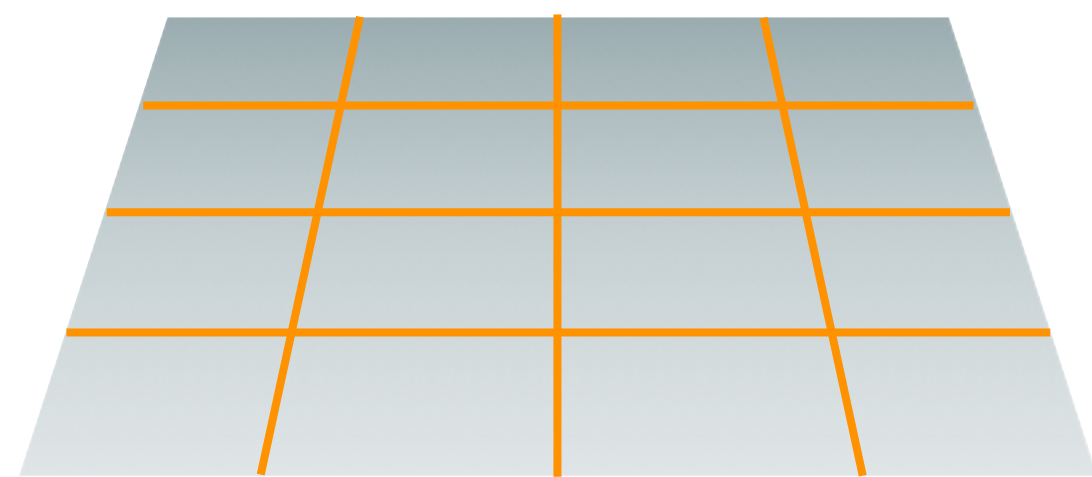
Three types of universes (if global isotropy)



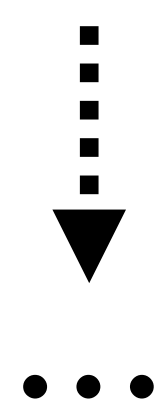
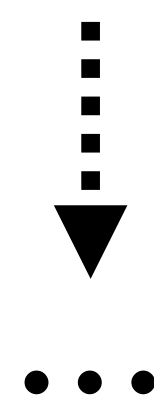
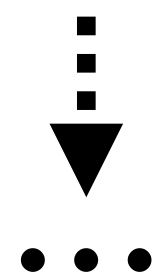
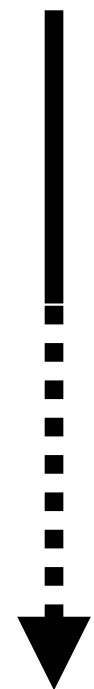
Euclidian ($K=0$)

Spherical ($K > 0$)

Hyperbolic ($K < 0$)



Different shapes
(topologies)

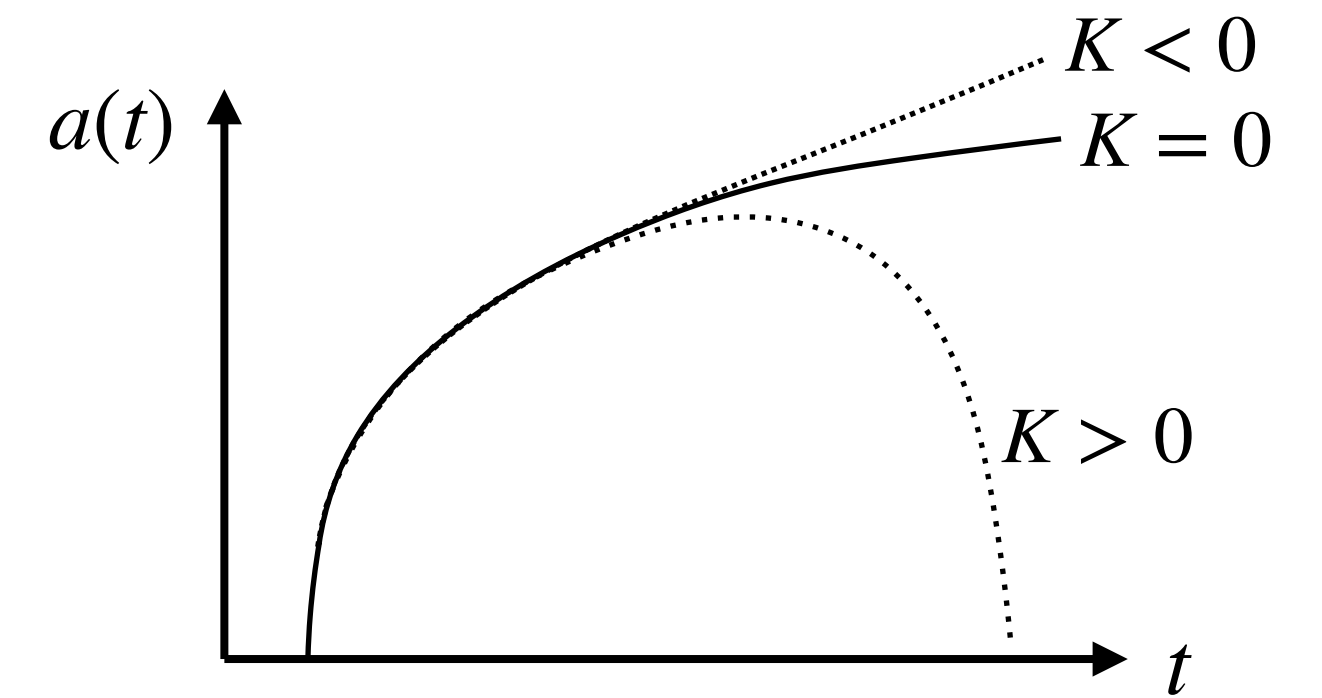


Cosmic topology : Study of the **shape** and **global curvature** of the Universe

Expansion in the standard model of cosmology

$$\Omega_{\text{matter}} + \Omega_{\text{rad}} + \Omega_{\Lambda} + \Omega_K = 1 \quad (\Omega_K := -K/(a^2 H^2))$$

Three types of universes (if global isotropy)

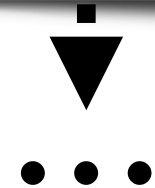
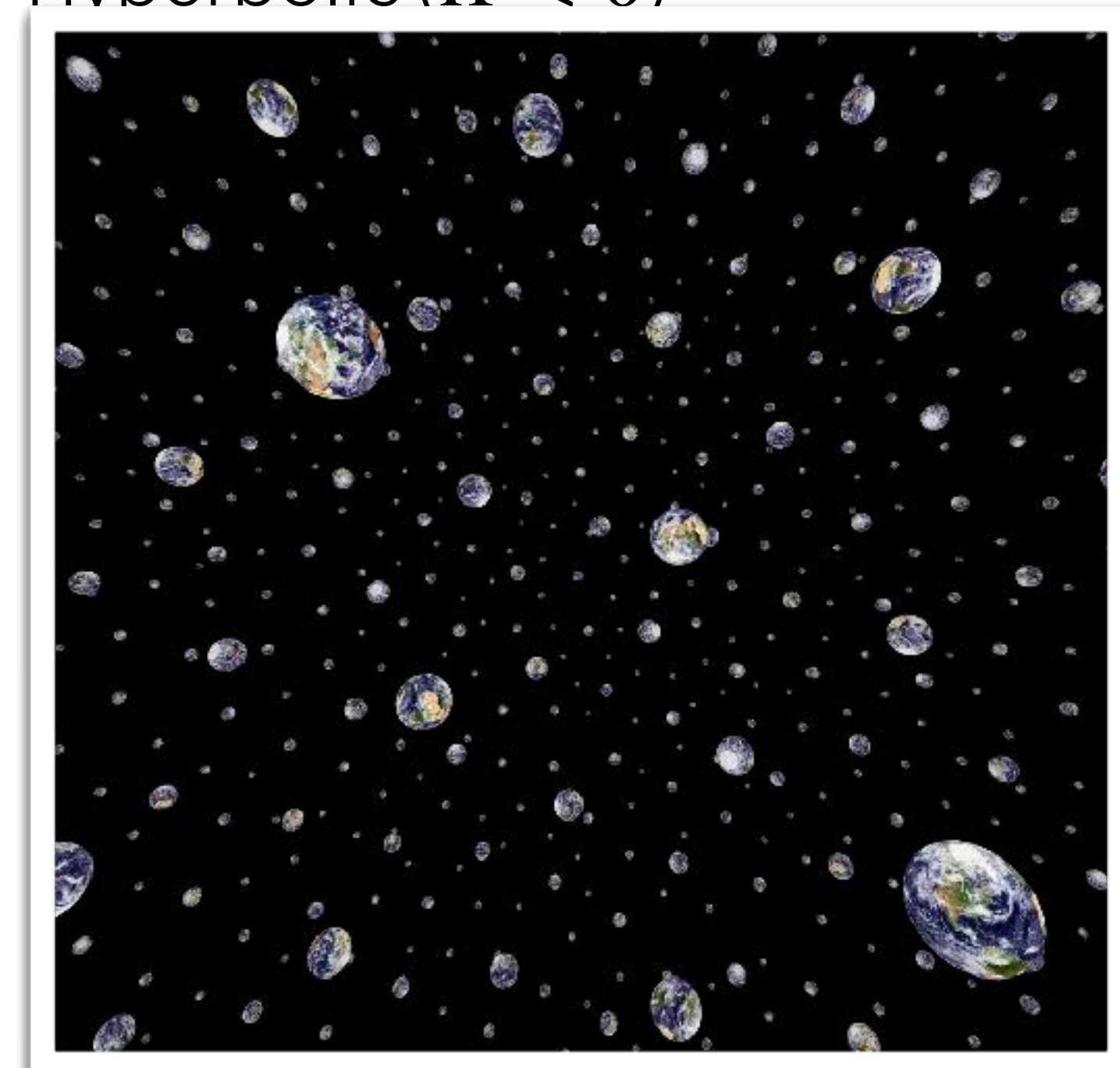
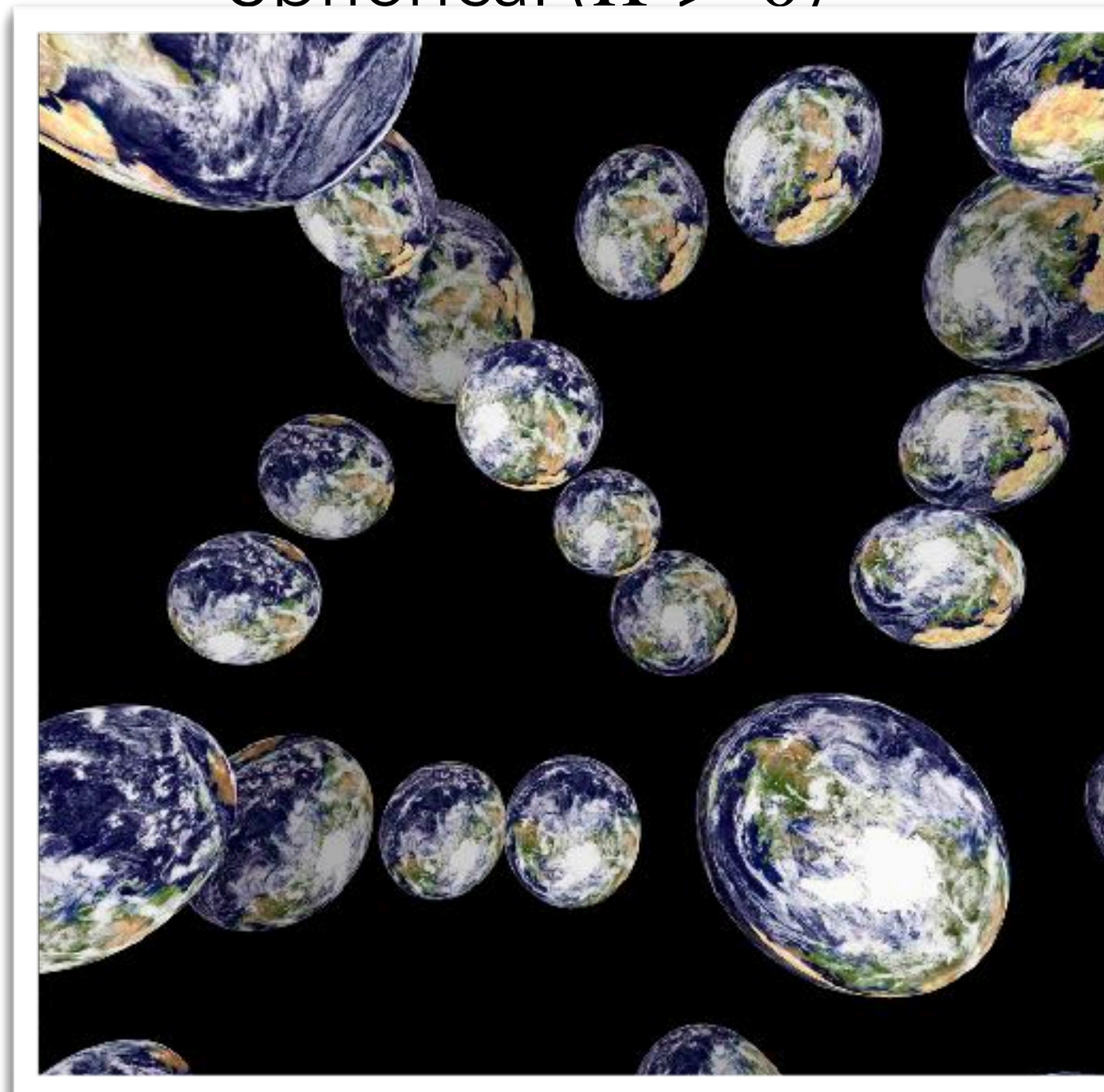
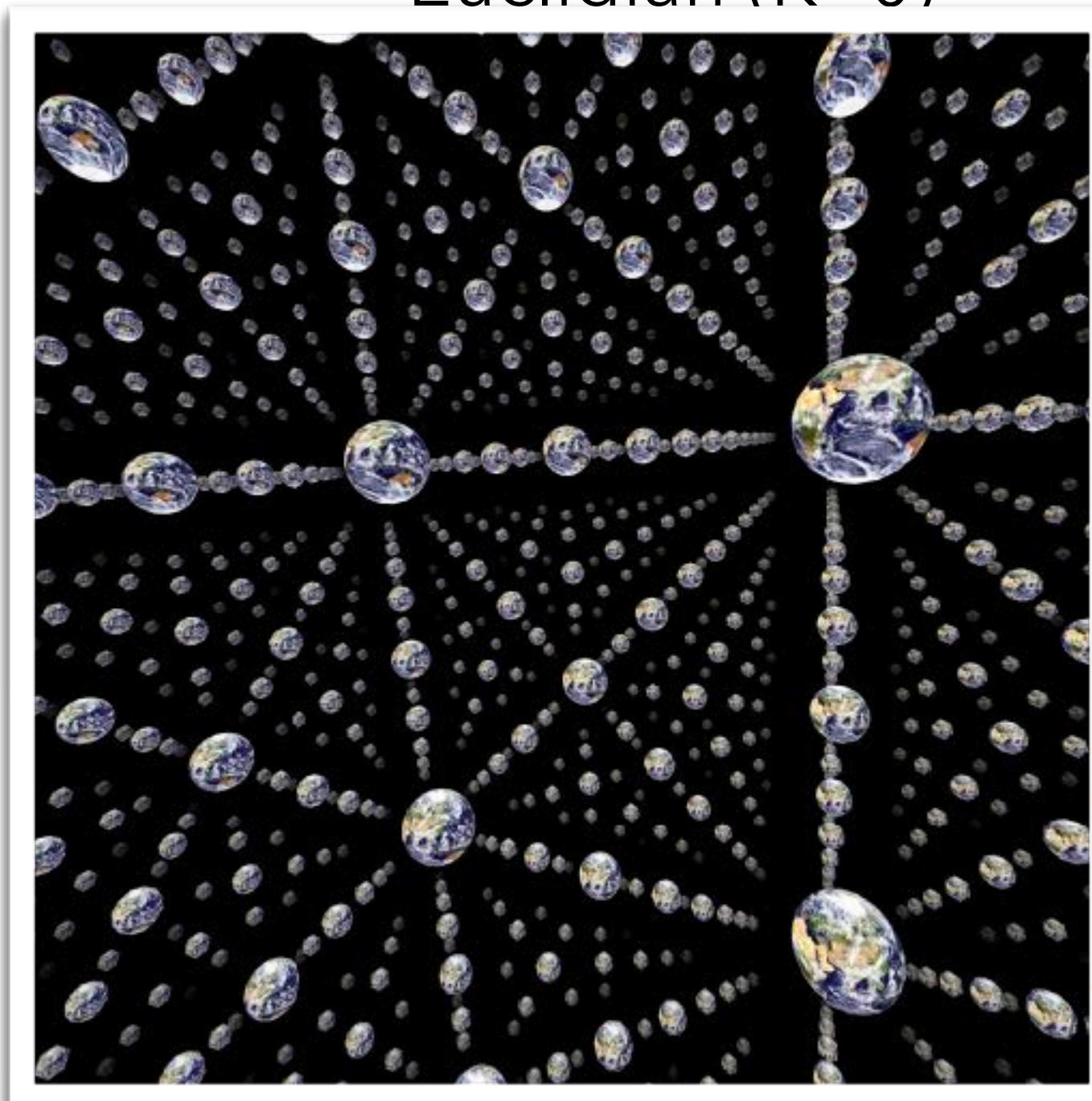
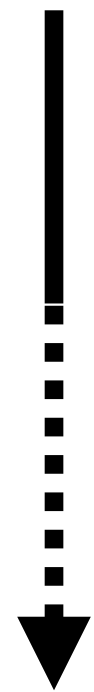


Euclidian ($K=0$)

Spherical ($K > 0$)

Hyperbolic ($K < 0$)

Different shapes
(topologies)

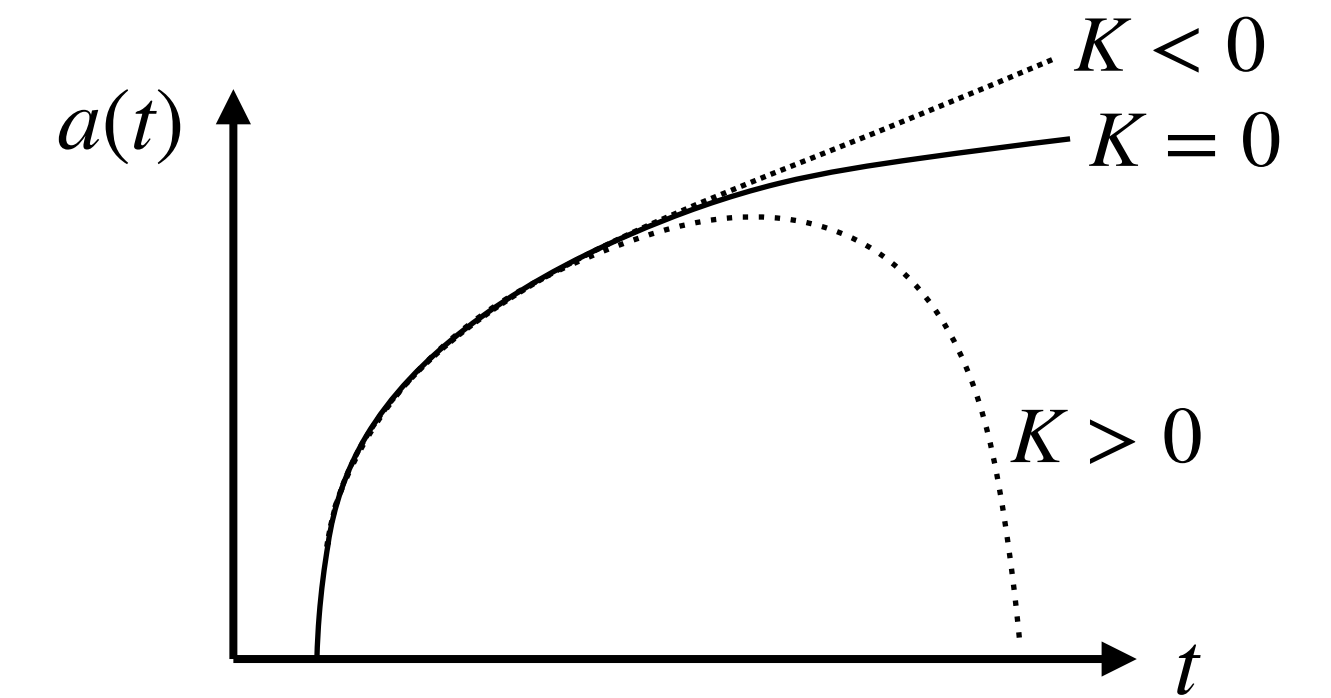


Cosmic topology : Study of the **shape** and **global curvature** of the Universe

Expansion in the standard model of cosmology

$$\Omega_{\text{matter}} + \Omega_{\text{rad}} + \Omega_{\Lambda} + \Omega_K = 1 \quad (\Omega_K := -K/(a^2 H^2))$$

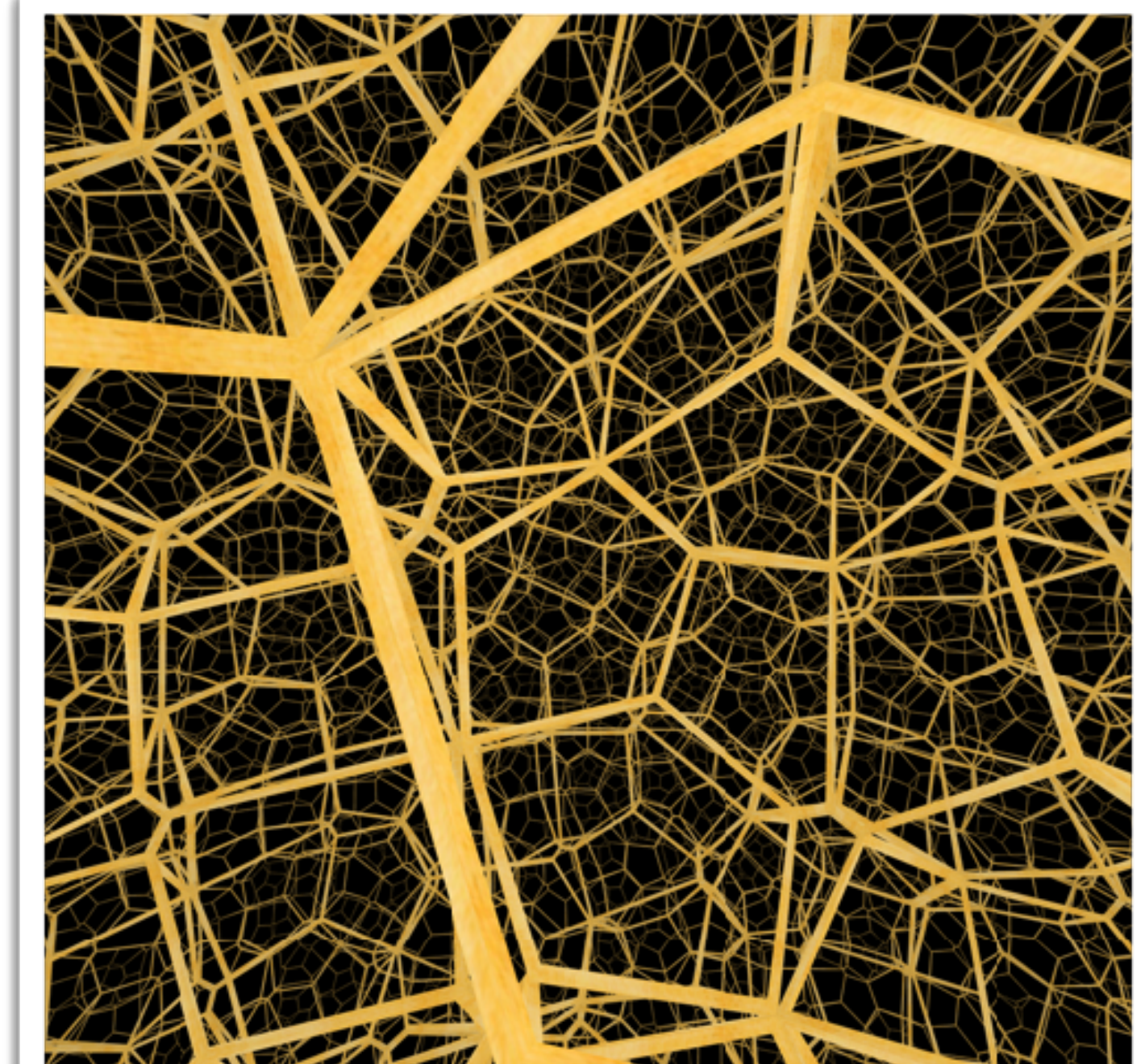
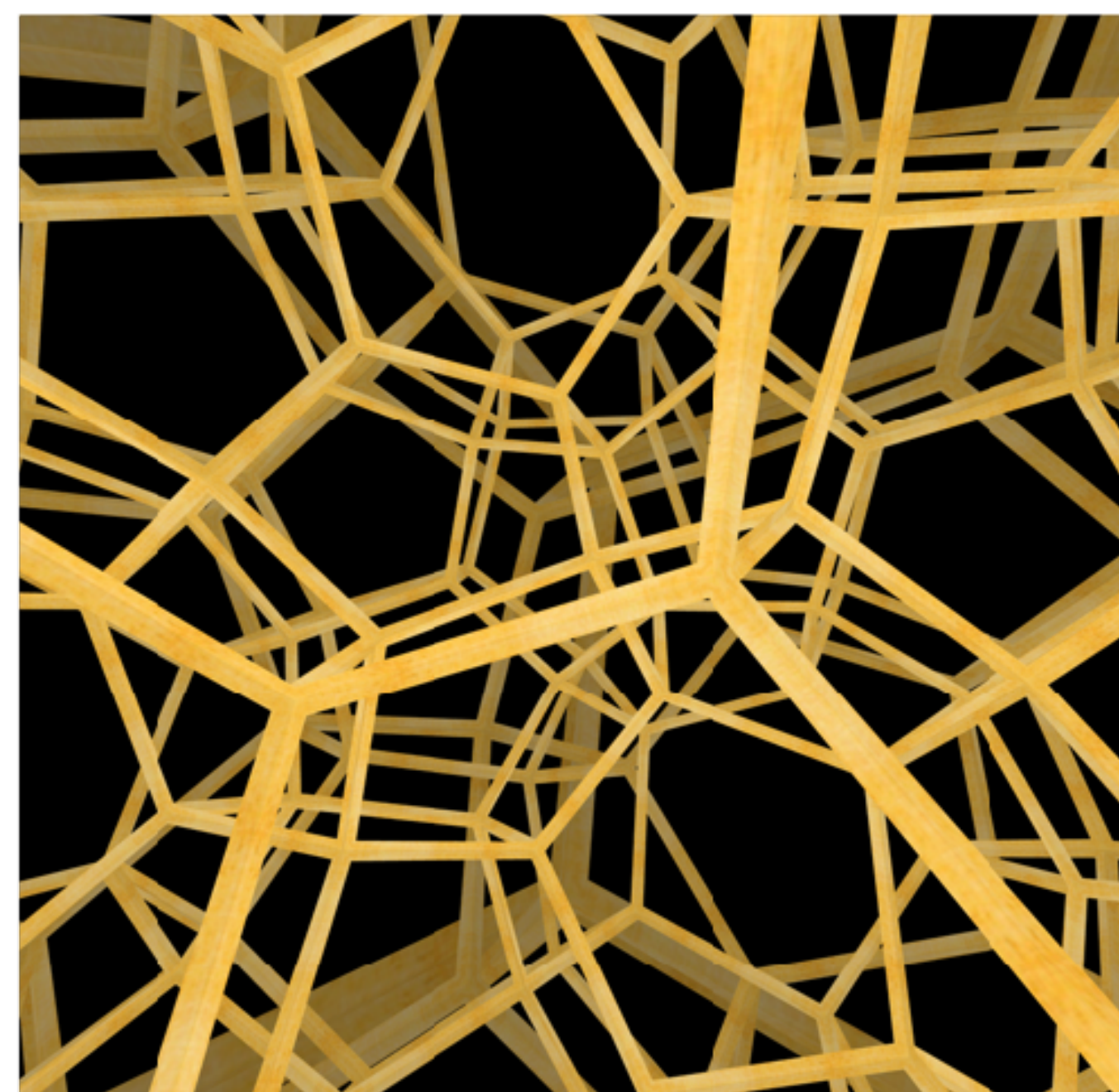
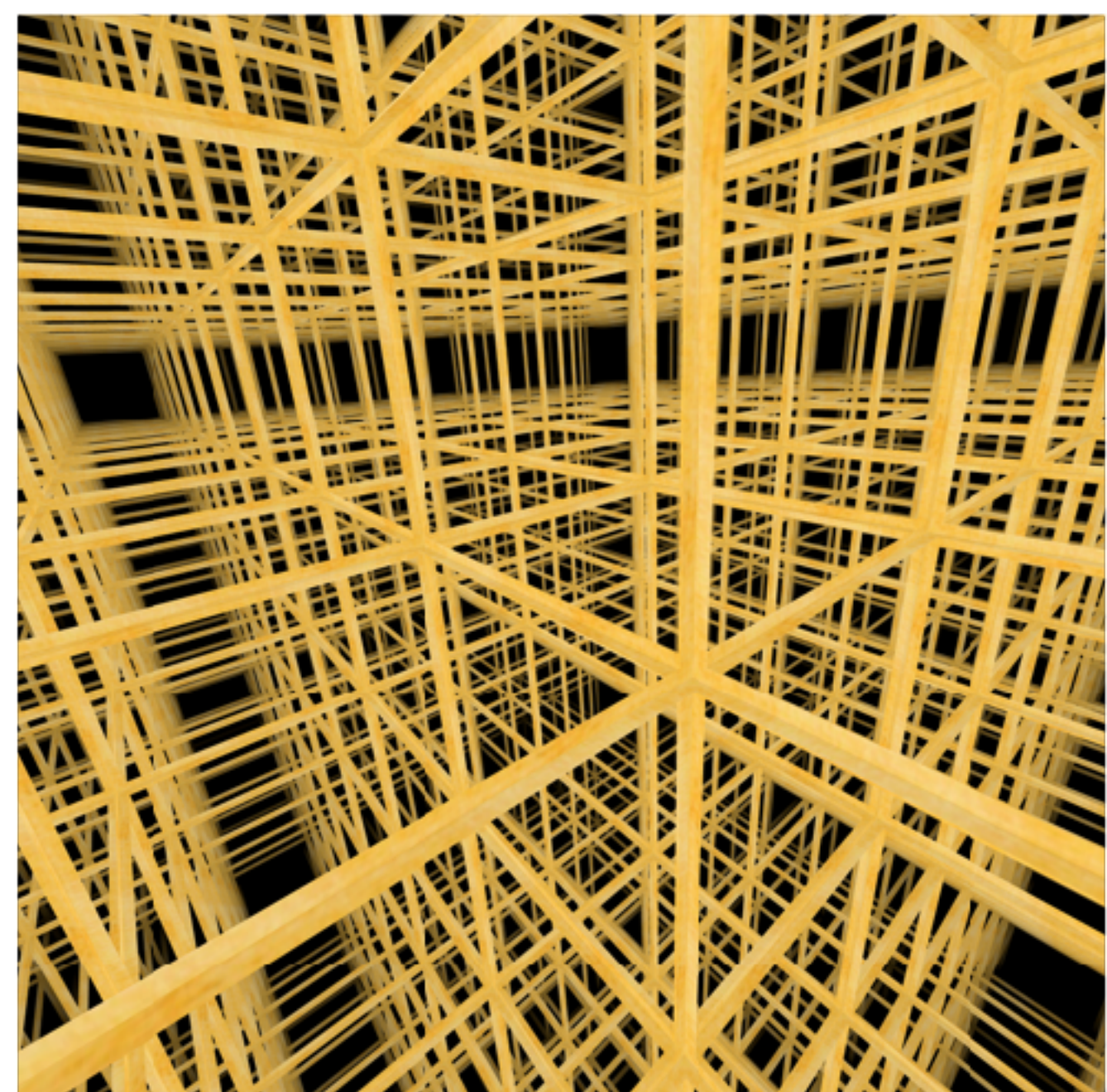
Three types of universes (if global isotropy)



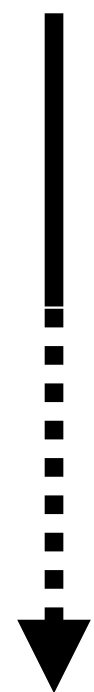
Euclidian ($K=0$)

Spherical ($K > 0$)

Hyperbolic ($K < 0$)



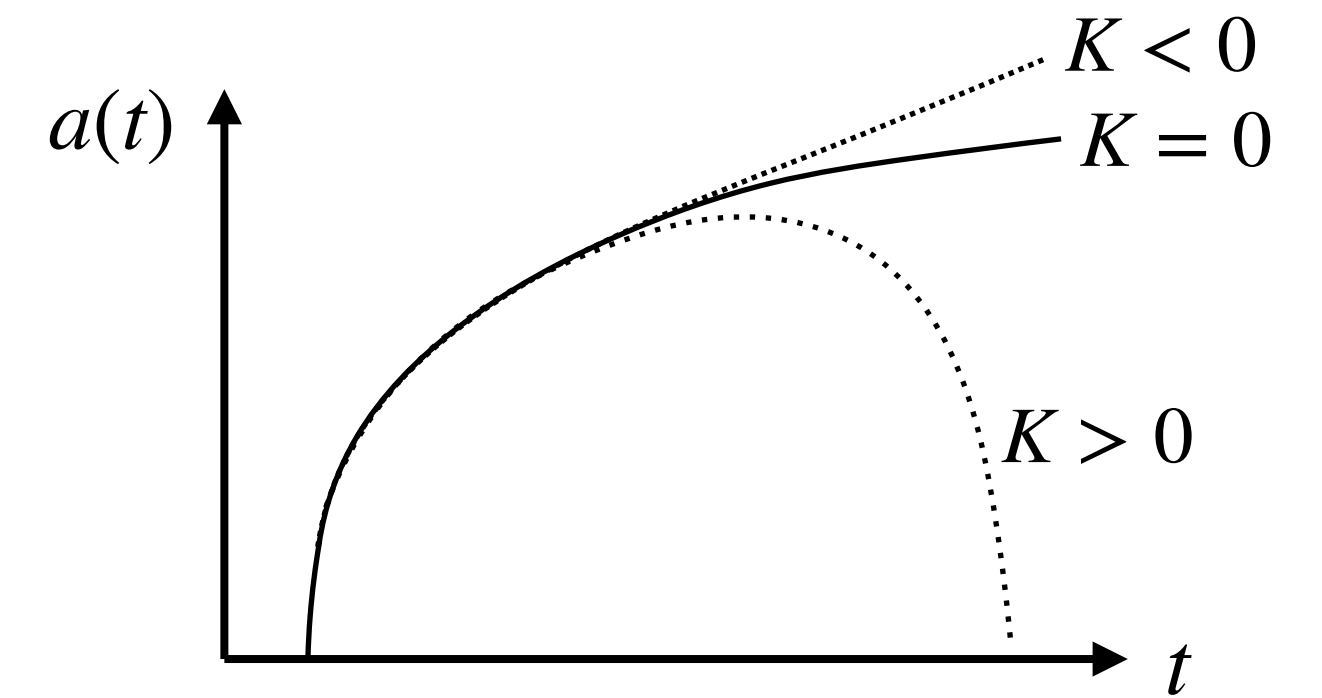
Different shapes
(topologies)



Cosmic topology : Study of the **shape** and **global curvature** of the Universe

Expansion in the standard model of cosmology

$$\Omega_{\text{matter}} + \Omega_{\text{rad}} + \Omega_{\Lambda} + \Omega_K = 1 \quad (\Omega_K := -K/(a^2 H^2))$$

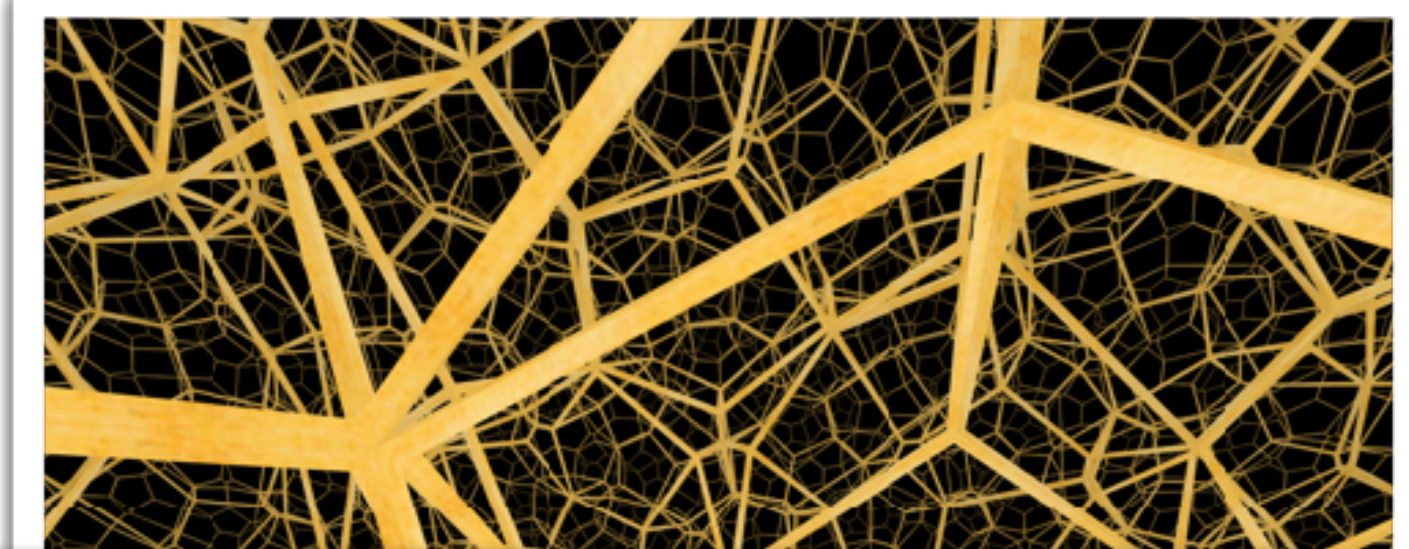
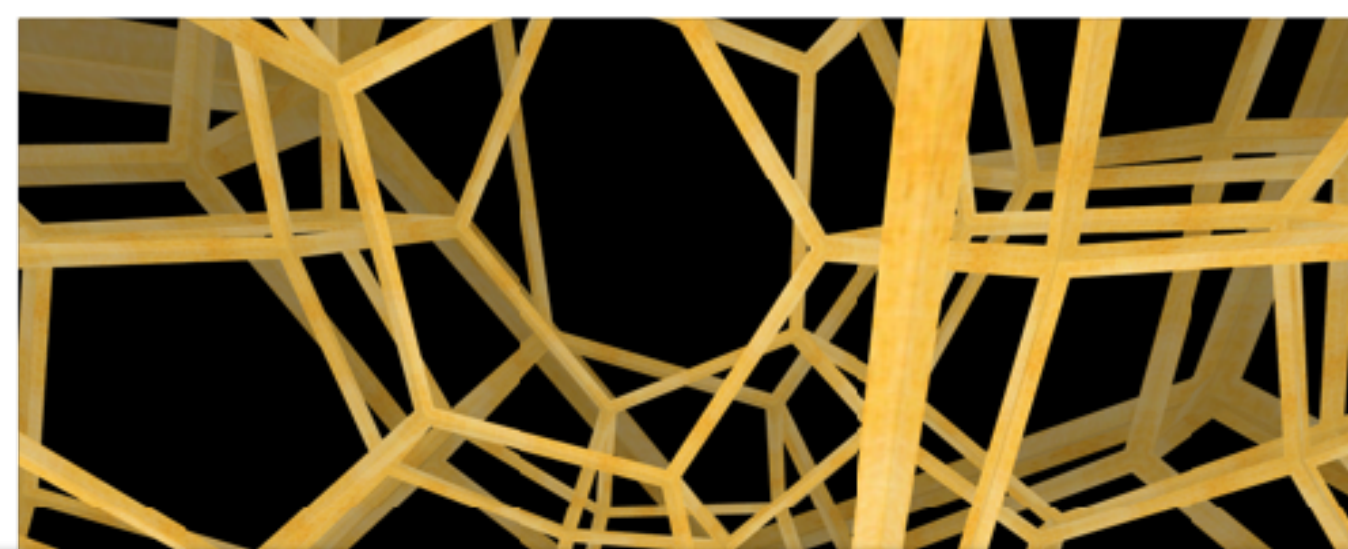
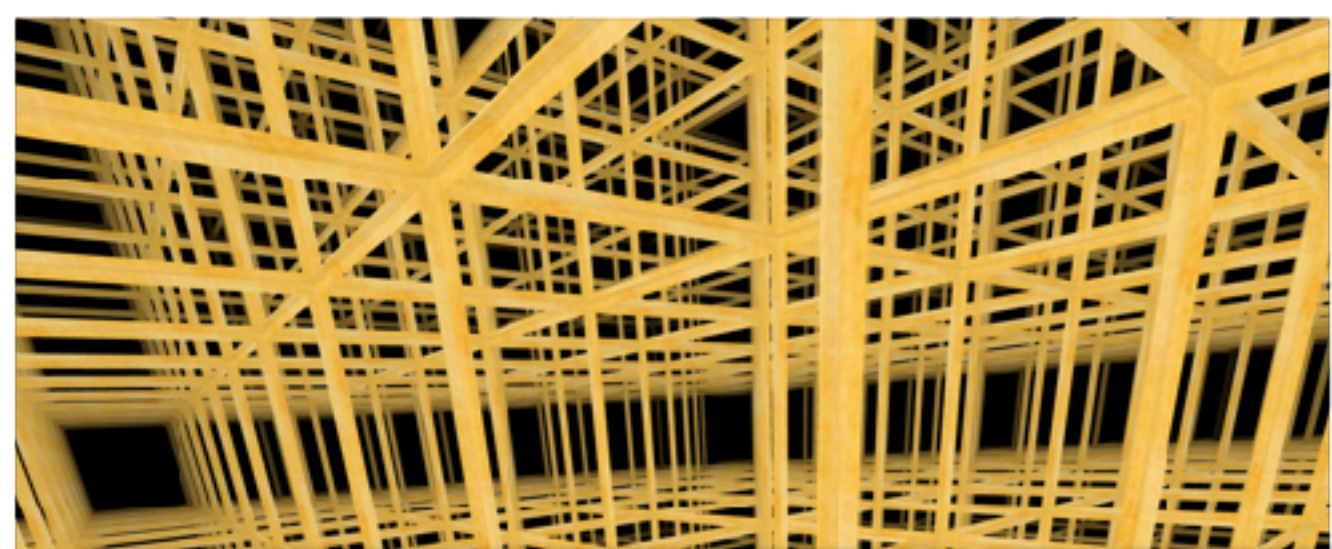


Three types of universes (if global isotropy)

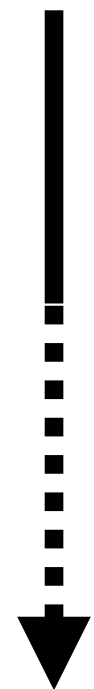
Euclidian ($K=0$)

Spherical ($K > 0$)

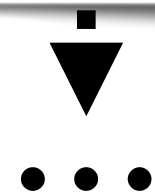
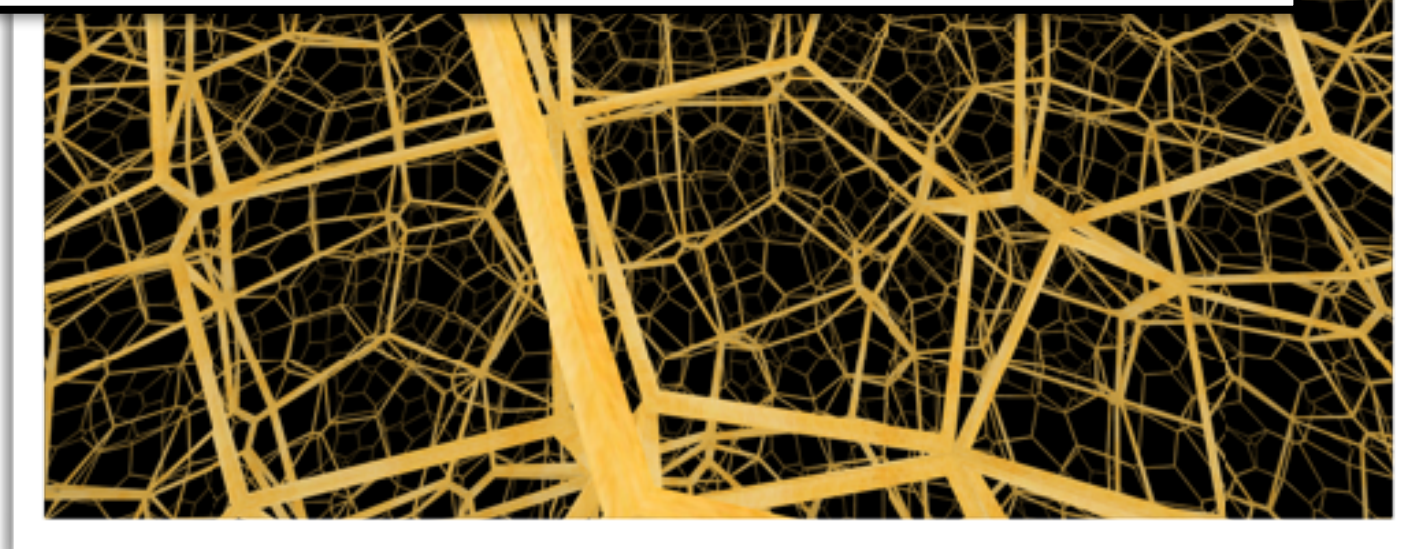
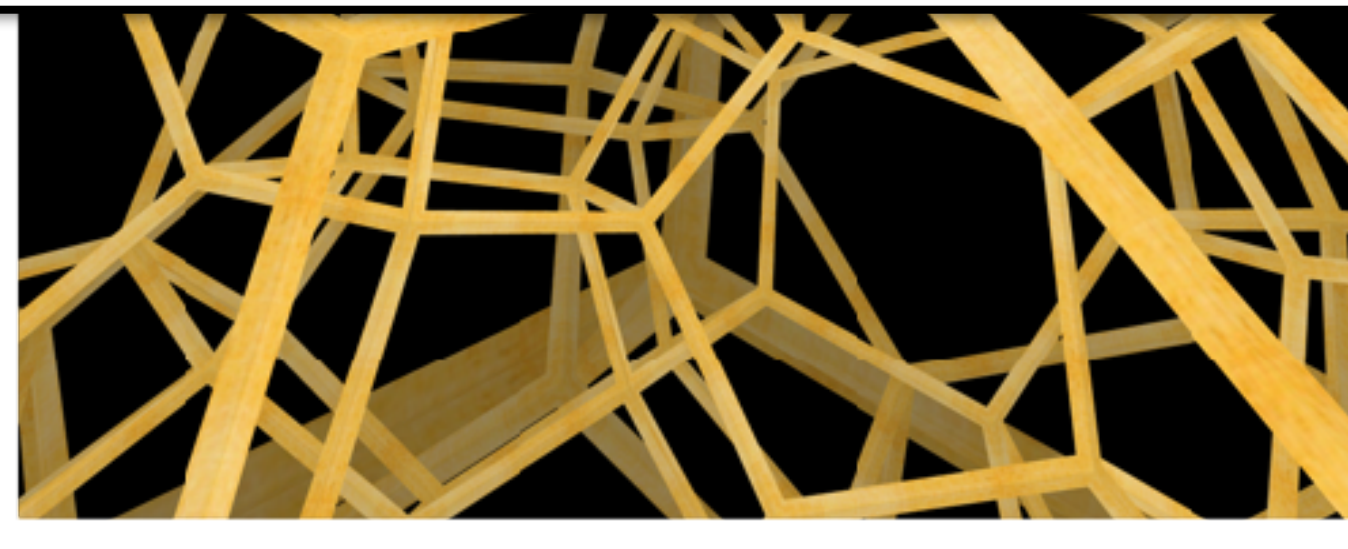
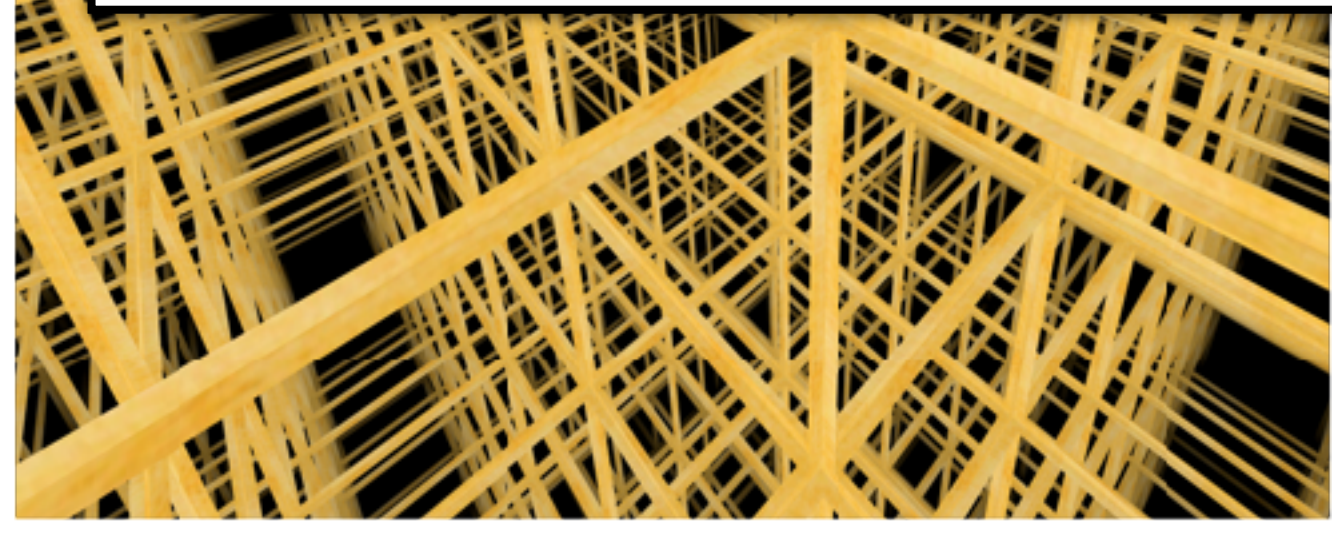
Hyperbolic ($K < 0$)



Different shapes
(topologies)



Not an exotic hypothesis: implicitly assumed in the Standard Model, as boundary conditions.



Classification of closed 3-manifolds Σ (Thurston's classification):

- Eight classes of topology, characterised by the **covering space** $\tilde{\Sigma}$:

$$\tilde{\Sigma} = \mathbb{E}^3, \mathbb{S}^3, \mathbb{H}^3, \mathbb{R} \times \mathbb{S}^2, \mathbb{R} \times \mathbb{H}^2, \tilde{SL}(2, \mathbb{R}), Nil, Sol$$

- Within each class, a topological space is characterised by a **discrete group** Γ , such that $\Sigma = \tilde{\Sigma}/\Gamma$

Classification of closed 3-manifolds Σ (Thurston's classification):

▸ Eight classes of topology, characterised by the **covering space** $\tilde{\Sigma}$:

$$\tilde{\Sigma} = \mathbb{E}^3, S^3, \mathbb{H}^3, \mathbb{R} \times S^2, \mathbb{R} \times \mathbb{H}^2, \tilde{SL}(2, \mathbb{R}), Nil, Sol$$

▸ Within each class, a topological space is characterised by a **discrete group** Γ , such that $\Sigma = \tilde{\Sigma}/\Gamma$


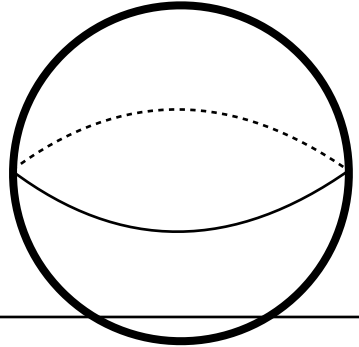
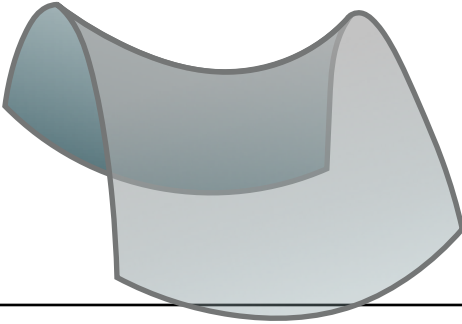
$\tilde{\Sigma}$	\mathbb{E}^3	S^3	\mathbb{H}^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times \mathbb{H}^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Σ	$\Gamma_1^{\mathbb{E}^3}$ $\Gamma_2^{\mathbb{E}^3}$ $\Gamma_3^{\mathbb{E}^3}$...	$\Gamma_1^{S^3}$ $\Gamma_2^{S^3}$ $\Gamma_3^{S^3}$...	$\Gamma_1^{\mathbb{H}^3}$ $\Gamma_2^{\mathbb{H}^3}$ $\Gamma_3^{\mathbb{H}^3}$...	$\Gamma_1^{\mathbb{R}^2 \times S^2}$ $\Gamma_2^{\mathbb{R}^2 \times S^2}$ $\Gamma_3^{\mathbb{R}^2 \times S^2}$...	$\Gamma_1^{\mathbb{R}^2 \times \mathbb{H}^2}$ $\Gamma_2^{\mathbb{R}^2 \times \mathbb{H}^2}$ $\Gamma_3^{\mathbb{R}^2 \times \mathbb{H}^2}$...	$\Gamma_1^{\tilde{SL}(2, \mathbb{R})}$ $\Gamma_2^{\tilde{SL}(2, \mathbb{R})}$ $\Gamma_3^{\tilde{SL}(2, \mathbb{R})}$...	Γ_1^{Nil} Γ_2^{Nil} Γ_3^{Nil} ...	Γ_1^{Sol} Γ_2^{Sol} Γ_3^{Sol} ...

Classification of closed 3-manifolds Σ (Thurston's classification):

▸ Eight classes of topology, characterised by the **covering space** $\tilde{\Sigma}$:

$$\tilde{\Sigma} = \mathbb{E}^3, \mathbb{S}^3, \mathbb{H}^3, \mathbb{R} \times \mathbb{S}^2, \mathbb{R} \times \mathbb{H}^2, \tilde{SL}(2, \mathbb{R}), Nil, Sol$$

▸ Within each class, a topological space is characterised by a **discrete group** Γ , such that $\Sigma = \tilde{\Sigma}/\Gamma$


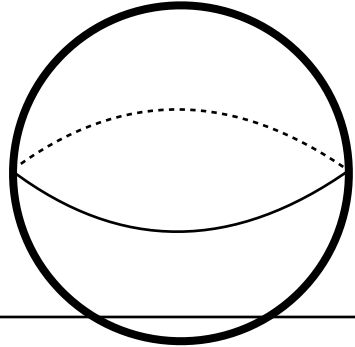
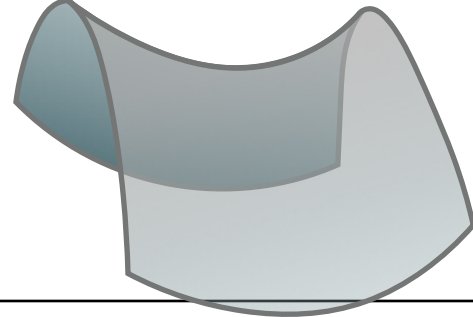
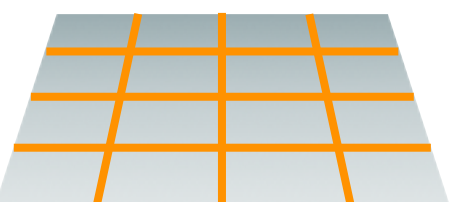

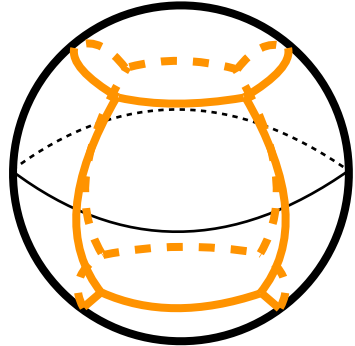
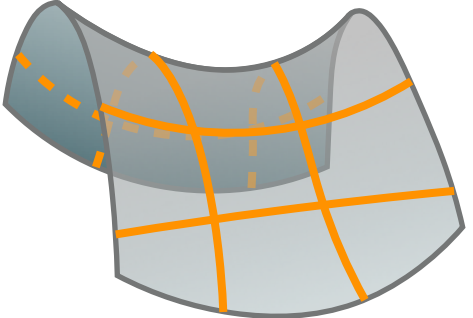
$\tilde{\Sigma}$				$\mathbb{R} \times \mathbb{S}^2$	$\mathbb{R} \times \mathbb{H}^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Σ				$\Gamma_1^{\mathbb{R}^2 \times \mathbb{S}^2}$ $\Gamma_2^{\mathbb{R}^2 \times \mathbb{S}^2}$ $\Gamma_3^{\mathbb{R}^2 \times \mathbb{S}^2}$...	$\Gamma_1^{\mathbb{R}^2 \times \mathbb{H}^2}$ $\Gamma_2^{\mathbb{R}^2 \times \mathbb{H}^2}$ $\Gamma_3^{\mathbb{R}^2 \times \mathbb{H}^2}$...	$\Gamma_1^{\tilde{SL}(2, \mathbb{R})}$ $\Gamma_2^{\tilde{SL}(2, \mathbb{R})}$ $\Gamma_3^{\tilde{SL}(2, \mathbb{R})}$...	Γ_1^{Nil} Γ_2^{Nil} Γ_3^{Nil} ...	Γ_1^{Sol} Γ_2^{Sol} Γ_3^{Sol} ...

Classification of closed 3-manifolds Σ (Thurston's classification):

▸ Eight classes of topology, characterised by the **covering space** $\tilde{\Sigma}$:

$$\tilde{\Sigma} = \mathbb{E}^3, \mathbb{S}^3, \mathbb{H}^3, \mathbb{R} \times \mathbb{S}^2, \mathbb{R} \times \mathbb{H}^2, \tilde{SL}(2, \mathbb{R}), Nil, Sol$$

▸ Within each class, a topological space is characterised by a **discrete group** Γ , such that $\Sigma = \tilde{\Sigma}/\Gamma$


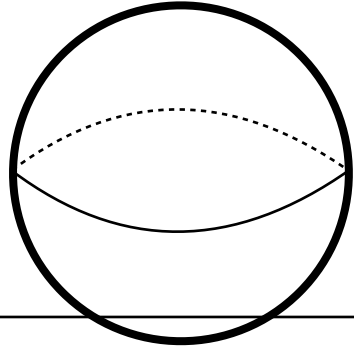
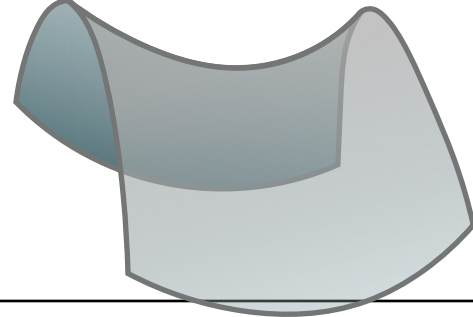
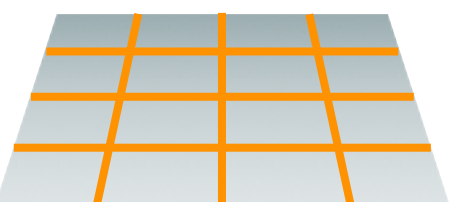

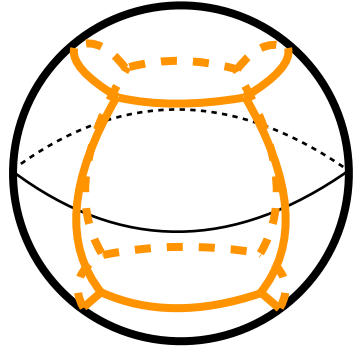
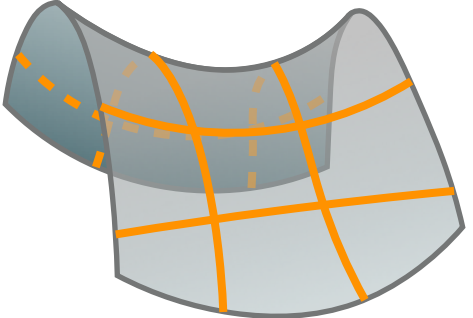
$\tilde{\Sigma}$				$\mathbb{R} \times \mathbb{S}^2$	$\mathbb{R} \times \mathbb{H}^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Σ	 			$\Gamma_1^{\mathbb{R}^2 \times \mathbb{S}^2}$ $\Gamma_2^{\mathbb{R}^2 \times \mathbb{S}^2}$ $\Gamma_3^{\mathbb{R}^2 \times \mathbb{S}^2}$...	$\Gamma_1^{\mathbb{R}^2 \times \mathbb{H}^2}$ $\Gamma_2^{\mathbb{R}^2 \times \mathbb{H}^2}$ $\Gamma_3^{\mathbb{R}^2 \times \mathbb{H}^2}$...	$\Gamma_1^{\tilde{SL}(2, \mathbb{R})}$ $\Gamma_2^{\tilde{SL}(2, \mathbb{R})}$ $\Gamma_3^{\tilde{SL}(2, \mathbb{R})}$...	Γ_1^{Nil} Γ_2^{Nil} Γ_3^{Nil} ...	Γ_1^{Sol} Γ_2^{Sol} Γ_3^{Sol} ...

Classification of closed 3-manifolds Σ (Thurston's classification):

▸ Eight classes of topology, characterised by the **covering space** $\tilde{\Sigma}$:

$$\tilde{\Sigma} = \mathbb{E}^3, \mathbb{S}^3, \mathbb{H}^3, \mathbb{R} \times \mathbb{S}^2, \mathbb{R} \times \mathbb{H}^2, \tilde{SL}(2, \mathbb{R}), Nil, Sol$$

▸ Within each class, a topological space is characterised by a **discrete group** Γ , such that $\Sigma = \tilde{\Sigma}/\Gamma$


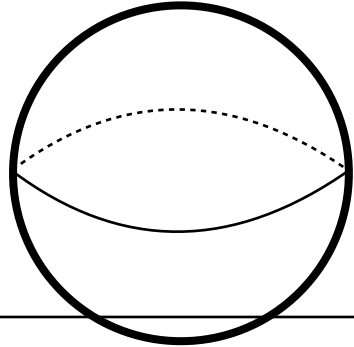
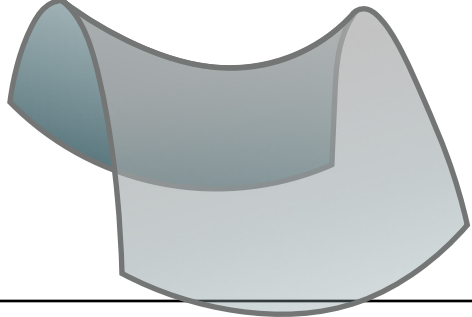
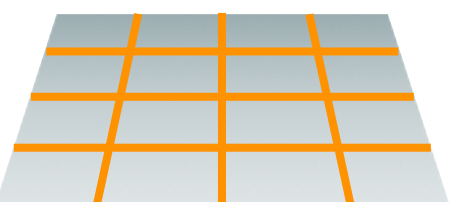

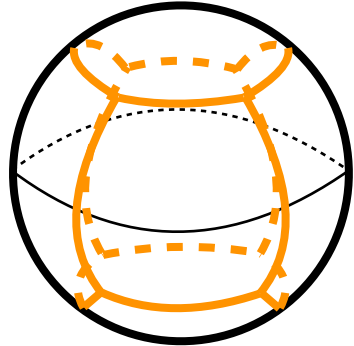
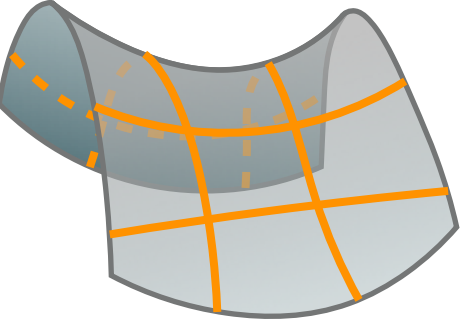
$\tilde{\Sigma}$				$\mathbb{R} \times \mathbb{S}^2$	$\mathbb{R} \times \mathbb{H}^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Σ	 			$\Gamma_1^{\mathbb{R}^2 \times \mathbb{S}^2}$ $\Gamma_2^{\mathbb{R}^2 \times \mathbb{S}^2}$ $\Gamma_3^{\mathbb{R}^2 \times \mathbb{S}^2}$...	$\Gamma_1^{\mathbb{R}^2 \times \mathbb{H}^2}$ $\Gamma_2^{\mathbb{R}^2 \times \mathbb{H}^2}$ $\Gamma_3^{\mathbb{R}^2 \times \mathbb{H}^2}$...	$\Gamma_1^{\tilde{SL}(2, \mathbb{R})}$ $\Gamma_2^{\tilde{SL}(2, \mathbb{R})}$ $\Gamma_3^{\tilde{SL}(2, \mathbb{R})}$...	Γ_1^{Nil} Γ_2^{Nil} Γ_3^{Nil} ...	Γ_1^{Sol} Γ_2^{Sol} Γ_3^{Sol} ...
Topological curvature	$\mathcal{R}_{ij}^{\mathbb{E}^3}$ 0	$\mathcal{R}_{ij}^{\mathbb{S}^3}$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin^2 \chi & 0 \\ 0 & 0 & \sin^2 \chi \sin^2 \theta \end{pmatrix}$	$\mathcal{R}_{ij}^{\mathbb{H}^3}$	$\mathcal{R}_{ij}^{\mathbb{R} \times \mathbb{S}^2}$	$\mathcal{R}_{ij}^{\mathbb{R} \times \mathbb{H}^2}$. . .	$\mathcal{R}_{ij}^{\tilde{SL}(2, \mathbb{R})}$	\mathcal{R}_{ij}^{Nil}	\mathcal{R}_{ij}^{Sol}

Classification of closed 3-manifolds Σ (Thurston's classification):

▸ Eight classes of topology, characterised by the **covering space** $\tilde{\Sigma}$:

$$\tilde{\Sigma} = \mathbb{E}^3, \mathbb{S}^3, \mathbb{H}^3, \mathbb{R} \times \mathbb{S}^2, \mathbb{R} \times \mathbb{H}^2, \tilde{SL}(2, \mathbb{R}), Nil, Sol$$

▸ Within each class, a topological space is characterised by a **discrete group** Γ , such that $\Sigma = \tilde{\Sigma}/\Gamma$

$\tilde{\Sigma}$				$\mathbb{R} \times \mathbb{S}^2$	$\mathbb{R} \times \mathbb{H}^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Σ	 			$\Gamma_1^{\mathbb{R}^2 \times \mathbb{S}^2}$ $\Gamma_2^{\mathbb{R}^2 \times \mathbb{S}^2}$ $\Gamma_3^{\mathbb{R}^2 \times \mathbb{S}^2}$...	$\Gamma_1^{\mathbb{R}^2 \times \mathbb{H}^2}$ $\Gamma_2^{\mathbb{R}^2 \times \mathbb{H}^2}$ $\Gamma_3^{\mathbb{R}^2 \times \mathbb{H}^2}$...	$\Gamma_1^{\tilde{SL}(2, \mathbb{R})}$ $\Gamma_2^{\tilde{SL}(2, \mathbb{R})}$ $\Gamma_3^{\tilde{SL}(2, \mathbb{R})}$...	Γ_1^{Nil} Γ_2^{Nil} Γ_3^{Nil} ...	Γ_1^{Sol} Γ_2^{Sol} Γ_3^{Sol} ...
Topological curvature	$\mathcal{R}_{ij}^{\mathbb{E}^3}$ 0	$\mathcal{R}_{ij}^{\mathbb{S}^3}$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin^2 \chi & 0 \\ 0 & 0 & \sin^2 \chi \sin^2 \theta \end{pmatrix}$	$\mathcal{R}_{ij}^{\mathbb{H}^3}$	$\mathcal{R}_{ij}^{\mathbb{R} \times \mathbb{S}^2}$	$\mathcal{R}_{ij}^{\mathbb{R} \times \mathbb{H}^2}$	$\mathcal{R}_{ij}^{\tilde{SL}(2, \mathbb{R})}$	\mathcal{R}_{ij}^{Nil}	\mathcal{R}_{ij}^{Sol}

Not necessarily the physical curvature.

$$S_{\text{grav}} = \int_{\mathcal{M}} g^{\mu\nu} \left(R_{\mu\nu}(\mathbf{g}) - \bar{R}_{\mu\nu}(\bar{\nabla}^{\text{topo}}) \right) dx^4$$

Motivations:

- * Existence of a Newtonian limit in any topology.
 - *Q.V. (PRD 2021; PRD 2022; CQG 2022; FoP 2024); Q.V. & B. Roukema (PRD 2023)*
- * From the Carrollian limit? (In prog.)
 - *Q.V. & H. Barzegar & J. Read (2025, preprint)*
- * Well-posed variational principle in any topology? (In prog.)

Cosmology: a universal dynamics

- * Expansion does not depend on Ω_K .
 - *Q.V. & V. Poulin (PRD 2023)*
- * Simple model for curved inflation.
 - *Q.V. & J. Larena (CQG 2024)*
- * Homogeneous but anisotropic solutions. (in prog.)
 - *Q.V. & H. Barzegar (in prep.)*

A bi-connection theory: (Q.V. FoP, 2024, arXiv:2204.13980)

- ▶ Spacetime manifold $\mathcal{M} = \mathbb{R} \times \Sigma$, with Σ a closed 3-manifold.
- ▶ A **physical** Lorentzian structure (g, ∇) and its Riemann curvature tensor $\mathbf{Riem}(\nabla)$.
- ▶ A **background**, symmetric connection $\bar{\nabla}^{\text{topo}}$ and its reference Riemann curvature tensor $\mathbf{Riem}(\bar{\nabla}^{\text{topo}})$.

$$S = \frac{1}{16\pi G} \int_{\mathcal{M}} dx^4 \sqrt{|g|} \left(R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} \right) g^{\mu\nu} + \mathcal{B} + S_{\text{matter}}$$

Similar to
Rosen (1980)

(i) Modified Einstein's equation

$$R_{\mu\nu} - \bar{R}_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right) + \Lambda g_{\mu\nu}$$

(ii) Conservation of $\bar{R}_{\mu\nu}$

$$\nabla^\nu \left(\bar{R}_{\nu\mu} - \frac{1}{2} g^{\sigma\gamma} \bar{R}_{\sigma\gamma} g_{\mu\nu} \right) = 0$$

A bi-connection theory: (Q.V. FoP, 2024, arXiv:2204.13980)

- ▶ Spacetime manifold $\mathcal{M} = \mathbb{R} \times \Sigma$, with Σ a closed 3-manifold.
- ▶ A **physical** Lorentzian structure (g, ∇) and its Riemann curvature tensor $\mathbf{Riem}(\nabla)$.
- ▶ A **background**, symmetric connection $\bar{\nabla}^{\text{topo}}$ and its reference Riemann curvature tensor $\mathbf{Riem}(\bar{\nabla}^{\text{topo}})$.

$$S = \frac{1}{16\pi G} \int_{\mathcal{M}} dx^4 \sqrt{|g|} \left(R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} \right) g^{\mu\nu} + \mathcal{B} + S_{\text{matter}}$$

Similar to
Rosen (1980)

(i) Modified Einstein's equation

$$R_{\mu\nu} - \bar{R}_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right) + \Lambda g_{\mu\nu}$$

(ii) Conservation of $\bar{R}_{\mu\nu}$

$$\nabla^\nu \left(\bar{R}_{\nu\mu} - \frac{1}{2} g^{\sigma\gamma} \bar{R}_{\sigma\gamma} g_{\mu\nu} \right) = 0$$

Definition of $\bar{\nabla}$: The reference connection $\bar{\nabla}$ is the (external Whitney) sum of any connection on \mathbb{R} and the maximally symmetric (Thurston) connection on Σ .

$$\bar{R}_{\mu\nu} := \begin{pmatrix} 0 & 0 \\ 0 & \mathcal{R}_{ij}^{\tilde{\Sigma}} \end{pmatrix}$$

$$\implies \text{If } \mathcal{M} = \mathbb{R} \times \mathbb{E}^3/\Gamma: \quad \bar{R}_{\mu\nu} = 0 \quad \text{and} \quad \text{topo-GR} \iff \text{GR.}$$

A bi-connection theory: (Q.V. FoP, 2024, arXiv:2204.13980)

- ▶ Spacetime manifold $\mathcal{M} = \mathbb{R} \times \Sigma$, with Σ a closed 3-manifold.
- ▶ A **physical** Lorentzian structure (g, ∇) and its Riemann curvature tensor $R_{\mu\nu}$.
- ▶ A **background**, symmetric connection $\bar{\nabla}^{\text{topo}}$ and its reference curvature tensor $Riem(\bar{\nabla}^{\text{topo}})$.

$$S = \frac{1}{16\pi G} \int_{\mathcal{M}} dx^4 \sqrt{|g|} \left(R_{\mu\nu} - \bar{R}_{\mu\nu} \right) + \mathcal{B} + S_{\text{matter}}$$

Similar to
Rosen (1980)

(i) Modified Einstein's equation

$$R_{\mu\nu} - \bar{R}_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)$$

(ii) Conservation of $\bar{R}_{\mu\nu}$

$$\nabla^\nu \left(\bar{R}_{\nu\mu} - \frac{1}{2} g^{\sigma\gamma} \bar{R}_{\sigma\gamma} g_{\mu\nu} \right) = 0$$

Definition of $\bar{\nabla}$: The reference connection $\bar{\nabla}$ is the (external Whitney) sum of any connection on \mathbb{R} and the maximally symmetric (Thurston) connection on Σ .

$$\bar{R}_{\mu\nu} := \begin{pmatrix} 0 & 0 \\ 0 & \mathcal{R}_{ij}^{\tilde{\Sigma}} \end{pmatrix}$$

$$\implies \text{If } \mathcal{M} = \mathbb{R} \times \mathbb{E}^3 / \Gamma: \quad \bar{R}_{\mu\nu} = 0 \quad \text{and} \quad \text{topo-GR} \iff \text{GR.}$$

Parameter-free

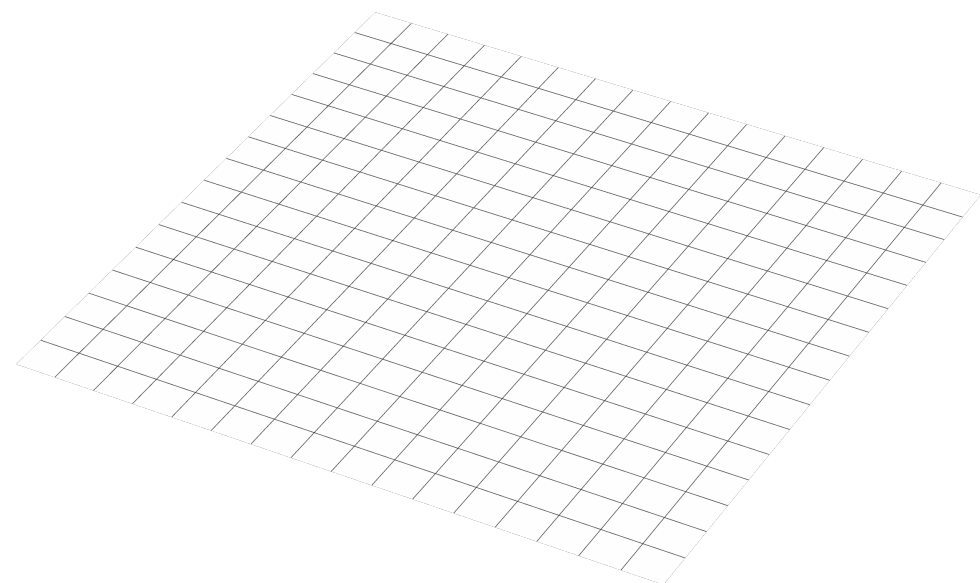
A bi-connection theory: (Q.V. FoP, 2024, arXiv:2204.13980)

- ▶ Spacetime manifold $\mathcal{M} = \mathbb{R} \times \Sigma$, with Σ a closed 3-manifold.
- ▶ A **physical** Lorentzian structure (g, ∇) and its Riemann curvature tensor $\mathbf{Riem}(\nabla)$.
- ▶ A **background**, symmetric connection $\bar{\nabla}^{\text{topo}}$ and its reference Riemann curvature tensor $\mathbf{Riem}(\bar{\nabla}^{\text{topo}})$.

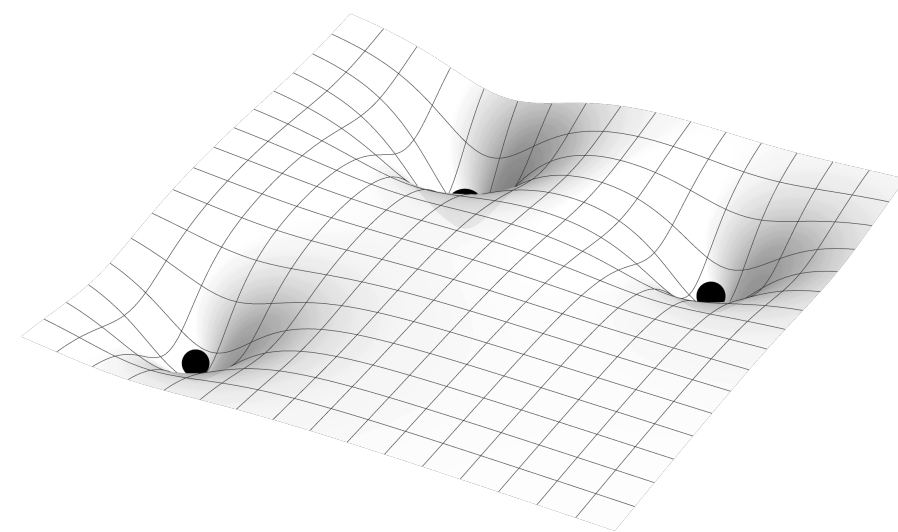
$$S = \frac{1}{16\pi G} \int_{\mathcal{M}} dx^4 \sqrt{|g|} \left(R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} \right) g^{\mu\nu} + \mathcal{B} + S_{\text{matter}}$$

Similar to
Rosen (1980)

$$\text{GR: } R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$



Vacuum: $R_{\mu\nu} = 0$



Matter



A bi-connection theory: (Q.V. FoP, 2024, arXiv:2204.13980)

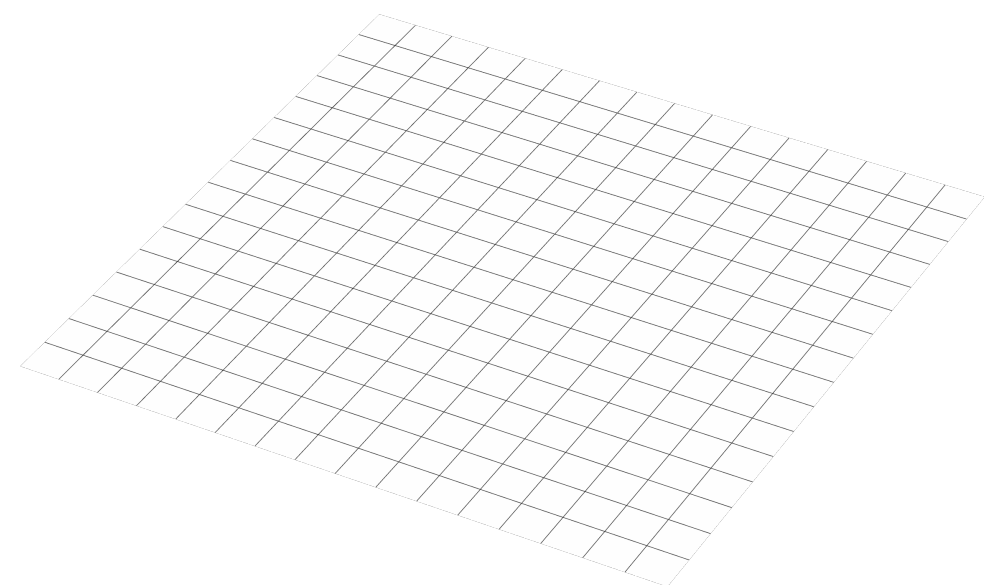
- ▶ Spacetime manifold $\mathcal{M} = \mathbb{R} \times \Sigma$, with Σ a closed 3-manifold.
- ▶ A **physical** Lorentzian structure (g, ∇) and its Riemann curvature tensor $Riem(\nabla)$.
- ▶ A **background**, symmetric connection $\bar{\nabla}^{topo}$ and its reference Riemann curvature tensor $Riem(\bar{\nabla}^{topo})$.

$$S = \frac{1}{16\pi G} \int_{\mathcal{M}} dx^4 \sqrt{|g|} \left(R_{\mu\nu} - \bar{R}_{\mu\nu}^{topo} \right) g^{\mu\nu} + \mathcal{B} + S_{matter}$$

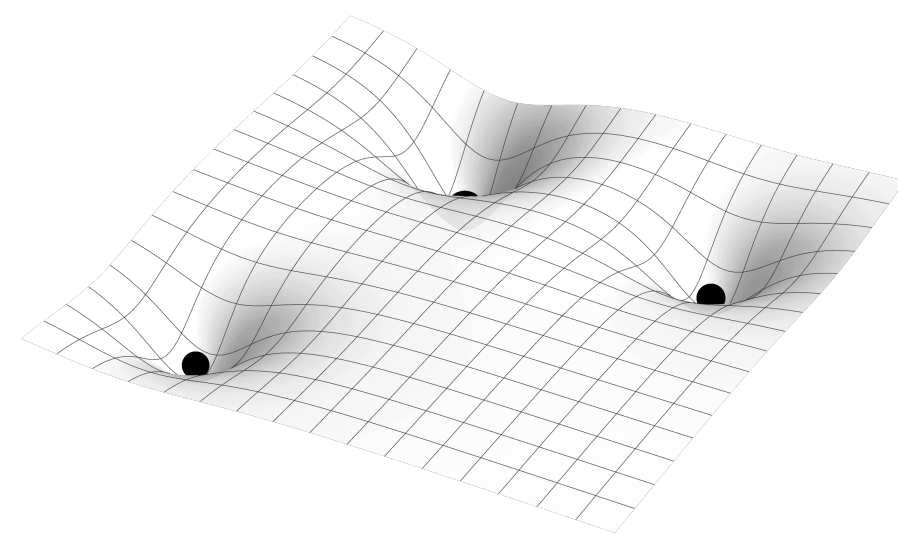
Similar to
Rosen (1980)

GR : $R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$

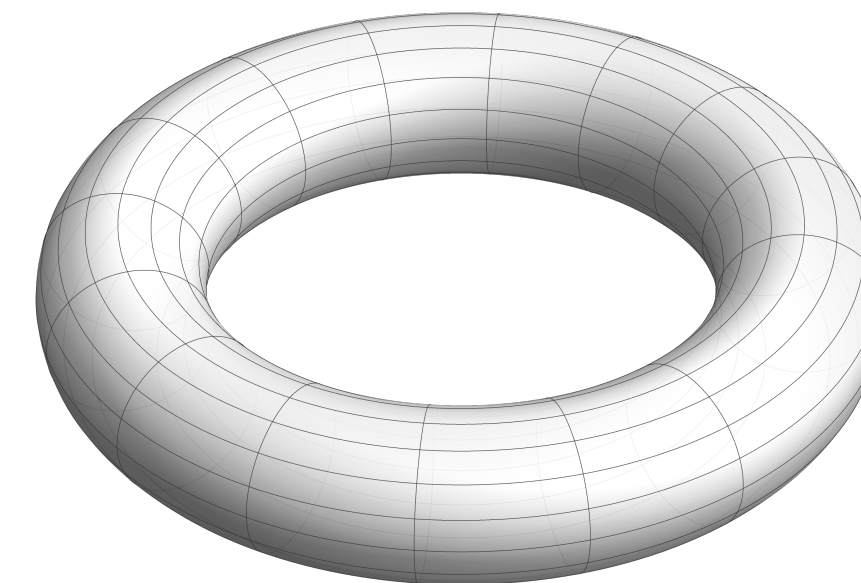
topo-GR : $R_{\mu\nu} - \bar{R}_{\mu\nu}^{topo} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$



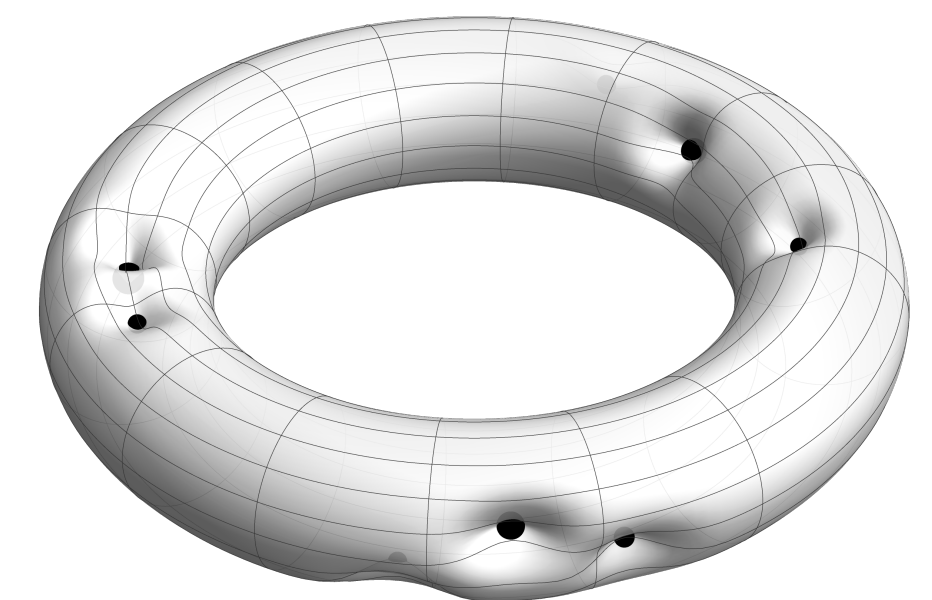
Vacuum: $R_{\mu\nu} = 0$



Matter



Vacuum : $R_{\mu\nu} = \bar{R}_{\mu\nu}^{topo}$



Matter

$$R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$

Motivations:

- * **Existence of a Newtonian limit in any topology.**
 - *Q.V. (PRD 2021; PRD 2022; CQG 2022; FoP 2024); Q.V. & B. Roukema (PRD 2023)*
- * From the Carrollian limit? (In prog.)
 - *Q.V. & H. Barzegar & J. Read (2025, preprint)*
- * Well-posed variational principle in any topology? (In prog.)

Cosmology: a universal dynamics

- * Expansion does not depend on Ω_K .
 - *Q.V. & V. Poulin (PRD 2023)*
- * Simple model for curved inflation.
 - *Q.V. & J. Larena (CQG 2024)*
- * Homogeneous but anisotropic solutions. (in prog.)
 - *Q.V. & H. Barzegar (in prep.)*

NEWTONIAN REGIME AND TOPOLOGY

$\tilde{\Sigma}$

\mathbb{E}^3

S^3

H^3

$\mathbb{R} \times S^2$

$\mathbb{R} \times H^2$

$\tilde{SL}(2, \mathbb{R})$

Nil

Sol

Relativistic regime

(i.e. Lorentz symmetry)

$$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$$

Non-relativistic regime

(i.e. Galilean symmetry)

NEWTONIAN REGIME AND TOPOLOGY

$\tilde{\Sigma}$	\mathbb{E}^3	S^3	H^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times H^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Relativistic regime (i.e. Lorentz symmetry)						$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$		
Non-relativistic regime (i.e. Galilean symmetry)	$\Delta\Phi = 4\pi G (\rho - \langle\rho\rangle),$ $\mathcal{R}_{ij} = 0$...	⋮						

NEWTONIAN REGIME AND TOPOLOGY

$\tilde{\Sigma}$

\mathbb{E}^3

S^3

H^3

$\mathbb{R} \times S^2$

$\mathbb{R} \times H^2$

$\tilde{SL}(2, \mathbb{R})$

Nil

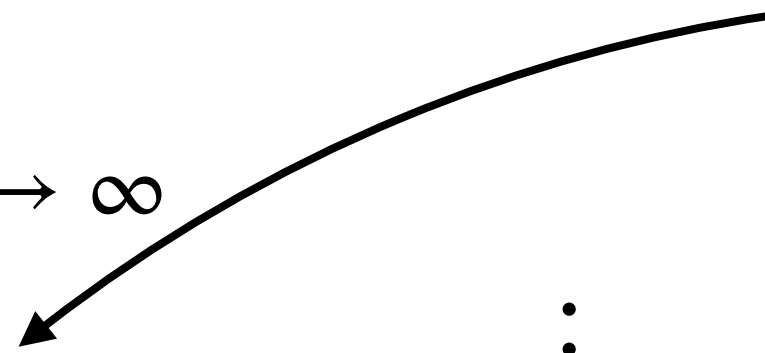
Sol

Relativistic regime

(i.e. Lorentz symmetry)

$$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$$

$c \rightarrow \infty$



$$\Delta\Phi = 4\pi G (\rho - \langle \rho \rangle),$$

$$\mathcal{R}_{ij} = 0$$

...

⋮

Non-relativistic regime

(i.e. Galilean symmetry)

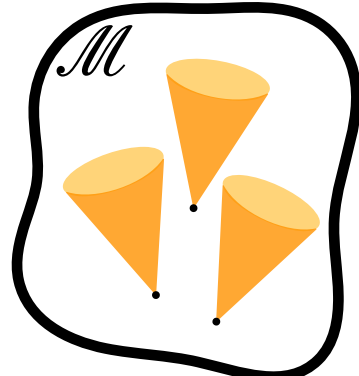
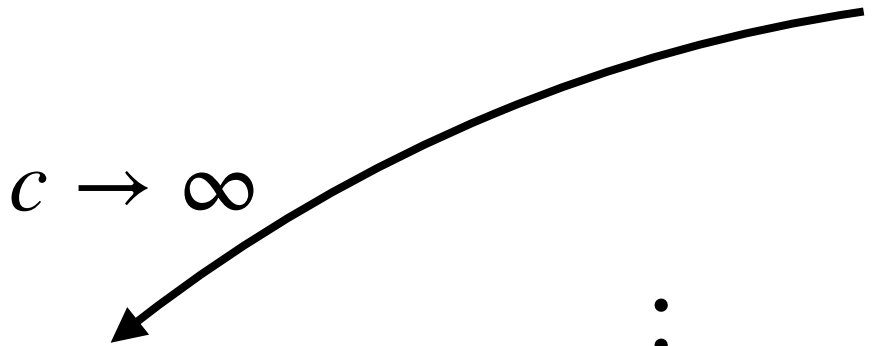
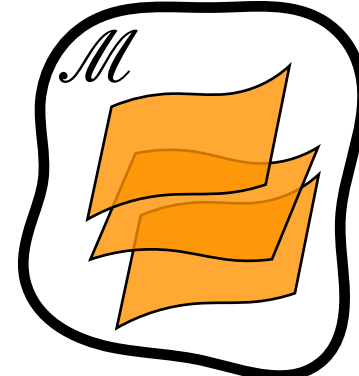
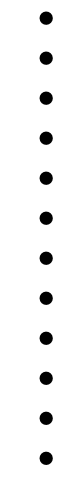
$\tilde{\Sigma}$	\mathbb{E}^3	S^3	H^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times H^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
<p>Relativistic regime (i.e. Lorentz symmetry)</p>	$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$							
<p>Non-relativistic regime (i.e. Galilean symmetry)</p>	$c \rightarrow \infty$							
	$\Delta\Phi = 4\pi G (\rho - \langle\rho\rangle),$ $\mathcal{R}_{ij} = 0$ \dots							
	$?? \downarrow c \rightarrow \infty$ $??$							

(1) How to define Newtonian gravitation on a non-Euclidean topology?

(2) Compatibility with Einstein's equation?

$\tilde{\Sigma}$	\mathbb{E}^3	S^3	H^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times H^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
<p>Relativistic regime (i.e. Lorentz symmetry)</p>	$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$							
<p>Non-relativistic regime (i.e. Galilean symmetry)</p>	$c \rightarrow \infty$							
	$\Delta\Phi = 4\pi G (\rho - \langle\rho\rangle),$ $\mathcal{R}_{ij} = 0$ \dots							
	$?? \downarrow c \rightarrow \infty$ $??$							

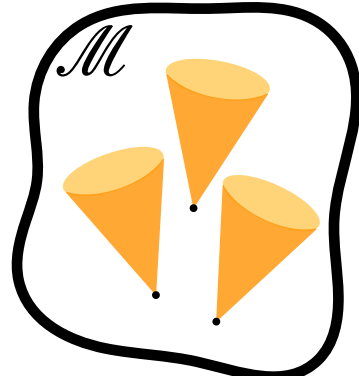
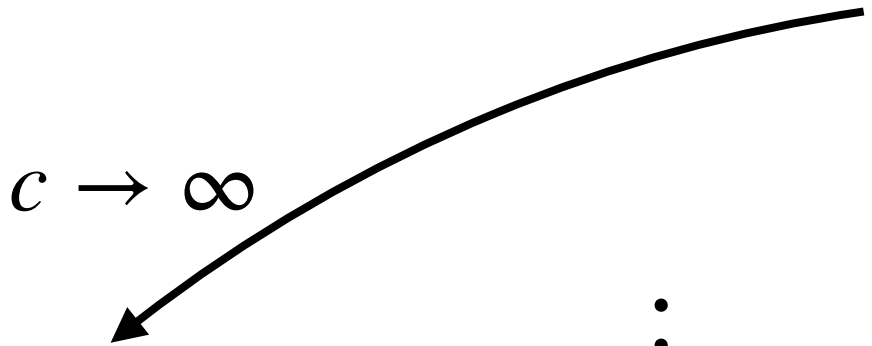
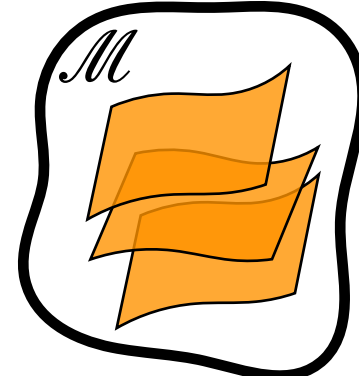
- (1) How to define Newtonian gravitation on a non-Euclidean topology?
 - (a) What is Newtonian gravitation?
- (2) Compatibility with Einstein's equation?

$\tilde{\Sigma}$	\mathbb{E}^3	S^3	H^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times H^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Relativistic regime (i.e. Lorentz symmetry) (g, ∇) 	$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$							
	$c \rightarrow \infty$ 							
Non-relativistic regime (i.e. Galilean symmetry) $(\tau, h, \hat{\nabla})$ 	$\hat{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_\mu \tau_\nu$ 							
	$?? \downarrow c \rightarrow \infty$ $??$							

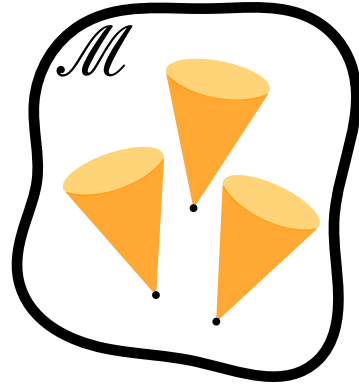
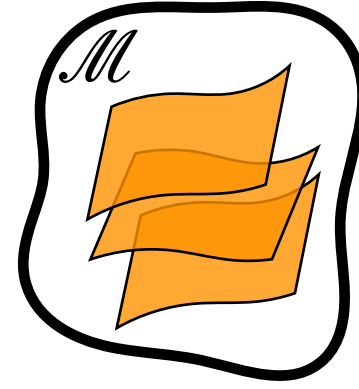
(1) How to define Newtonian gravitation on a non-Euclidean topology?

(a) What is Newtonian gravitation? \longrightarrow With Galilean structures: $(\tau, h, \hat{\nabla})$ with $\tau_\nu h^{\nu\mu} = 0, \hat{\nabla} h = 0 = \hat{\nabla} \tau$

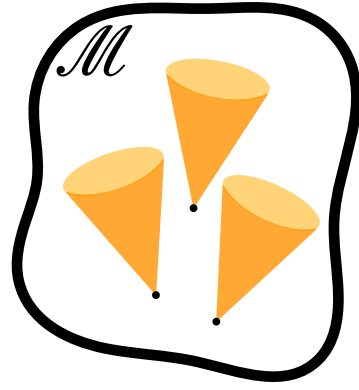
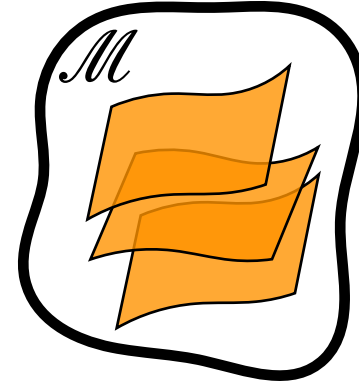
(2) Compatibility with Einstein's equation?

$\tilde{\Sigma}$	\mathbb{E}^3	S^3	H^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times H^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Relativistic regime (i.e. Lorentz symmetry) (g, ∇) 	$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$							
	$c \rightarrow \infty$ 							
Non-relativistic regime (i.e. Galilean symmetry) $(\tau, h, \hat{\nabla})$ 	$\hat{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_\mu \tau_\nu$							
	$\hat{R}_{\mu\nu} - \bar{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_\mu \tau_\nu$							

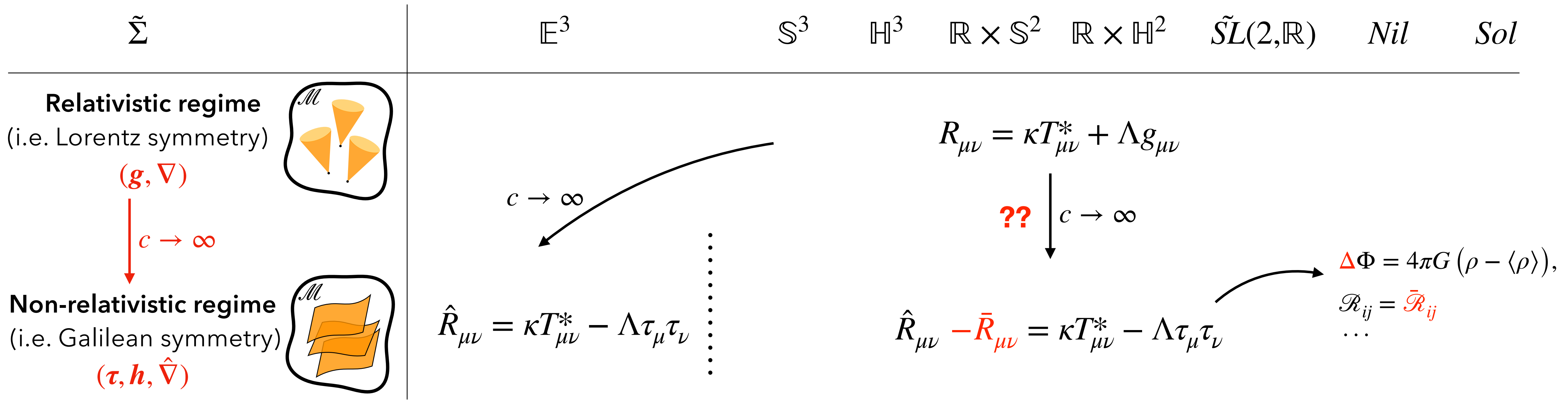
- (1) How to define Newtonian gravitation on a non-Euclidean topology? \longrightarrow ~ Same equations, with $\mathcal{R}_{ij} \neq 0$.
 - (a) What is Newtonian gravitation? \longrightarrow With Galilean structures: $(\tau, h, \hat{\nabla})$ with $\tau_\nu h^{\nu\mu} = 0, \hat{\nabla} h = 0 = \hat{\nabla} \tau$
- (2) Compatibility with Einstein's equation?

$\tilde{\Sigma}$	\mathbb{E}^3	S^3	H^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times H^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Relativistic regime (i.e. Lorentz symmetry) (g, ∇) 	$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$							
	$c \rightarrow \infty$							
Non-relativistic regime (i.e. Galilean symmetry) $(\tau, h, \hat{\nabla})$ 	$\hat{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_\mu \tau_\nu$							
	$\hat{R}_{\mu\nu} - \bar{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_\mu \tau_\nu$							
	$\Delta\Phi = 4\pi G(\rho - \langle\rho\rangle),$ $\mathcal{R}_{ij} = \bar{\mathcal{R}}_{ij}$ \dots							

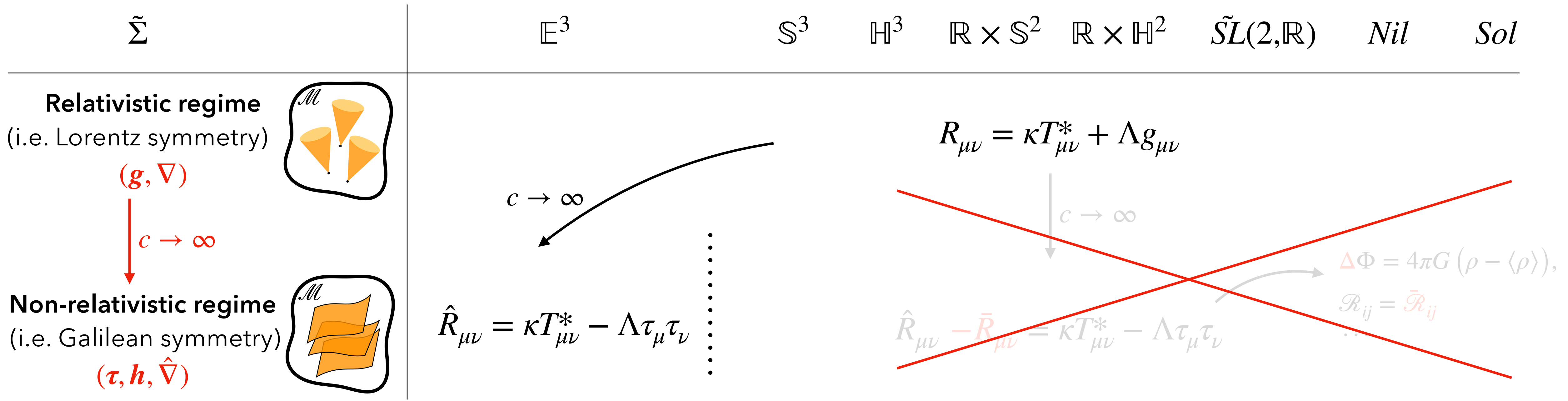
- (1) How to define Newtonian gravitation on a non-Euclidean topology? \longrightarrow ~ Same equations, with $\mathcal{R}_{ij} \neq 0$.
 - (a) What is Newtonian gravitation? \longrightarrow With Galilean structures: $(\tau, h, \hat{\nabla})$ with $\tau_\nu h^{\nu\mu} = 0, \hat{\nabla} h = 0 = \hat{\nabla} \tau$
- (2) Compatibility with Einstein's equation?

$\tilde{\Sigma}$	\mathbb{E}^3	S^3	H^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times H^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Relativistic regime (i.e. Lorentz symmetry) (g, ∇) 	$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$							
	$c \rightarrow \infty$							
Non-relativistic regime (i.e. Galilean symmetry) $(\tau, h, \hat{\nabla})$ 	$\hat{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_\mu \tau_\nu$							
	$\hat{R}_{\mu\nu} - \bar{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_\mu \tau_\nu$							
	$\Delta\Phi = 4\pi G(\rho - \langle\rho\rangle),$ $\mathcal{R}_{ij} = \bar{\mathcal{R}}_{ij}$ \dots							

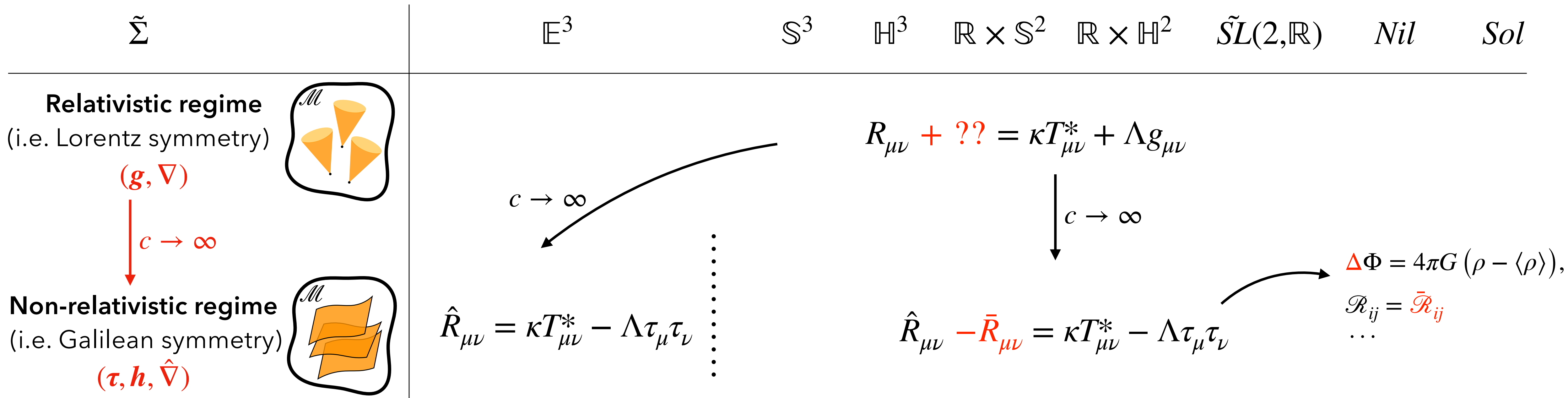
- (1) How to define Newtonian gravitation on a non-Euclidean topology? \longrightarrow ~ Same equations, with $\mathcal{R}_{ij} \neq 0$.
 - (a) What is Newtonian gravitation? \longrightarrow With Galilean structures: $(\tau, h, \hat{\nabla})$ with $\tau_\nu h^{\nu\mu} = 0, \hat{\nabla} h = 0 = \hat{\nabla} \tau$
- (2) Compatibility with Einstein's equation?
 - (b) How to define the Newtonian limit in any topology?



- (1) How to define Newtonian gravitation on a non-Euclidean topology? \longrightarrow ~ Same equations, with $\mathcal{R}_{ij} \neq 0$.
 - (a) What is Newtonian gravitation? \longrightarrow With Galilean structures: $(\tau, h, \hat{\nabla})$ with $\tau_\nu h^{\nu\mu} = 0, \hat{\nabla} h = 0 = \hat{\nabla} \tau$
- (2) Compatibility with Einstein's equation?
 - (b) How to define the Newtonian limit in any topology? \longrightarrow With Galilean structures.



- (1) How to define Newtonian gravitation on a non-Euclidean topology? \longrightarrow ~ Same equations, with $\mathcal{R}_{ij} \neq 0$.
 - (a) What is Newtonian gravitation? \longrightarrow With Galilean structures: $(\tau, h, \hat{\nabla})$ with $\tau_\nu h^{\nu\mu} = 0, \hat{\nabla} h = 0 = \hat{\nabla} \tau$
- (2) Compatibility with Einstein's equation? **NO!**
 - (b) How to define the Newtonian limit in any topology? \longrightarrow With Galilean structures.



(1) How to define Newtonian gravitation on a non-Euclidean topology? \longrightarrow ~ Same equations, with $\mathcal{R}_{ij} \neq 0$.

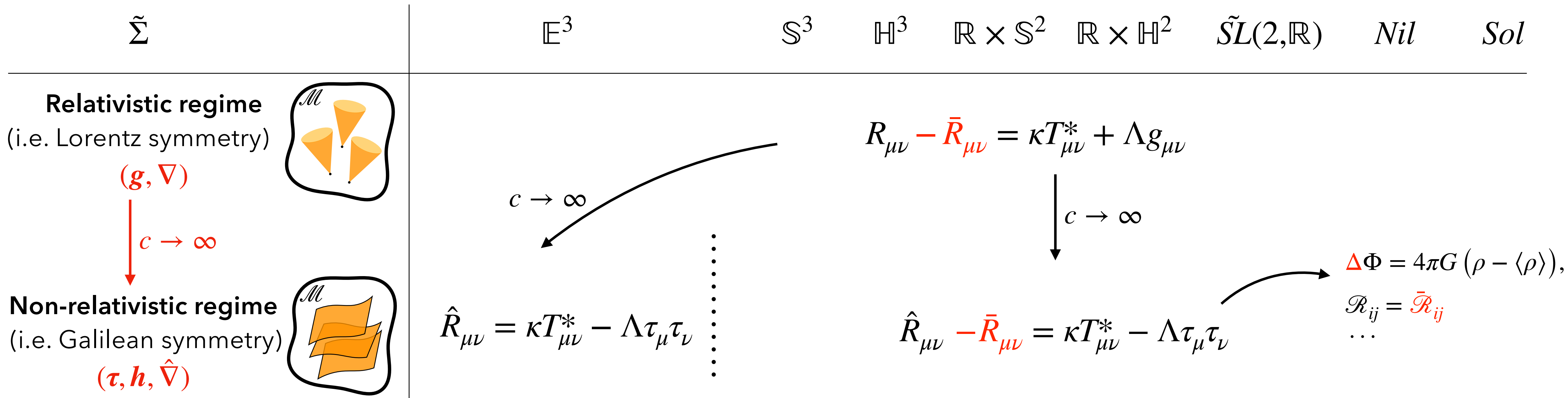
(a) What is Newtonian gravitation? \longrightarrow With Galilean structures: $(\tau, h, \hat{\nabla})$ with $\tau_\nu h^{\nu\mu} = 0, \hat{\nabla} h = 0 = \hat{\nabla} \tau$

(2) Compatibility with Einstein's equation? **NO!**

(b) How to define the Newtonian limit in any topology? \longrightarrow With Galilean structures.

(3) What relativistic equation admits a Newtonian limit in any topology?

New principle: Existence of a non-relativistic limit of gravitation in any topology.



(1) How to define Newtonian gravitation on a non-Euclidean topology? \longrightarrow ~ Same equations, with $\mathcal{R}_{ij} \neq 0$.

(a) What is Newtonian gravitation? \longrightarrow With Galilean structures: $(\tau, h, \hat{\nabla})$ with $\tau_\nu h^{\nu\mu} = 0, \hat{\nabla} h = 0 = \hat{\nabla} \tau$

(2) Compatibility with Einstein's equation? **NO!**

(b) How to define the Newtonian limit in any topology? \longrightarrow With Galilean structures.

(3) What relativistic equation admits a Newtonian limit in any topology?

New principle: Existence of a non-relativistic limit of gravitation in any topology.

$$R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$

Motivations:

- * Existence of a Newtonian limit in any topology.
 - *Q.V. (PRD 2021; PRD 2022; CQG 2022; FoP 2024); Q.V. & B. Roukema (PRD 2023)*
- * **From the Carrollian limit? (In prog.)**
 - *Q.V. & H. Barzegar & J. Read (2025, preprint)*
- * Well-posed variational principle in any topology? (In prog.)

Cosmology: a universal dynamics

- * Expansion does not depend on Ω_K .
 - *Q.V. & V. Poulin (PRD 2023)*
- * Simple model for curved inflation.
 - *Q.V. & J. Larena (CQG 2024)*
- * Homogeneous but anisotropic solutions. (in prog.)
 - *Q.V. & H. Barzegar (in prep.)*

CARROLLIAN REGIME AND TOPOLOGY

$\tilde{\Sigma}$

\mathbb{E}^3

S^3

H^3

$\mathbb{R} \times S^2$

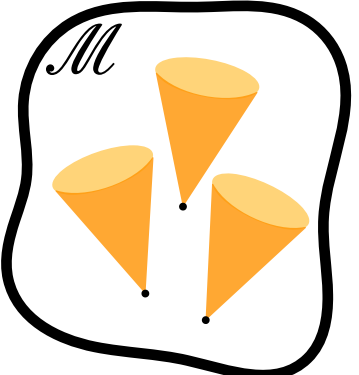
$\mathbb{R} \times H^2$

$\tilde{SL}(2, \mathbb{R})$

Nil

Sol

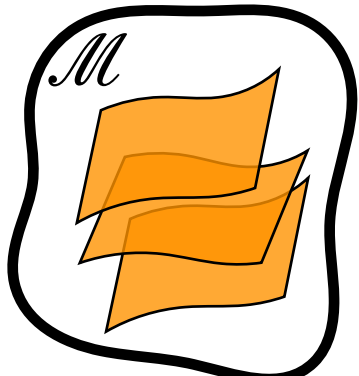
Relativistic regime
(i.e. Lorentz symmetry)



(g, ∇)

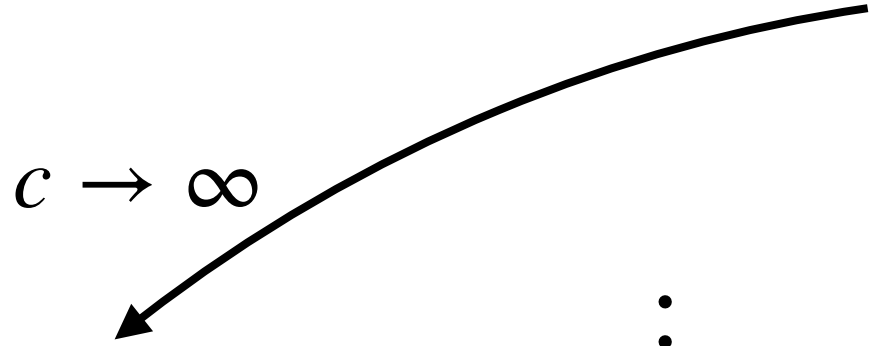
$c \rightarrow \infty$

Non-relativistic regime
(i.e. Galilean symmetry)



$(\tau, h, \hat{\nabla})$

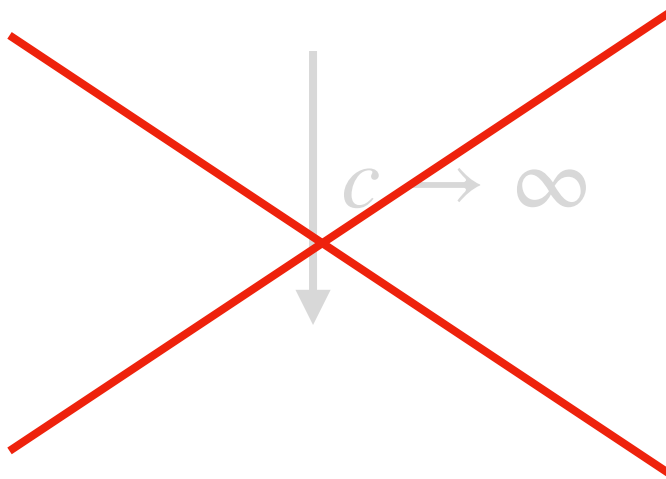
$c \rightarrow \infty$



$$\hat{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_\mu \tau_\nu$$

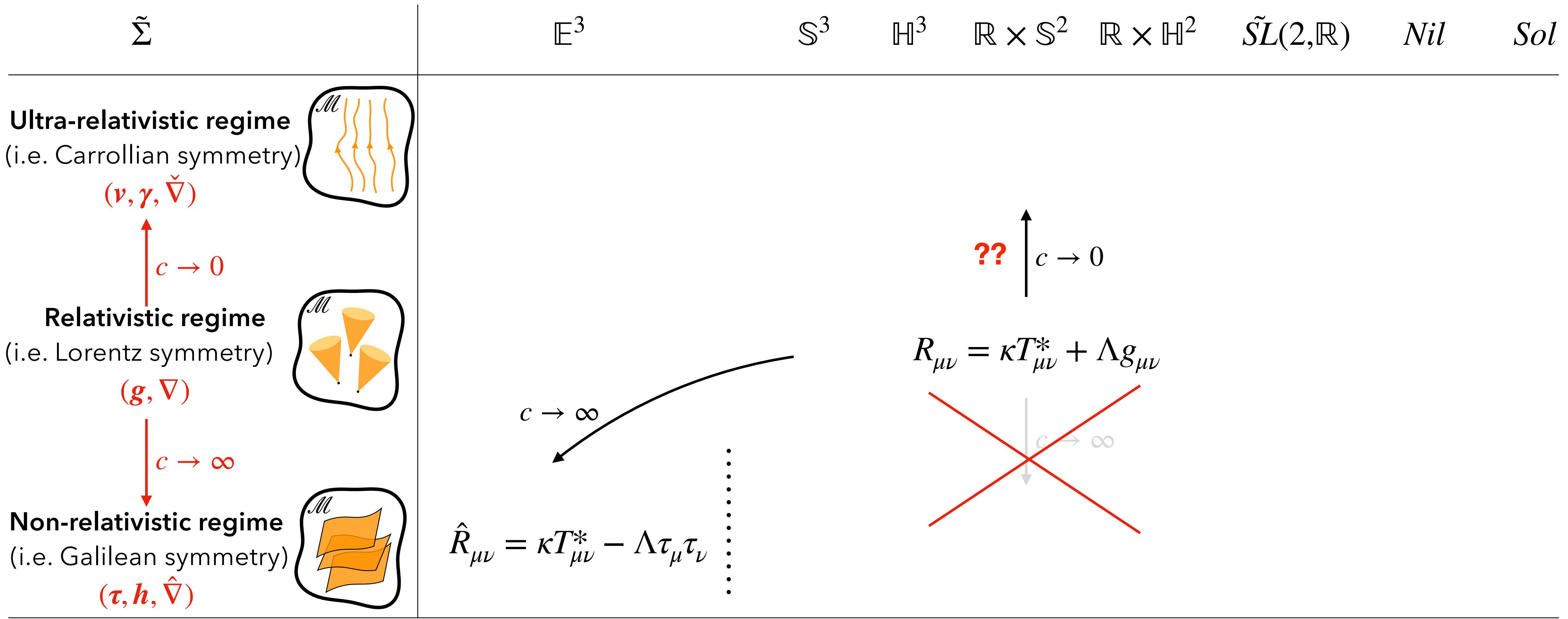
⋮

$$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$$

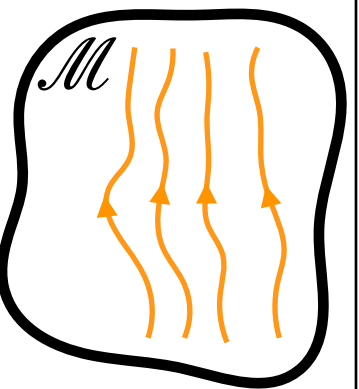


CARROLLIAN REGIME AND TOPOLOGY

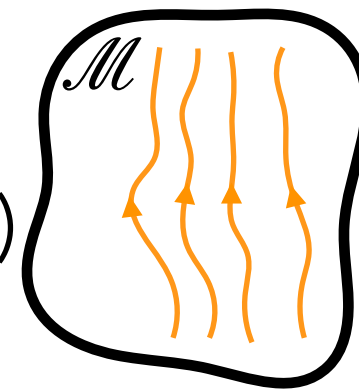
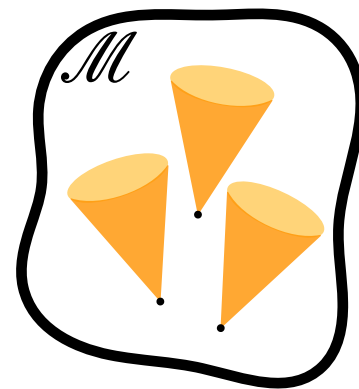
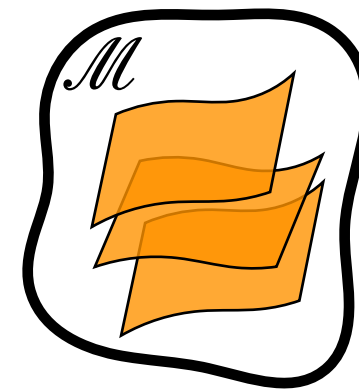
$\tilde{\Sigma}$	\mathbb{E}^3	S^3	H^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times H^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
<p>Ultra-relativistic regime (i.e. Carrollian symmetry)</p> <p>$(\nu, \gamma, \check{\nabla})$</p> <p>$c \rightarrow 0$</p> <p>Relativistic regime (i.e. Lorentz symmetry)</p> <p>(g, ∇)</p> <p>$c \rightarrow \infty$</p> <p>Non-relativistic regime (i.e. Galilean symmetry)</p> <p>$(\tau, h, \hat{\nabla})$</p>								
	$\hat{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_\mu \tau_\nu$							
	 $R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$ 							
	$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$							



• Is the Carrollian limit of Einstein's equation defined for any topology?

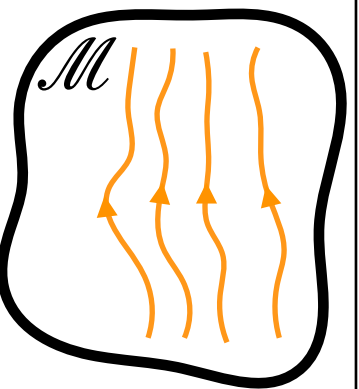
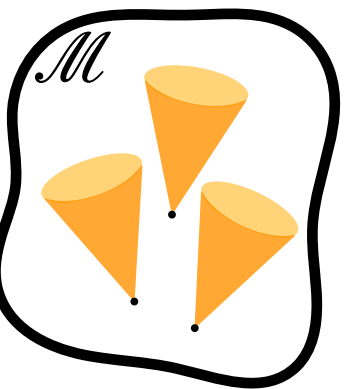
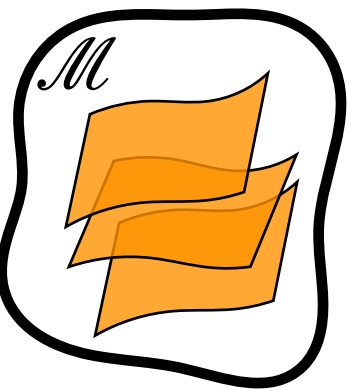
$\tilde{\Sigma}$	\mathbb{E}^3	S^3	H^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times H^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>	
<p>Ultra-relativistic regime (i.e. Carrollian symmetry)</p> <p>$(\mathbf{v}, \gamma, \check{\nabla})$</p> <p>$\uparrow c \rightarrow 0$</p> <p>Relativistic regime (i.e. Lorentz symmetry)</p> <p>(\mathbf{g}, ∇)</p> <p>$\downarrow c \rightarrow \infty$</p> <p>Non-relativistic regime (i.e. Galilean symmetry)</p> <p>$(\boldsymbol{\tau}, \mathbf{h}, \hat{\nabla})$</p>		$\mathbf{K} := -\frac{1}{2} \mathcal{L}_v \gamma$ $\begin{cases} K_{\mu\nu} K^{\mu\nu} - K^2 = \rho \\ D_\nu K^{\mu\nu} - D^\mu K = 0 \end{cases}$	$\mathcal{L}_v K_{\mu\nu} = -2K_\mu{}^\sigma K_{\nu\sigma} + K K_{\mu\nu}$	<p>??</p> <p>$\uparrow c \rightarrow 0$</p>	$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$	<p>$\downarrow c \rightarrow \infty$</p>	$\hat{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_\mu \tau_\nu$ <p>⋮</p>		

• Is the Carrollian limit of Einstein's equation defined for any topology?

$\tilde{\Sigma}$	\mathbb{E}^3	S^3	H^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times H^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
<p>Ultra-relativistic regime (i.e. Carrollian symmetry)</p> <p>$(\mathbf{v}, \boldsymbol{\gamma}, \check{\nabla})$</p> <p>$c \rightarrow 0$</p> 	$\mathbf{K} := -\frac{1}{2} \mathcal{L}_{\mathbf{v}} \boldsymbol{\gamma}$ $\begin{cases} K_{\mu\nu} K^{\mu\nu} - K^2 = \rho \\ D_{\nu} K^{\mu\nu} - D^{\mu} K = 0 \end{cases}$					$\mathcal{L}_{\mathbf{v}} K_{\mu\nu} = -2K_{\mu}^{\sigma} K_{\nu\sigma} + K K_{\mu\nu}$ $\mathcal{R}_{ij} \neq 0$		
<p>Relativistic regime (i.e. Lorentz symmetry)</p> <p>(\mathbf{g}, ∇)</p> <p>$c \rightarrow \infty$</p> 	$c \rightarrow \infty$					$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$		
<p>Non-relativistic regime (i.e. Galilean symmetry)</p> <p>$(\boldsymbol{\tau}, \mathbf{h}, \hat{\nabla})$</p> 	$\hat{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_{\mu} \tau_{\nu}$					$c \rightarrow \infty$		

• Is the Carrollian limit of Einstein's equation defined for any topology?

Yes

$\tilde{\Sigma}$	\mathbb{E}^3	S^3	\mathbb{H}^3	$\mathbb{R} \times S^2$	$\mathbb{R} \times \mathbb{H}^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
<p>Ultra-relativistic regime (i.e. Carrollian symmetry)</p> <p>$(\mathbf{v}, \boldsymbol{\gamma}, \check{\nabla})$</p> <p>$c \rightarrow 0$</p> <p>Relativistic regime (i.e. Lorentz symmetry)</p> <p>(\mathbf{g}, ∇)</p> <p>$c \rightarrow \infty$</p> <p>Non-relativistic regime (i.e. Galilean symmetry)</p> <p>$(\boldsymbol{\tau}, \mathbf{h}, \hat{\nabla})$</p>		$\mathbf{K} := -\frac{1}{2} \mathcal{L}_{\mathbf{v}} \boldsymbol{\gamma} \quad \begin{cases} K_{\mu\nu} K^{\mu\nu} - K^2 = \rho \\ D_{\nu} K^{\mu\nu} - D^{\mu} K = 0 \end{cases}$	$\mathcal{L}_{\mathbf{v}} K_{\mu\nu} = -2K_{\mu}^{\sigma} K_{\nu\sigma} + K K_{\mu\nu}$	$\mathcal{R}_{ij} \neq 0$	$R_{\mu\nu} = \kappa T_{\mu\nu}^* + \Lambda g_{\mu\nu}$	$\hat{R}_{\mu\nu} = \kappa T_{\mu\nu}^* - \Lambda \tau_{\mu} \tau_{\nu}$		
				<p>??</p> <p>$c \rightarrow 0$</p>	<p>$c \rightarrow \infty$</p>			

• Is the Carrollian limit of Einstein's equation defined for any topology?

Yes

• $\bar{\nabla}^{\text{topo}}$ is a Galilean, a Carrollian and a Lorentzian connection: i.e. $\exists(\boldsymbol{\tau}, \mathbf{h}), (\mathbf{v}, \boldsymbol{\gamma}), \mathbf{g}, \bar{\nabla} \boldsymbol{\tau} = \bar{\nabla} \mathbf{h} = \bar{\nabla} \mathbf{v} = \bar{\nabla} \boldsymbol{\gamma} = \bar{\nabla} \mathbf{g} = 0$.

$$R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$

Motivations:

- * Existence of a Newtonian limit in any topology.
 - *Q.V. (PRD 2021; PRD 2022; CQG 2022; FoP 2024); Q.V. & B. Roukema (PRD 2023)*
- * From the Carrollian limit? (In prog.)
 - *Q.V. & H. Barzegar & J. Read (2025, preprint)*
- * **Well-posed variational principle in any topology? (In prog.)**

Cosmology: a universal dynamics

- * Expansion does not depend on Ω_K .
 - *Q.V. & V. Poulin (PRD 2023)*
- * Simple model for curved inflation.
 - *Q.V. & J. Larena (CQG 2024)*
- * Homogeneous but anisotropic solutions. (in prog.)
 - *Q.V. & H. Barzegar (in prep.)*

A background connection is often needed in general relativity:

- ▶ Well-posed, finite valued, action (e.g., [Dyers et al., 2009](#)):

$$S_{\text{EH+GHY}} = \int_{\mathcal{M}} \sqrt{-g} g^{\mu\nu} R_{\mu\nu} dx^4$$

A background connection is often needed in general relativity:

- ▶ Well-posed, finite valued, action (e.g., [Dyers et al., 2009](#)):

$$S_{\text{EH+GHY}} = \int_{\mathcal{M}} \sqrt{-g} g^{\mu\nu} R_{\mu\nu} dx^4 + 2 \int_{\partial\mathcal{M}} \sqrt{|h|} n^\mu h^{\alpha\beta} \left(\partial_\mu g_{\alpha\beta} - \partial_\alpha g_{\beta\mu} \right) dx^3$$

—> Definitions of energy, mass, entropy.

A background connection is often needed in general relativity:

- ▶ Well-posed, finite valued, action (e.g., [Dyers et al., 2009](#)):

$$S_{\text{EH+GHY}} = \int_{\mathcal{M}} \sqrt{-g} g^{\mu\nu} R_{\mu\nu} dx^4 + 2 \int_{\partial\mathcal{M}} \sqrt{|h|} n^\mu h^{\alpha\beta} \left(\bar{\nabla}_\mu g_{\alpha\beta} - \bar{\nabla}_\alpha g_{\beta\mu} \right) dx^3 \quad \text{with } \bar{R}^\alpha_{\beta\mu\nu} = 0$$

→ Definitions of energy, mass, entropy.

A background connection is often needed in general relativity:

- ▶ Well-posed, finite valued, action (e.g., [Dyers et al., 2009](#)):

$$S_{\text{EH+GHY}} = \int_{\mathcal{M}} \sqrt{-g} g^{\mu\nu} R_{\mu\nu} dx^4 + 2 \int_{\partial\mathcal{M}} \sqrt{|h|} n^\mu h^{\alpha\beta} \left(\bar{\nabla}_\mu g_{\alpha\beta} - \bar{\nabla}_\alpha g_{\beta\mu} \right) dx^3 \quad \text{with } \bar{R}^\alpha_{\beta\mu\nu} = 0$$

→ Definitions of energy, mass, entropy.

- ▶ Equivalent to a first order covariant variational principle (e.g., [Erdmenger et al., 2024](#)):

$$S_{\mathcal{C}} = \int_{\mathcal{M}} \sqrt{|g|} \mathcal{C}^\alpha_{\mu[\beta} \mathcal{C}^\beta_{\nu]\alpha} g^{\mu\nu} \quad \text{with the non-metricity } \mathcal{C}^\alpha_{\mu\nu} := \Gamma^\alpha_{\mu\nu} - \bar{\Gamma}^\alpha_{\mu\nu}$$



$$R_{\mu\nu} = \kappa T^*_{\mu\nu} + \Lambda g_{\mu\nu}$$

A background connection is often needed in general relativity:

- ▶ Well-posed, finite valued, action (e.g., [Dyers et al., 2009](#)):

$$S_{\text{EH+GHY}} = \int_{\mathcal{M}} \sqrt{-g} g^{\mu\nu} R_{\mu\nu} dx^4 + 2 \int_{\partial\mathcal{M}} \sqrt{|h|} n^\mu h^{\alpha\beta} \left(\bar{\nabla}_\mu g_{\alpha\beta} - \bar{\nabla}_\alpha g_{\beta\mu} \right) dx^3 \quad \text{with } \bar{R}^\alpha_{\beta\mu\nu} = 0$$

→ Definitions of energy, mass, entropy.

- ▶ Equivalent to a first order covariant variational principle (e.g., [Erdmenger et al., 2024](#)):

$$S_{\mathcal{C}} = \int_{\mathcal{M}} \sqrt{|g|} \mathcal{C}^\alpha_{\mu[\beta} \mathcal{C}^\beta_{\nu]\alpha} g^{\mu\nu} \quad \text{with the non-metricity } \mathcal{C}^\alpha_{\mu\nu} := \Gamma^\alpha_{\mu\nu} - \bar{\Gamma}^\alpha_{\mu\nu}$$

$$R_{\mu\nu} = \kappa T^*_{\mu\nu} + \Lambda g_{\mu\nu}$$

→ $\bar{\nabla}$ always taken to be flat: $\bar{R}^\alpha_{\beta\mu\nu} = 0$ (Affine manifold)

Does it constrain the topology?

A background connection is often needed in general relativity:

- ▶ Well-posed, finite valued, action (e.g., Dyers et al., 2009):

$$S_{\text{EH+GHY}} = \int_{\mathcal{M}} \sqrt{-g} g^{\mu\nu} R_{\mu\nu} dx^4 + 2 \int_{\partial\mathcal{M}} \sqrt{|h|} n^\mu h^{\alpha\beta} \left(\bar{\nabla}_\mu g_{\alpha\beta} - \bar{\nabla}_\alpha g_{\beta\mu} \right) dx^3 \quad \text{with } \bar{R}^\alpha_{\beta\mu\nu} = 0$$

→ Definitions of energy, mass, entropy.

- ▶ Equivalent to a first order covariant variational principle (e.g., Erdmenger et al., 2024):

$$S_{\mathcal{C}} = \int_{\mathcal{M}} \sqrt{|g|} \mathcal{C}^\alpha_{\mu[\beta} \mathcal{C}^\beta_{\nu]\alpha} g^{\mu\nu} \quad \text{with the non-metricity } \mathcal{C}^\alpha_{\mu\nu} := \Gamma^\alpha_{\mu\nu} - \bar{\Gamma}^\alpha_{\mu\nu}$$

$$R_{\mu\nu} = \kappa T^*_{\mu\nu} + \Lambda g_{\mu\nu}$$

→ $\bar{\nabla}$ always taken to be flat: $\bar{R}^\alpha_{\beta\mu\nu} = 0$ (Affine manifold)

Does it constrain the topology?

Theorem: If $\bar{\nabla}$ is *complete*, the spacetime universal cover is homeomorphic to \mathbb{R}^n . (⇒ (e.g.) $\mathbb{R} \times S^3$ impossible)

A background connection is often needed in general relativity:

- ▶ Well-posed, finite valued, action (e.g., Dyers et al., 2009):

$$S_{\text{EH+GHY}} = \int_{\mathcal{M}} \sqrt{-g} g^{\mu\nu} R_{\mu\nu} dx^4 + 2 \int_{\partial\mathcal{M}} \sqrt{|h|} n^\mu h^{\alpha\beta} \left(\bar{\nabla}_\mu g_{\alpha\beta} - \bar{\nabla}_\alpha g_{\beta\mu} \right) dx^3 \quad \text{with } \bar{R}^\alpha_{\beta\mu\nu} = 0$$

→ Definitions of energy, mass, entropy.

- ▶ Equivalent to a first order covariant variational principle (e.g., Erdmenger et al., 2024):

$$S_{\mathcal{C}} = \int_{\mathcal{M}} \sqrt{|g|} \mathcal{C}^\alpha_{\mu[\beta} \mathcal{C}^\beta_{\nu]\alpha} g^{\mu\nu} \quad \text{with the non-metricity } \mathcal{C}^\alpha_{\mu\nu} := \Gamma^\alpha_{\mu\nu} - \bar{\Gamma}^\alpha_{\mu\nu}$$

$$R_{\mu\nu} = \kappa T^*_{\mu\nu} + \Lambda g_{\mu\nu}$$

→ $\bar{\nabla}$ always taken to be flat: ~~$\bar{R}^\alpha_{\beta\mu\nu} = 0$~~ (Affine manifold)

Does it constrain the topology?

Theorem: If $\bar{\nabla}$ is *complete*, the spacetime universal cover is homeomorphic to \mathbb{R}^n . (⇒ (e.g.) $\mathbb{R} \times S^3$ impossible)

Hypothesis: $\bar{\Gamma}^\alpha_{\mu\nu}$ should be adapted as function of the spacetime topology, i.e. $\bar{R}_{\mu\nu} \neq 0$ for non-Euclidean topologies.

A background connection is often needed in general relativity:

- ▶ Well-posed, finite valued, action (e.g., Dyers et al., 2009):

$$S_{\text{EH+GHY}} = \int_{\mathcal{M}} \sqrt{-g} g^{\mu\nu} R_{\mu\nu} dx^4 + 2 \int_{\partial\mathcal{M}} \sqrt{|h|} n^\mu h^{\alpha\beta} \left(\bar{\nabla}_\mu g_{\alpha\beta} - \bar{\nabla}_\alpha g_{\beta\mu} \right) dx^3 \quad \text{with } \bar{R}^\alpha_{\beta\mu\nu} = 0$$

→ Definitions of energy, mass, entropy.

- ▶ Equivalent to a first order covariant variational principle (e.g., Erdmenger et al., 2024):

$$S_{\mathcal{C}} = \int_{\mathcal{M}} \sqrt{|g|} \mathcal{C}^\alpha_{\mu[\beta} \mathcal{C}^\beta_{\nu]\alpha} g^{\mu\nu} \quad \text{with the non-metricity } \mathcal{C}^\alpha_{\mu\nu} := \Gamma^\alpha_{\mu\nu} - \bar{\Gamma}^\alpha_{\mu\nu}$$

$$R_{\mu\nu} - \bar{R}_{\mu\nu} = \kappa T^*_{\mu\nu} + \Lambda g_{\mu\nu} \quad (\text{topo-GR})$$

→ $\bar{\nabla}$ always taken to be flat: ~~$\bar{R}^\alpha_{\beta\mu\nu} = 0$~~ (Affine manifold)

Does it constrain the topology?

Theorem: If $\bar{\nabla}$ is *complete*, the spacetime universal cover is homeomorphic to \mathbb{R}^n . (⇒ (e.g.) $\mathbb{R} \times S^3$ impossible)

Hypothesis: $\bar{\Gamma}^\alpha_{\mu\nu}$ should be adapted as function of the spacetime topology, i.e. $\bar{R}_{\mu\nu} \neq 0$ for non-Euclidean topologies.

$$R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$

Motivations:

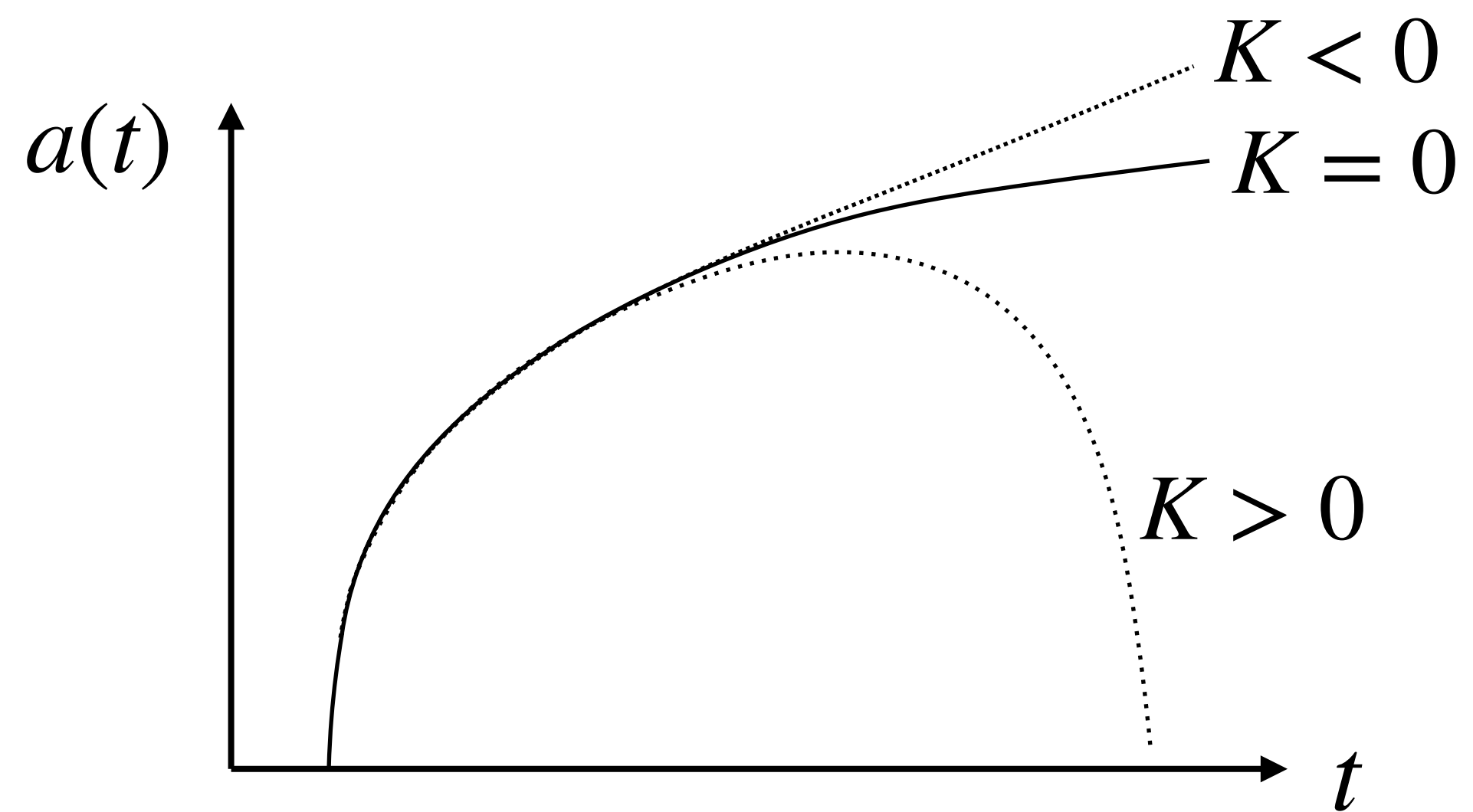
- * Existence of a Newtonian limit in any topology.
 - *Q.V. (PRD 2021; PRD 2022; CQG 2022; FoP 2024); Q.V. & B. Roukema (PRD 2023)*
- * From the Carrollian limit? (In prog.)
 - *Q.V. & H. Barzegar & J. Read (2025, preprint)*
- * Well-posed variational principle in any topology? (In prog.)

Cosmology: a universal dynamics

- * **Expansion does not depend on Ω_K .**
 - *Q.V. & V. Poulin (PRD 2023)*
- * Simple model for curved inflation.
 - *Q.V. & J. Larena (CQG 2024)*
- * Homogeneous but anisotropic solutions. (in prog.)
 - *Q.V. & H. Barzegar (in prep.)*

Exact homogeneous and isotropic solution: [Vigneron & Poulin \(2023\)](#)

$$\text{GR: } \begin{cases} \Omega_{\neq K} + \Omega_K = 1, \\ q = \Omega_m/2 + \Omega_{\text{rad}} - \Omega_\Lambda \end{cases}$$

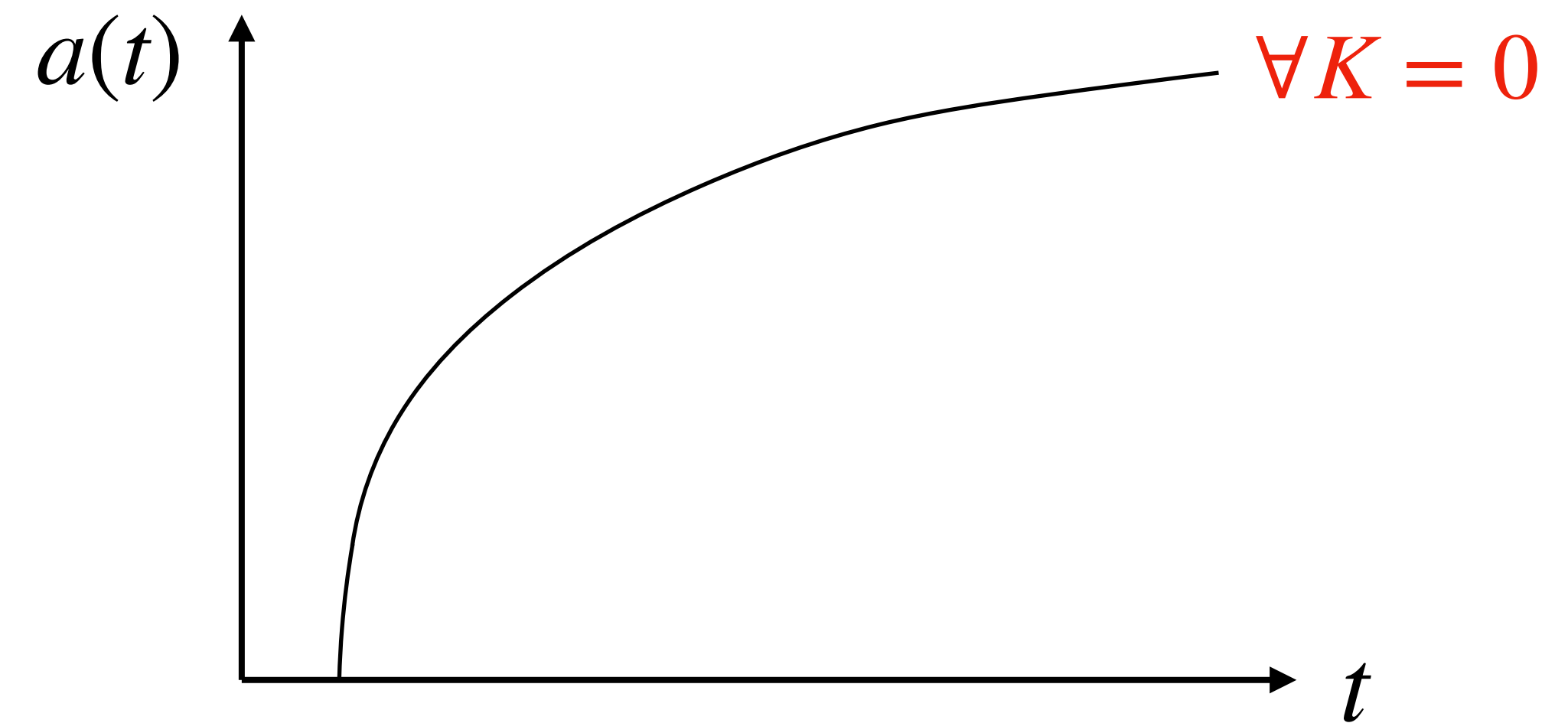
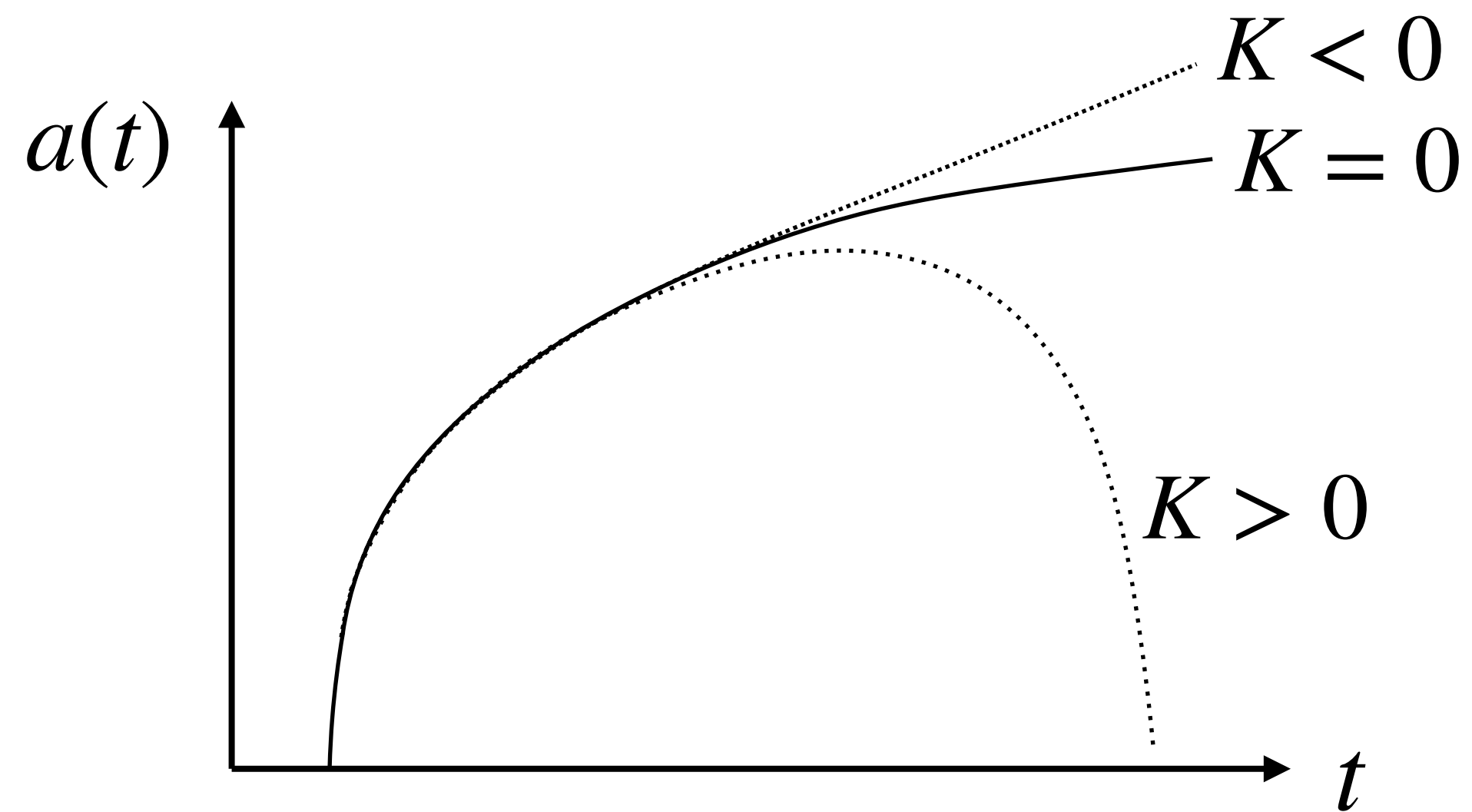


Exact homogeneous and isotropic solution: [Vigneron & Poulin \(2023\)](#)

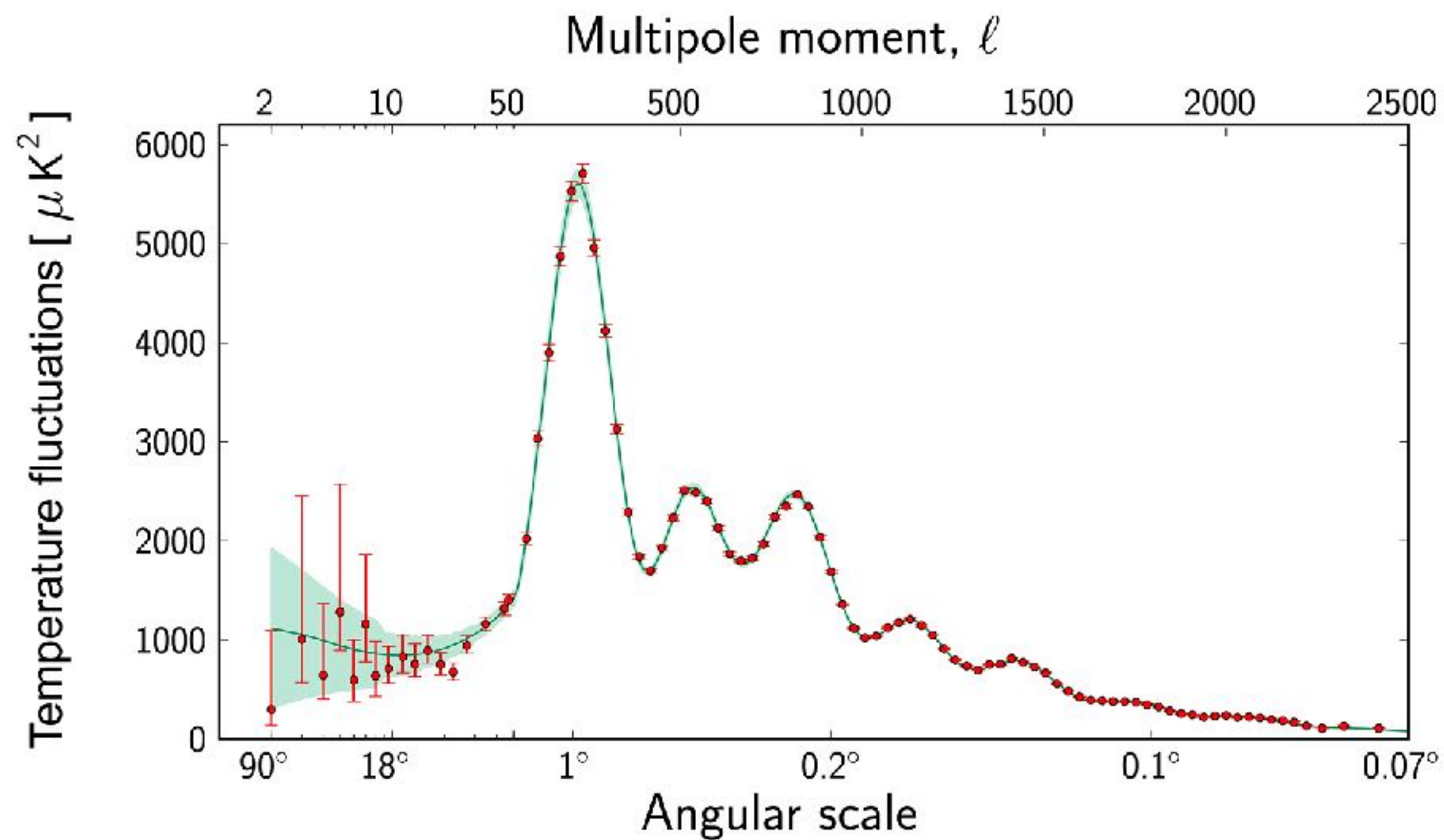
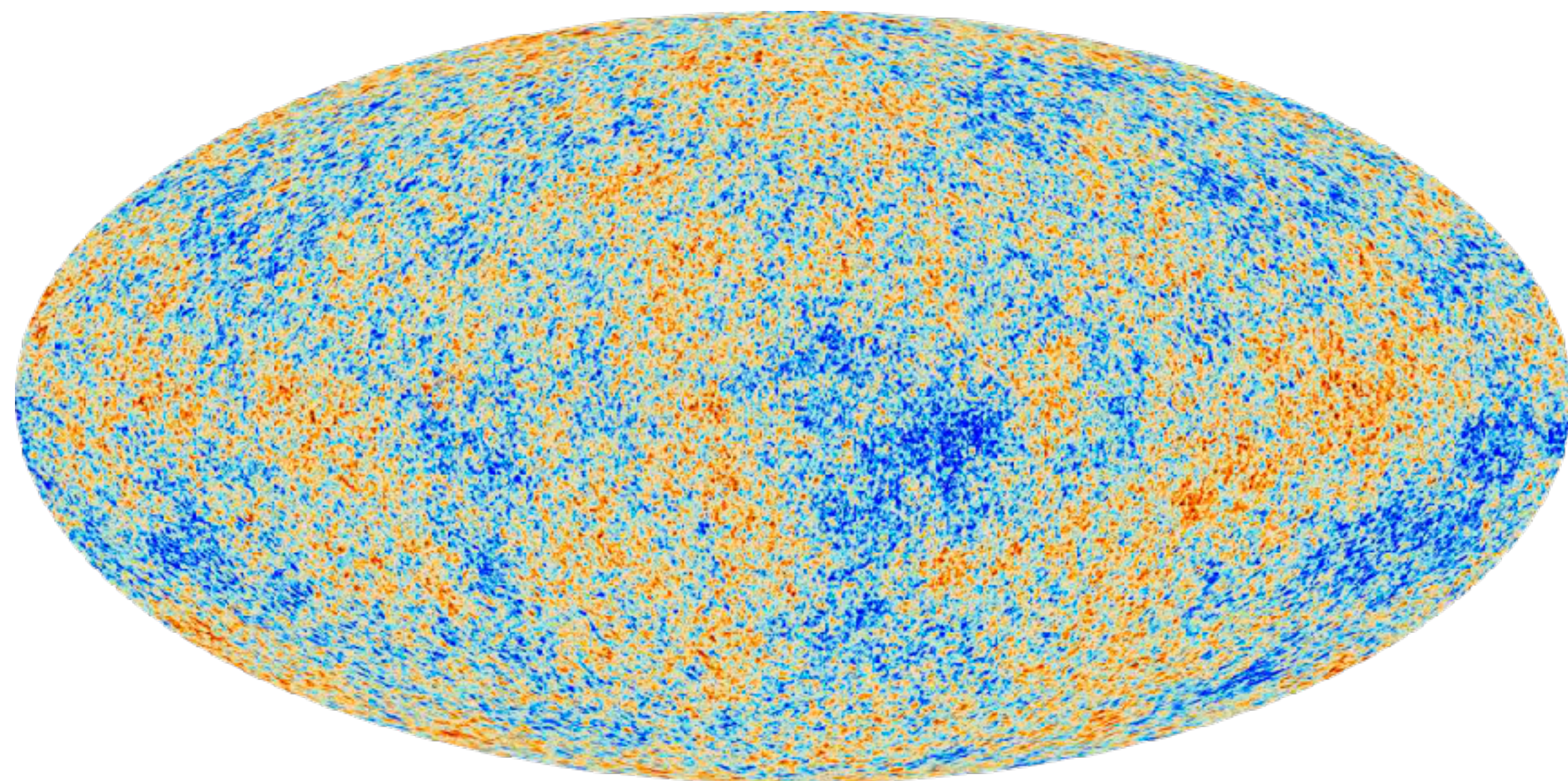
$$\text{GR: } \begin{cases} \Omega_{\neq K} + \Omega_K = 1, \\ q = \Omega_m/2 + \Omega_{\text{rad}} - \Omega_\Lambda \end{cases}$$

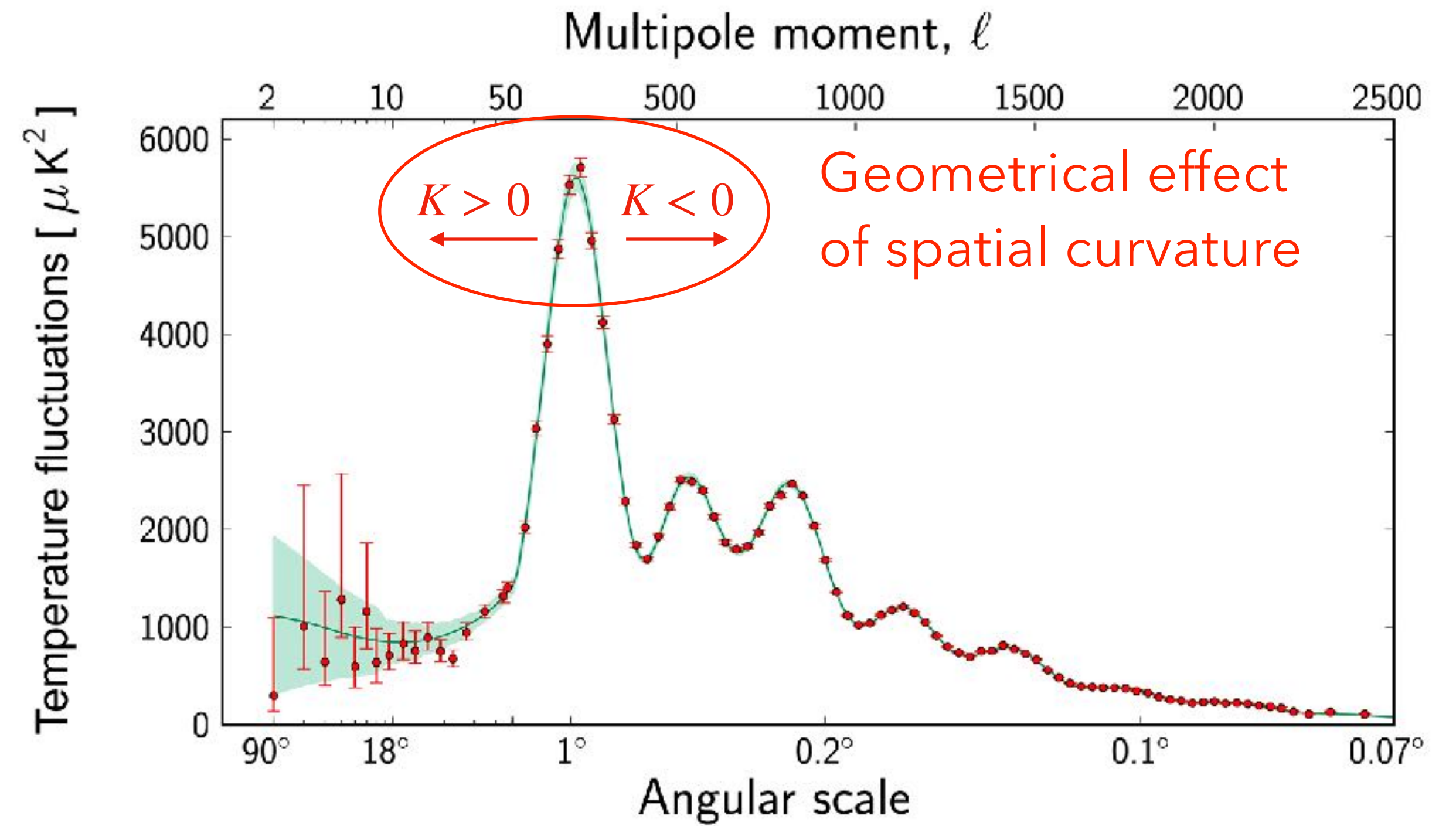
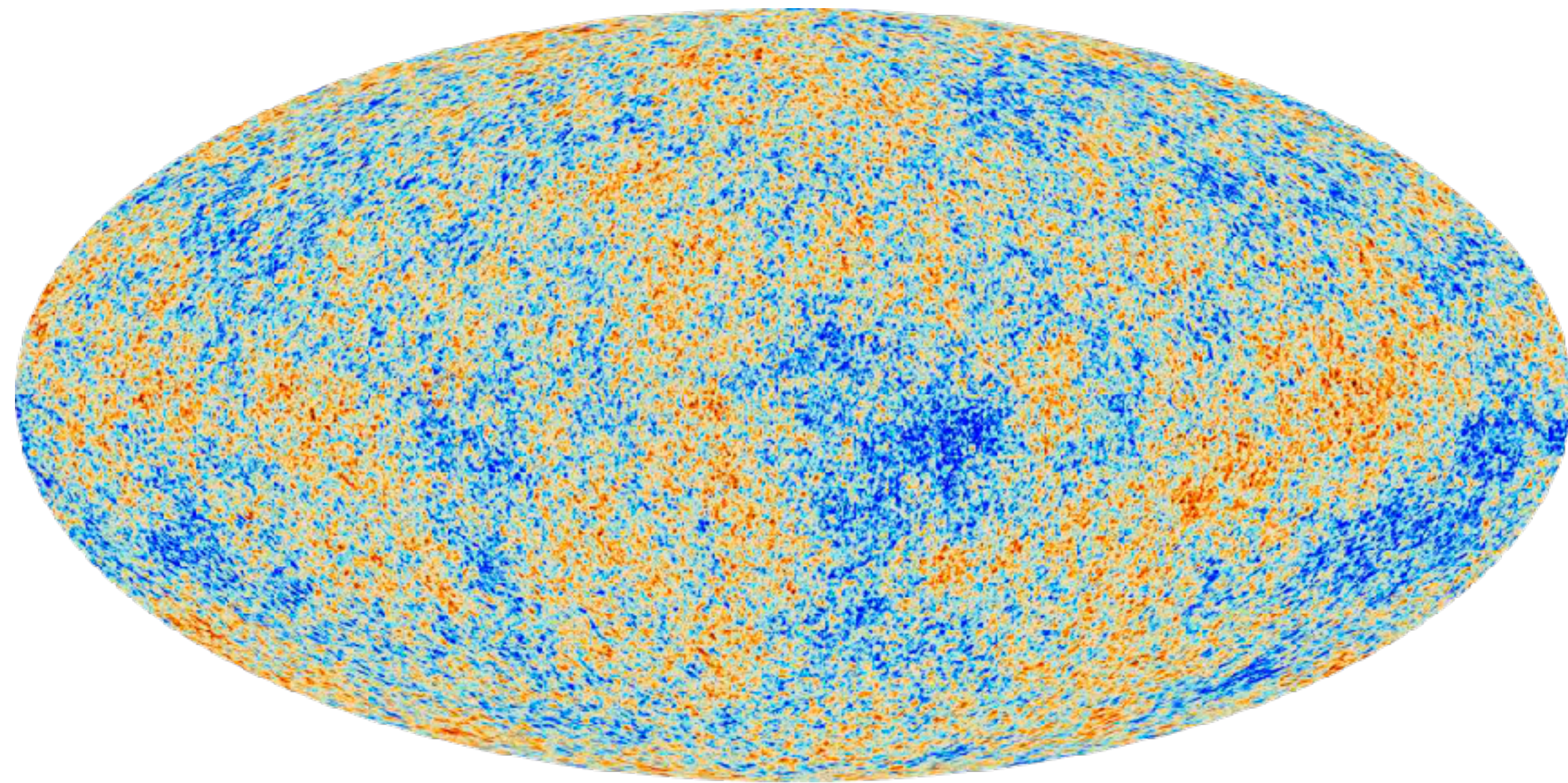


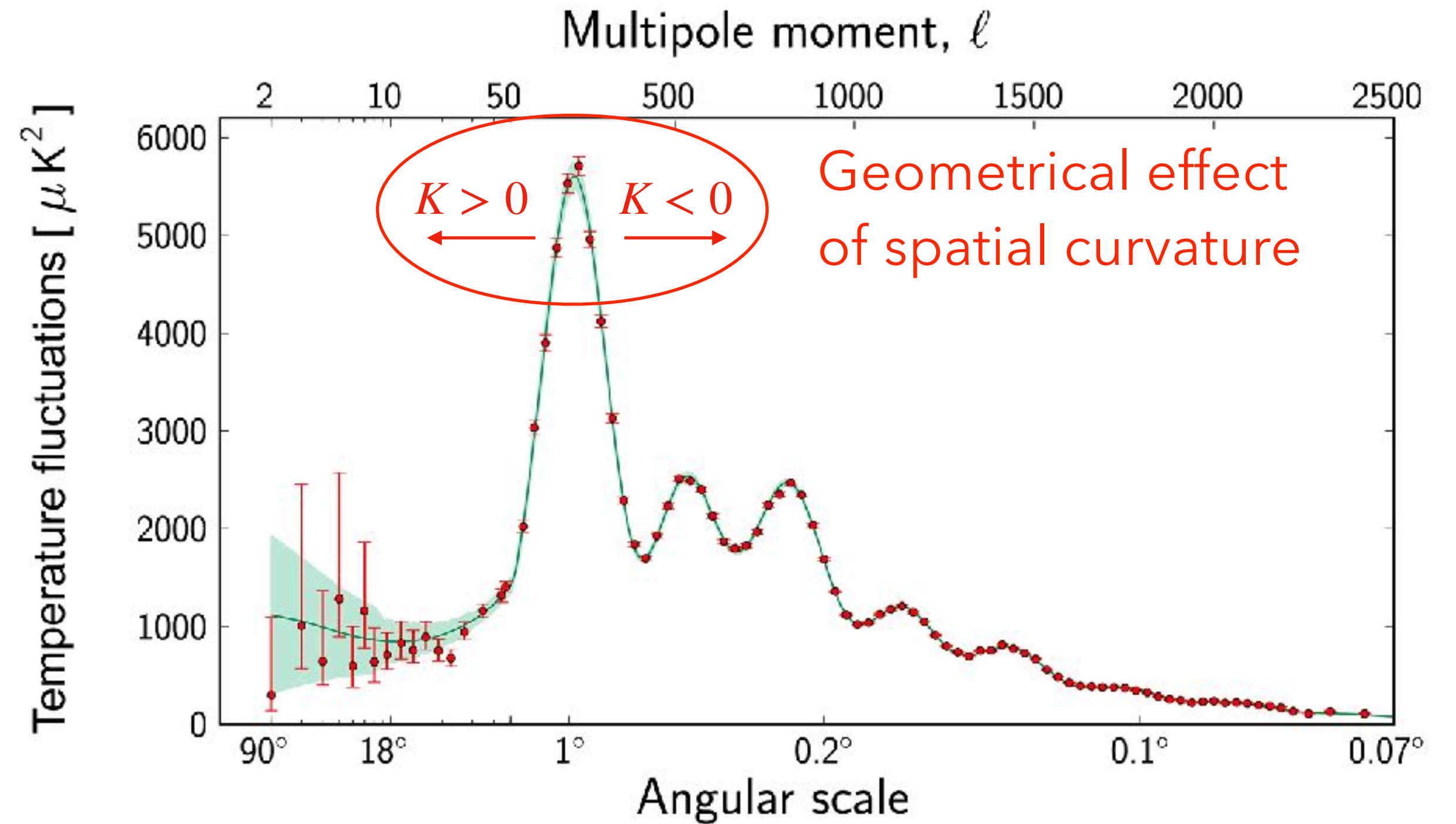
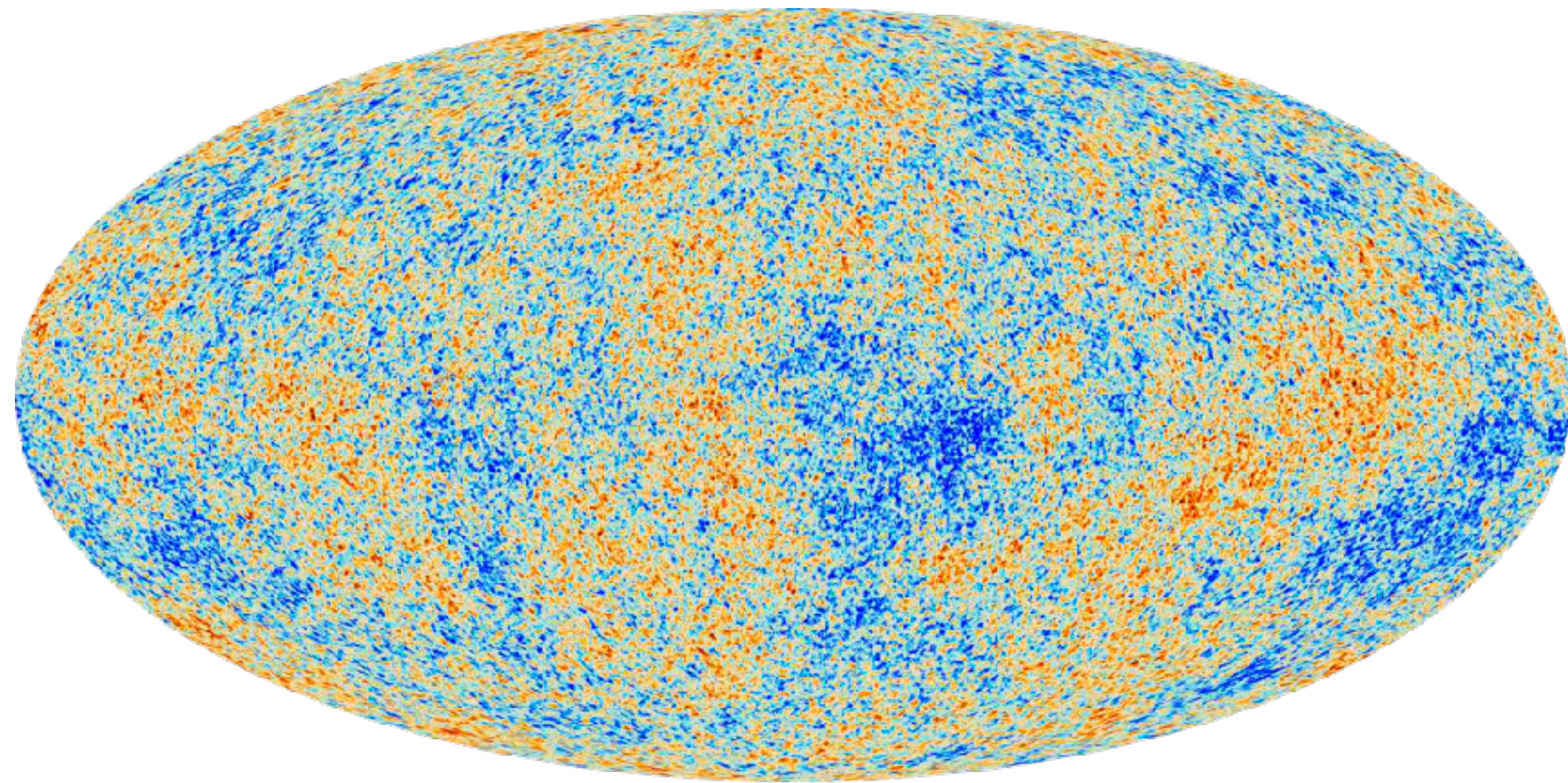
$$\text{topo-GR: } \begin{cases} \Omega_{\neq K} = 1, \quad \forall \Omega_K \\ q = \Omega_m/2 + \Omega_{\text{rad}} - \Omega_\Lambda \end{cases}$$



Question: What is the value of Ω_K from observations with this new model?







Results: No significant differences compared to the Λ CDM model, i.e. $\Omega_K \sim 0$.

$$R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$

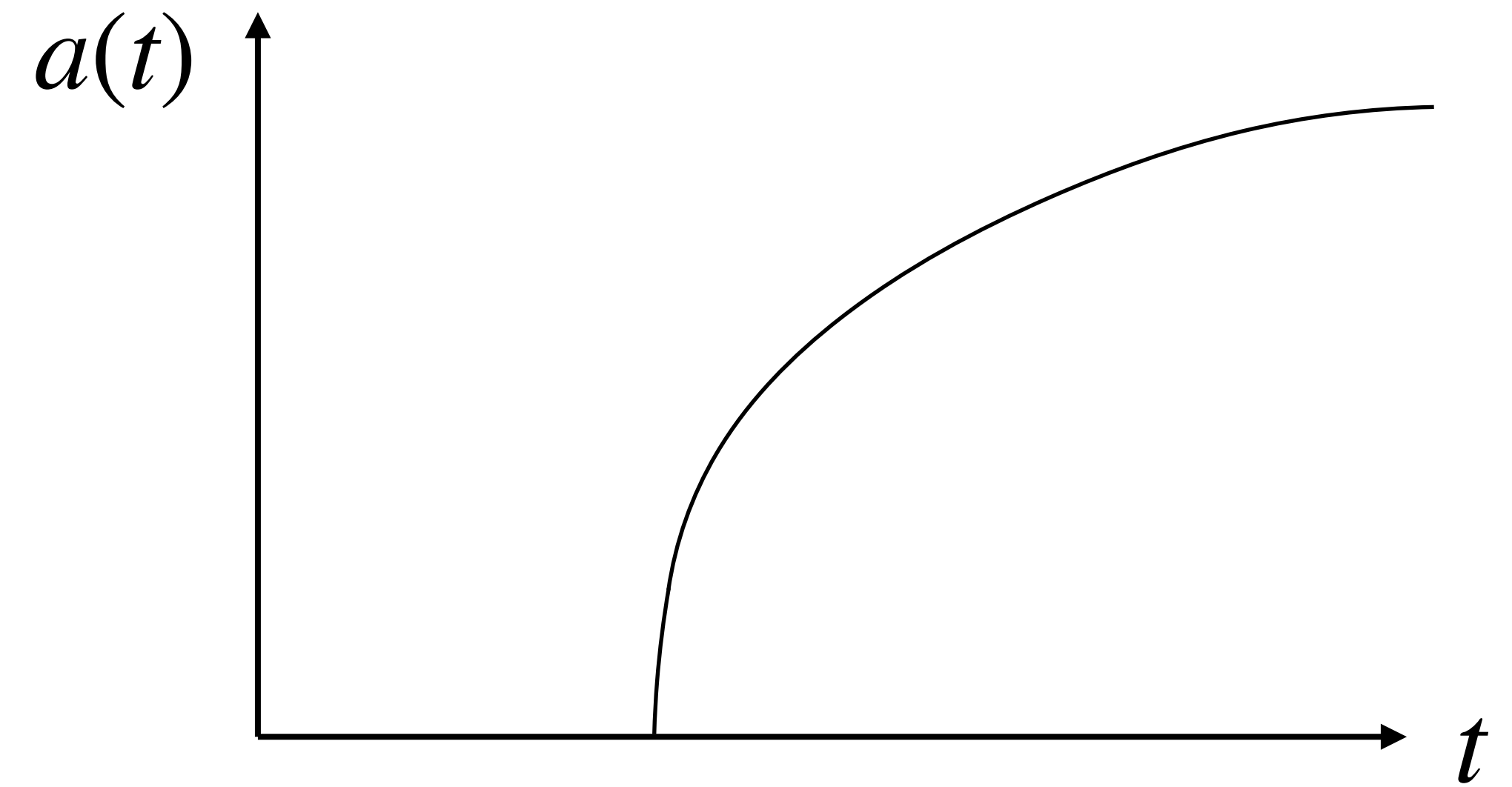
Motivations:

- * Existence of a Newtonian limit in any topology.
 - *Q.V. (PRD 2021; PRD 2022; CQG 2022; FoP 2024); Q.V. & B. Roukema (PRD 2023)*
- * From the Carrollian limit? (In prog.)
 - *Q.V. & H. Barzegar & J. Read (2025, preprint)*
- * Well-posed variational principle in any topology? (In prog.)

Cosmology: a universal dynamics

- * Expansion does not depend on Ω_K .
 - *Q.V. & V. Poulin (PRD 2023)*
- * **Simple model for curved inflation.**
 - *Q.V. & J. Larena (CQG 2024)*
- * Homogeneous but anisotropic solutions. (in prog.)
 - *Q.V. & H. Barzegar (in prep.)*

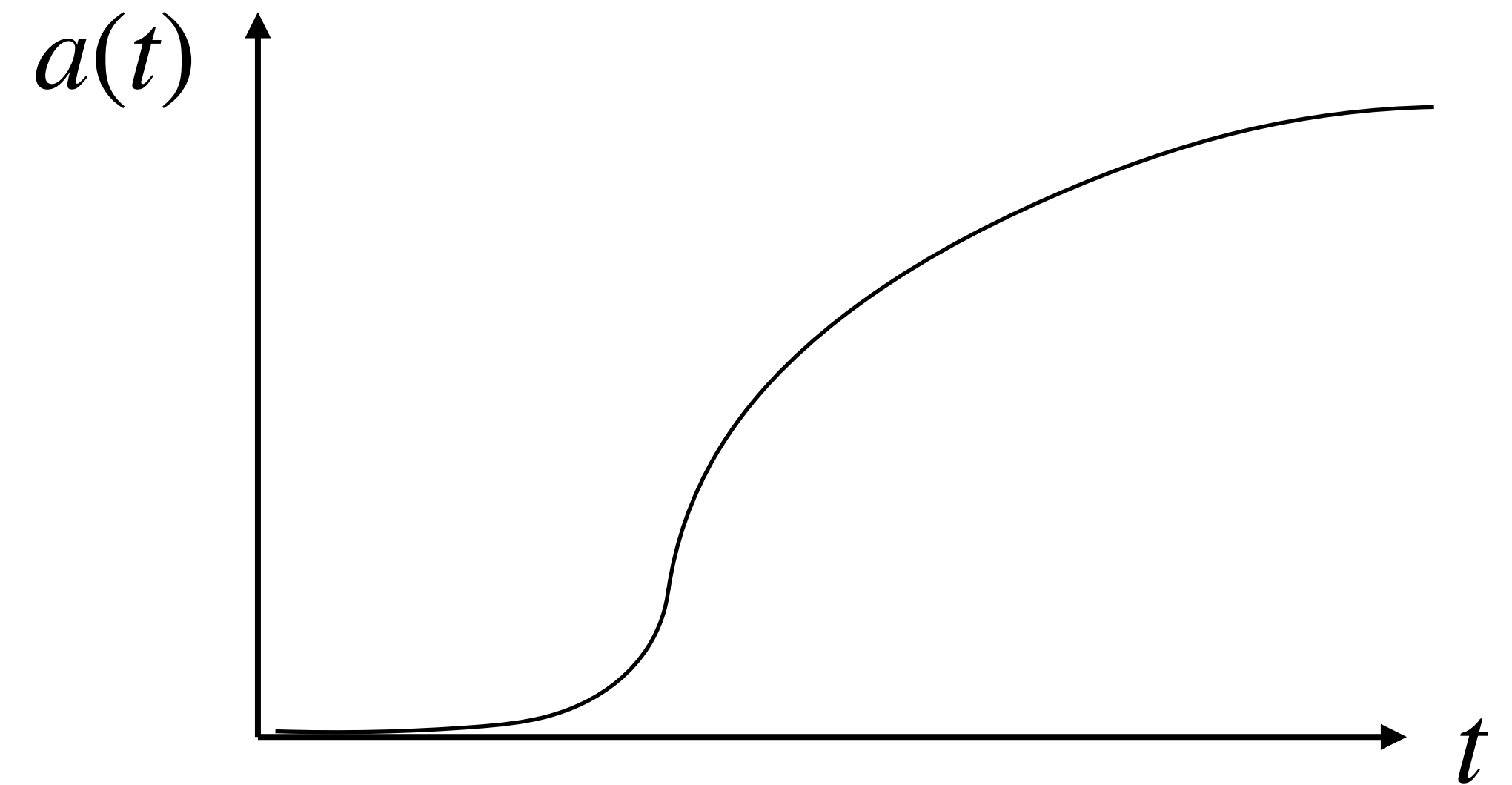
GENERAL RELATIVITY ($K = 0$)



GENERAL RELATIVITY ($K = 0$)

1. Matter described by a scalar field φ :

$$T^{\mu}_{\nu} = \nabla^{\mu}\varphi \nabla_{\nu}\varphi - \frac{1}{2}\nabla^{\alpha}\varphi \nabla_{\alpha}\varphi \delta^{\mu}_{\nu} - V\delta^{\mu}_{\nu}$$



Provides an origin for the causality of the CMB

GENERAL RELATIVITY ($K = 0$)

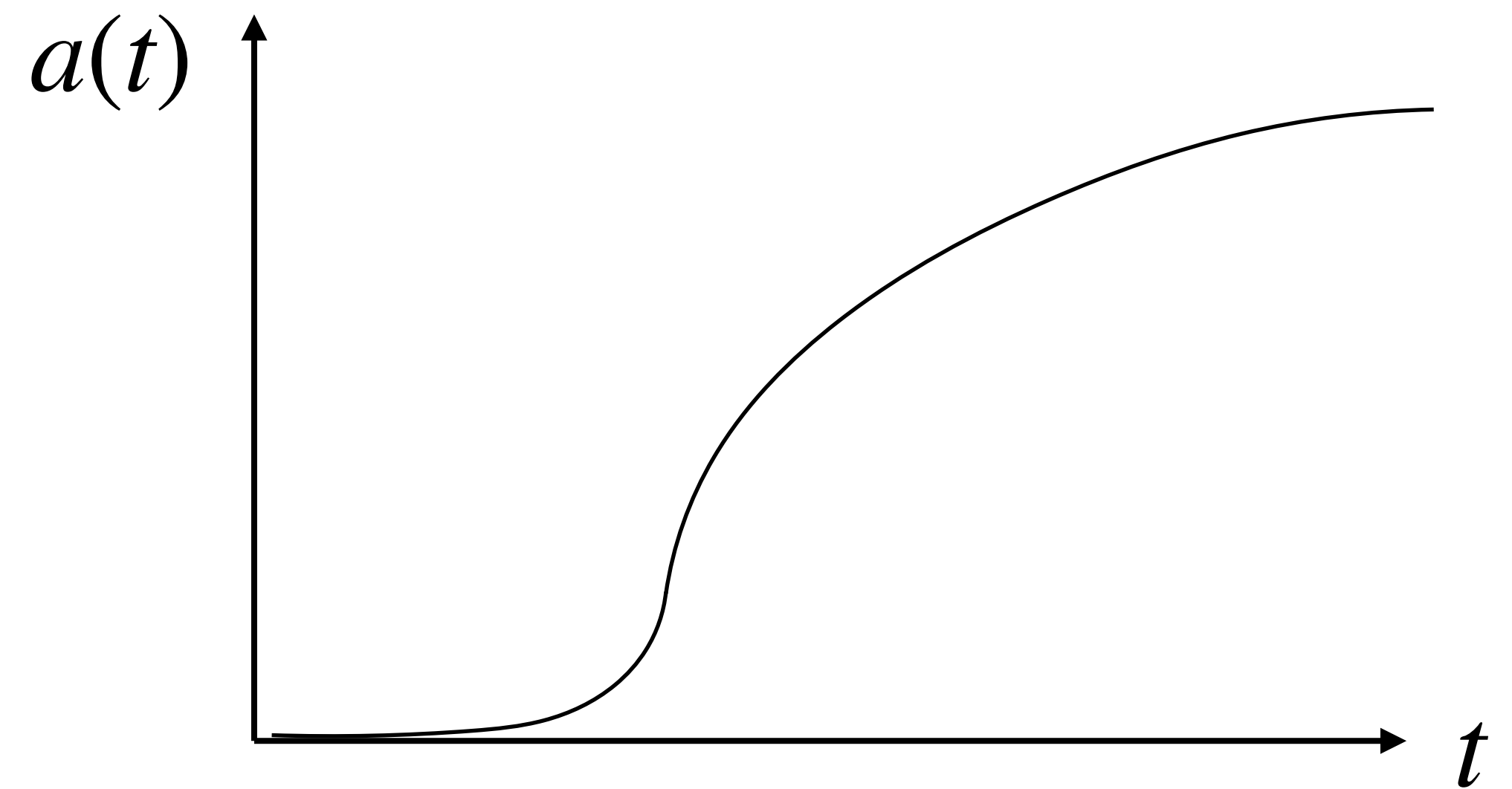
1. Matter described by a scalar field φ :

$$T^{\mu}_{\nu} = \nabla^{\mu}\varphi \nabla_{\nu}\varphi - \frac{1}{2}\nabla^{\alpha}\varphi \nabla_{\alpha}\varphi \delta^{\mu}_{\nu} - V\delta^{\mu}_{\nu}$$

2. Weak field equations

$$\Delta\Psi = 4\pi G a^2 \delta\rho_{\Delta},$$

...



Provides an origin for the causality of the CMB

GENERAL RELATIVITY ($K = 0$)

1. Matter described by a scalar field φ :

$$T^{\mu}_{\nu} = \nabla^{\mu}\varphi \nabla_{\nu}\varphi - \frac{1}{2}\nabla^{\alpha}\varphi \nabla_{\alpha}\varphi \delta^{\mu}_{\nu} - V\delta^{\mu}_{\nu}$$

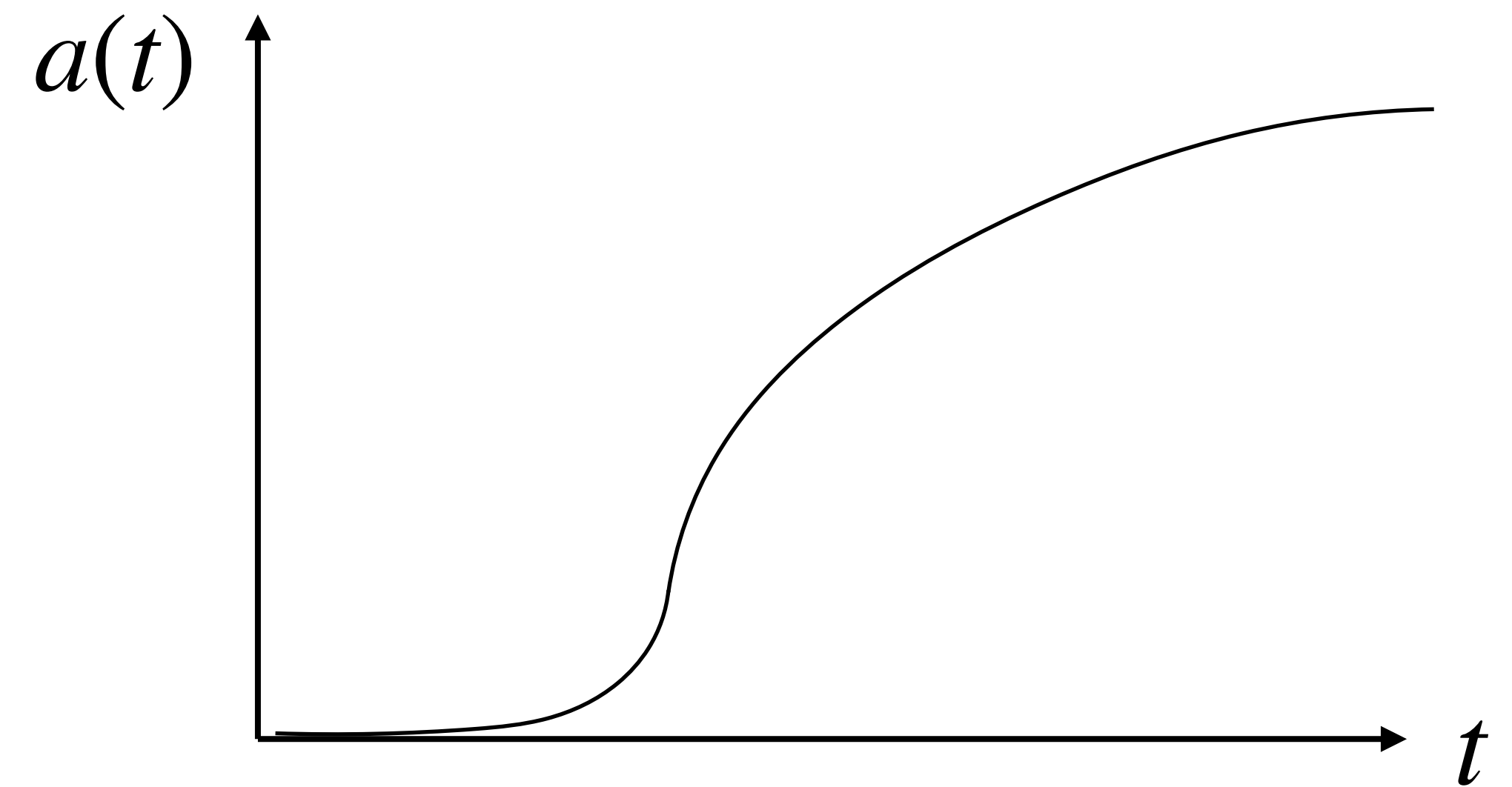
2. Weak field equations

$$\Delta\Psi = 4\pi G a^2 \delta\rho_{\Delta},$$

...

3. Curvature perturbation \mathcal{R} solution of a wave equation:

$$\mathcal{R}'' + \frac{2z'}{z}\mathcal{R}' - \Delta\mathcal{R} = 0$$



Acceleration

Provides an origin for the causality of the CMB

GENERAL RELATIVITY ($K = 0$)

1. Matter described by a scalar field φ :

$$T^\mu{}_\nu = \nabla^\mu \varphi \nabla_\nu \varphi - \frac{1}{2} \nabla^\alpha \varphi \nabla_\alpha \varphi \delta^\mu_\nu - V \delta^\mu_\nu$$

2. Weak field equations

$$\Delta \Psi = 4\pi G a^2 \delta \rho_\Delta,$$

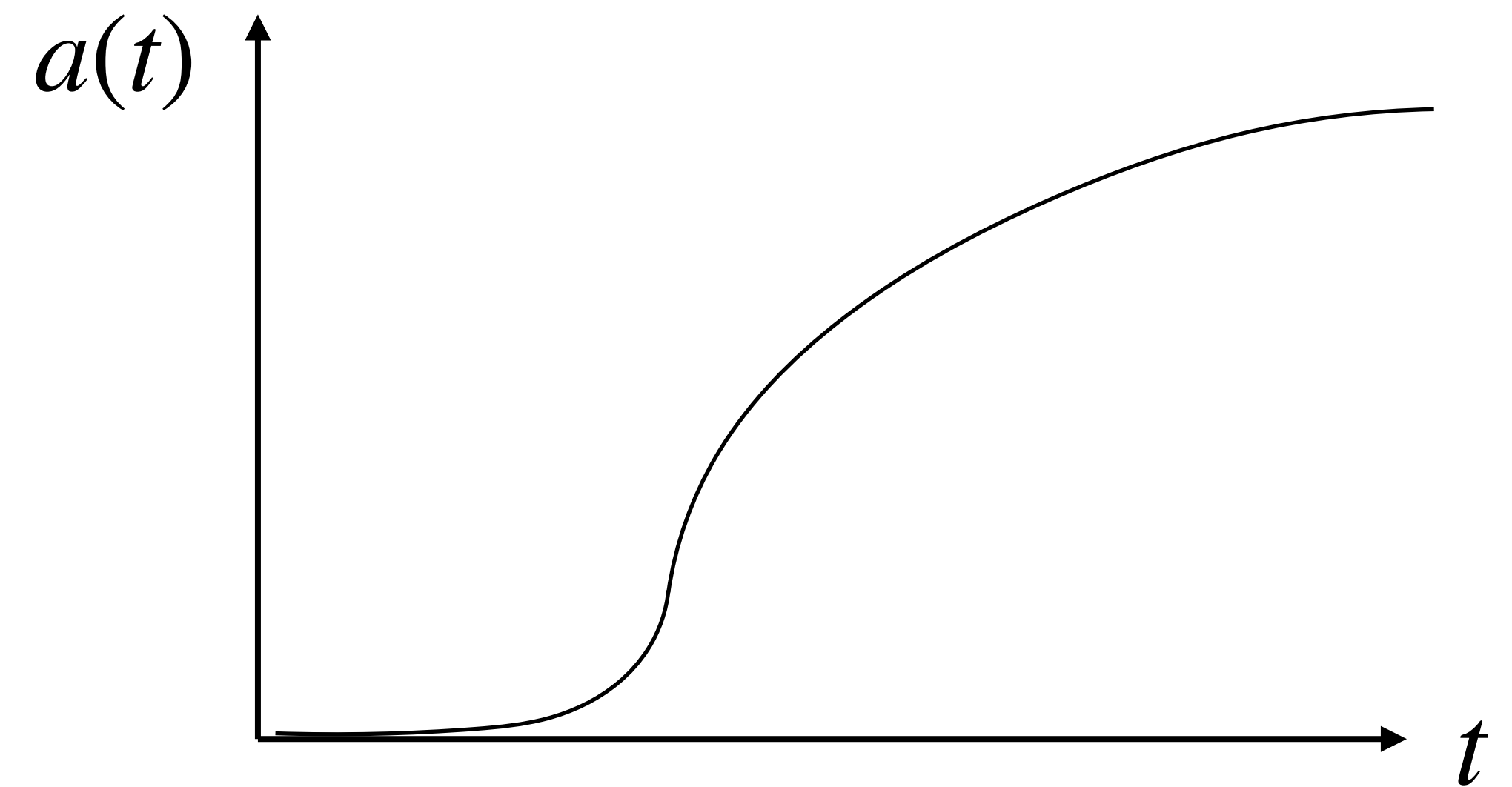
...

3. Curvature perturbation \mathcal{R} solution of a wave equation:

$$\mathcal{R}'' + \frac{2z'}{z} \mathcal{R}' - \Delta \mathcal{R} = 0$$

4. Canonical quantization in the limit $t \rightarrow 0$

→ Lowest energy state defines initial conditions



Acceleration

Provides an origin for the
causality of the CMB

GENERAL RELATIVITY ($K = 0$)

1. Matter described by a scalar field φ :

$$T^\mu{}_\nu = \nabla^\mu \varphi \nabla_\nu \varphi - \frac{1}{2} \nabla^\alpha \varphi \nabla_\alpha \varphi \delta^\mu_\nu - V \delta^\mu_\nu$$

2. Weak field equations

$$\Delta \Psi = 4\pi G a^2 \delta \rho_\Delta,$$

...

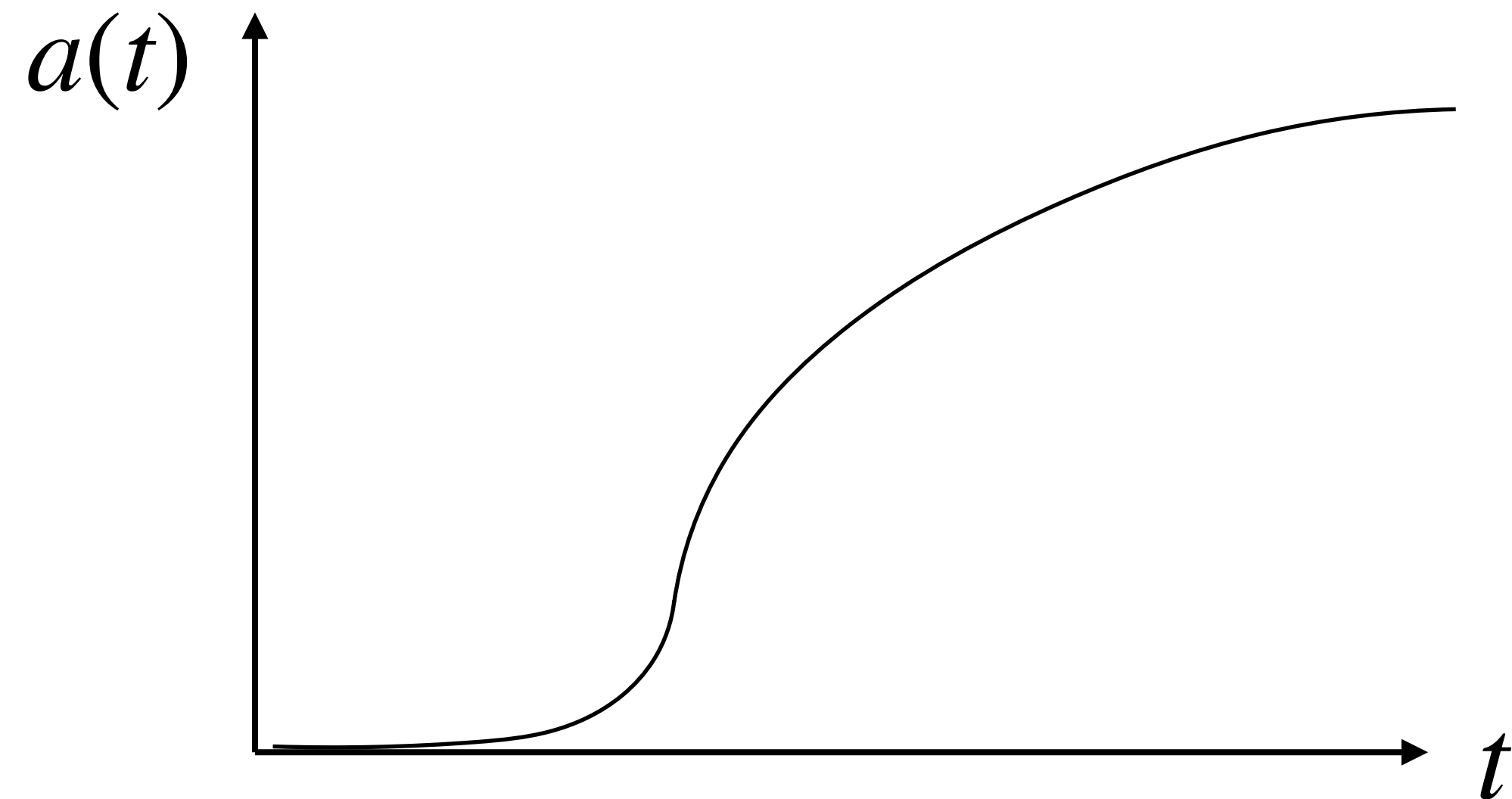
3. Curvature perturbation \mathcal{R} solution of a wave equation:

$$\mathcal{R}'' + \frac{2z'}{z} \mathcal{R}' - \Delta \mathcal{R} = 0$$

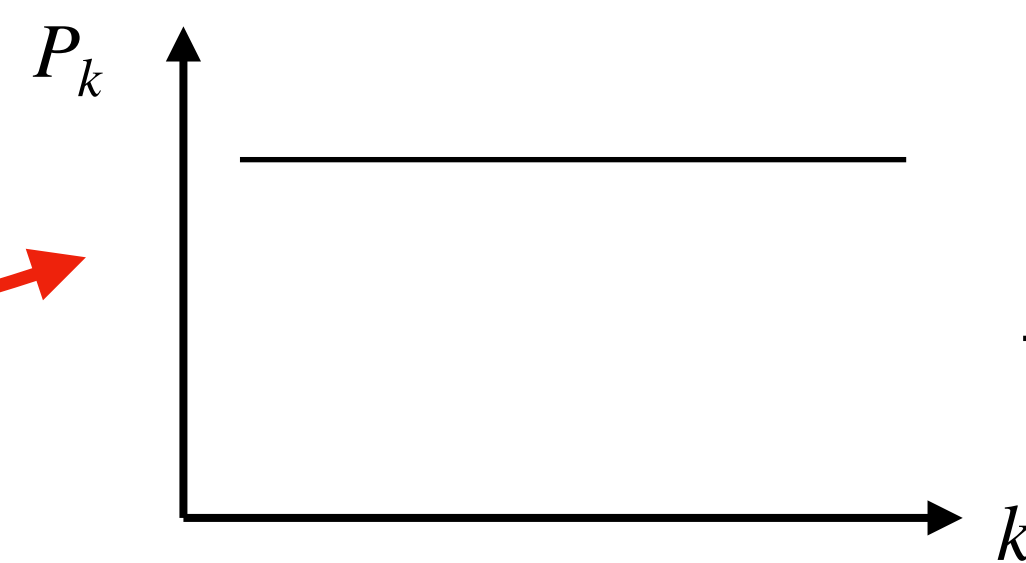
4. Canonical quantization in the limit $t \rightarrow 0$

→ Lowest energy state defines initial conditions

5. Scale-invariant primordial power spectrum as observed on the CMB.

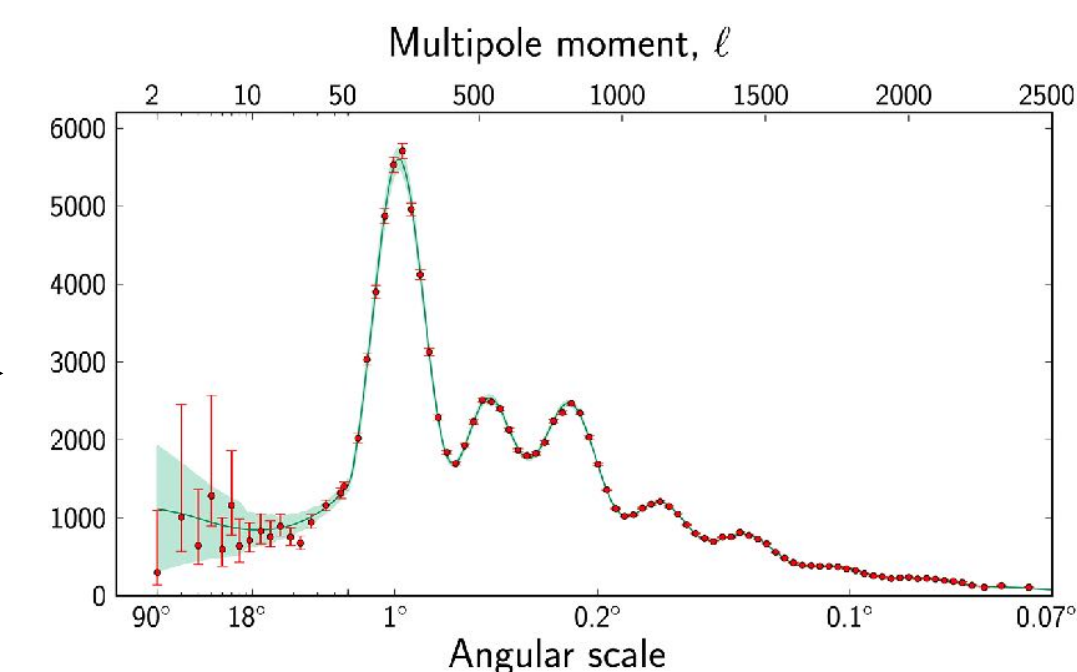


Provides an origin for the causality of the CMB



Flat initial power spectrum

Evolution



CMB power spectrum

GENERAL RELATIVITY ($K = 0$)

1. Matter described by a scalar field φ :

$$T^\mu{}_\nu = \nabla^\mu \varphi \nabla_\nu \varphi - \frac{1}{2} \nabla^\alpha \varphi \nabla_\alpha \varphi \delta^\mu_\nu - V \delta^\mu_\nu$$

2. Weak field equations

$$\Delta \Psi = 4\pi G a^2 \delta \rho_\Delta,$$

...

3. Curvature perturbation \mathcal{R} solution of a wave equation:

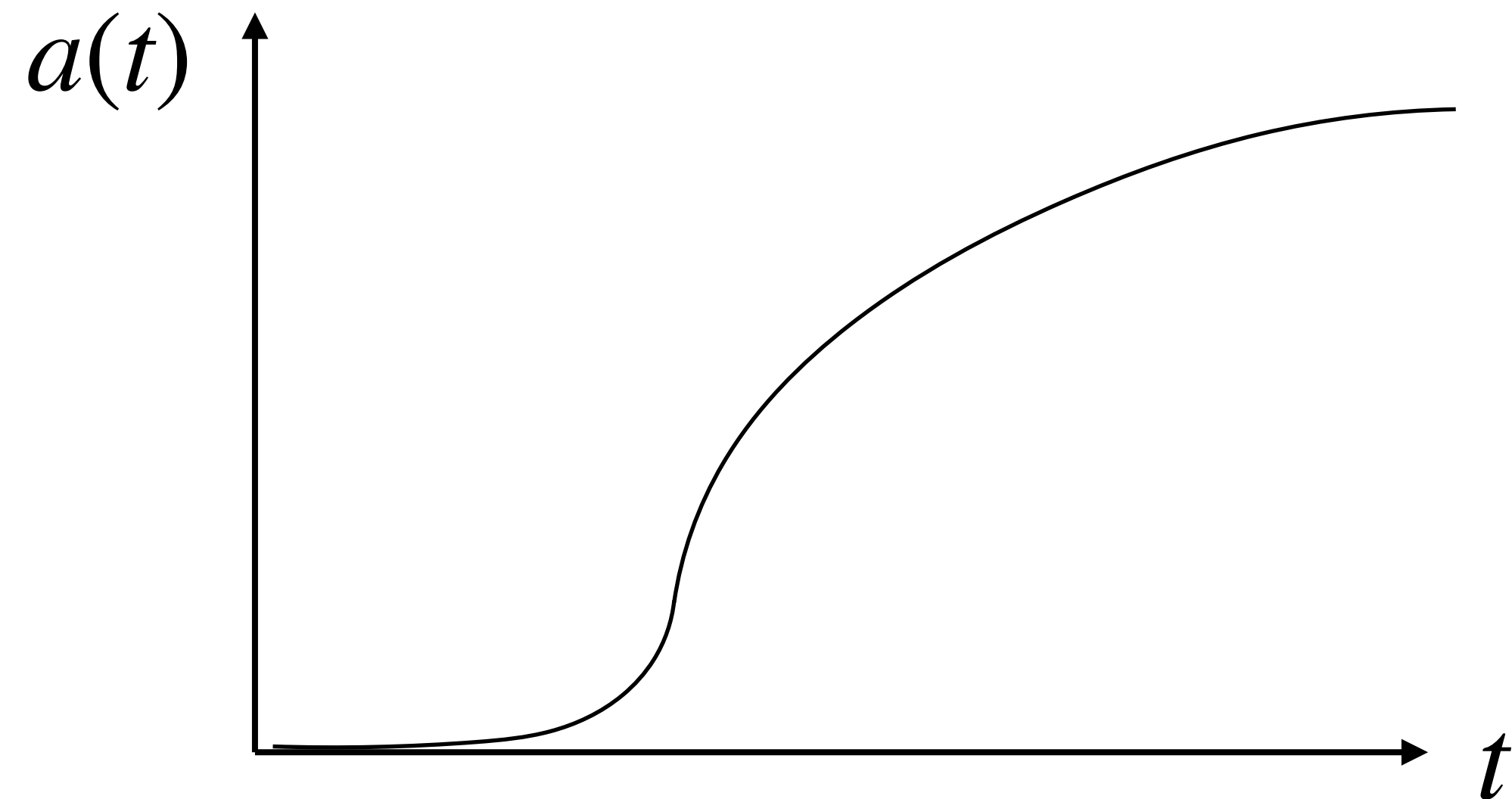
$$\mathcal{R}'' + \frac{2z'}{z} \mathcal{R}' - \Delta \mathcal{R} = 0$$

4. Canonical quantization in the limit $t \rightarrow 0$

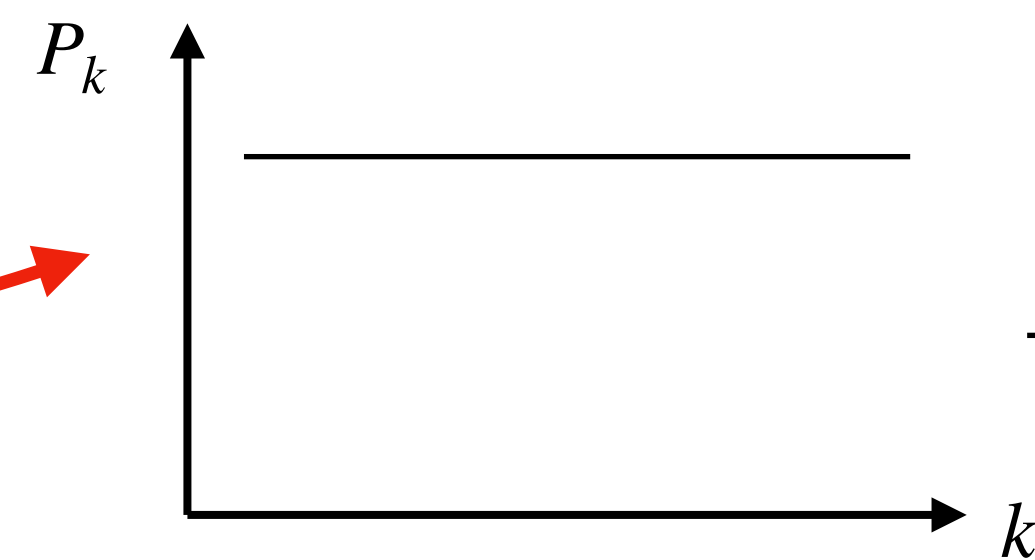
→ Lowest energy state defines initial conditions

5. Scale-invariant primordial power spectrum as observed on the CMB.

⇒ Simple, natural, model explaining CMB anisotropies.

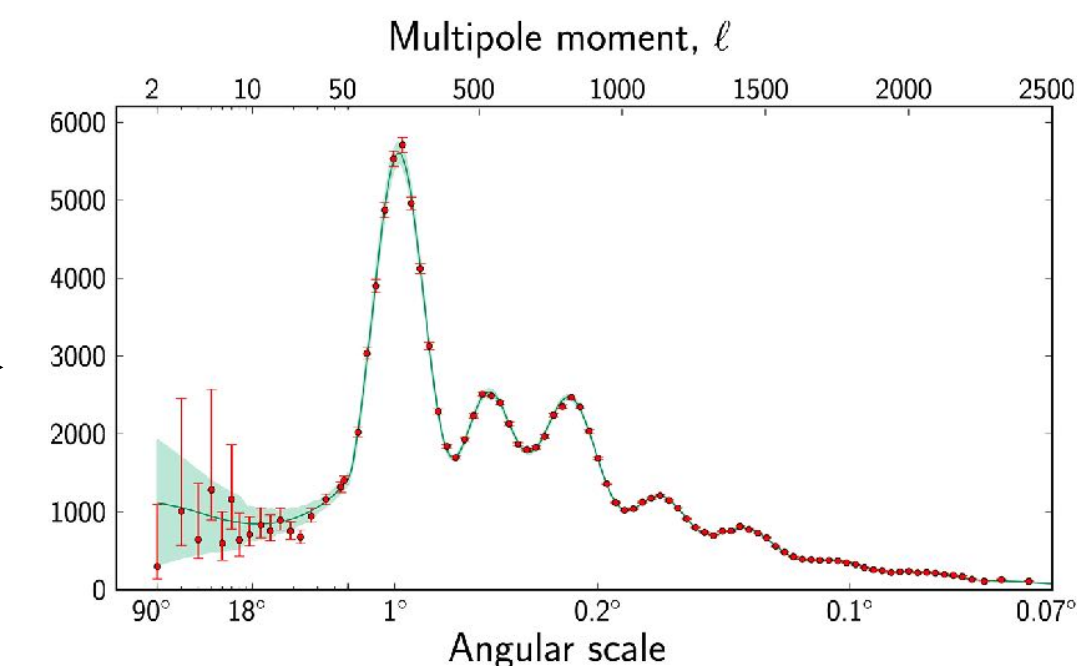


Provides an origin for the causality of the CMB



Flat initial power spectrum

Evolution

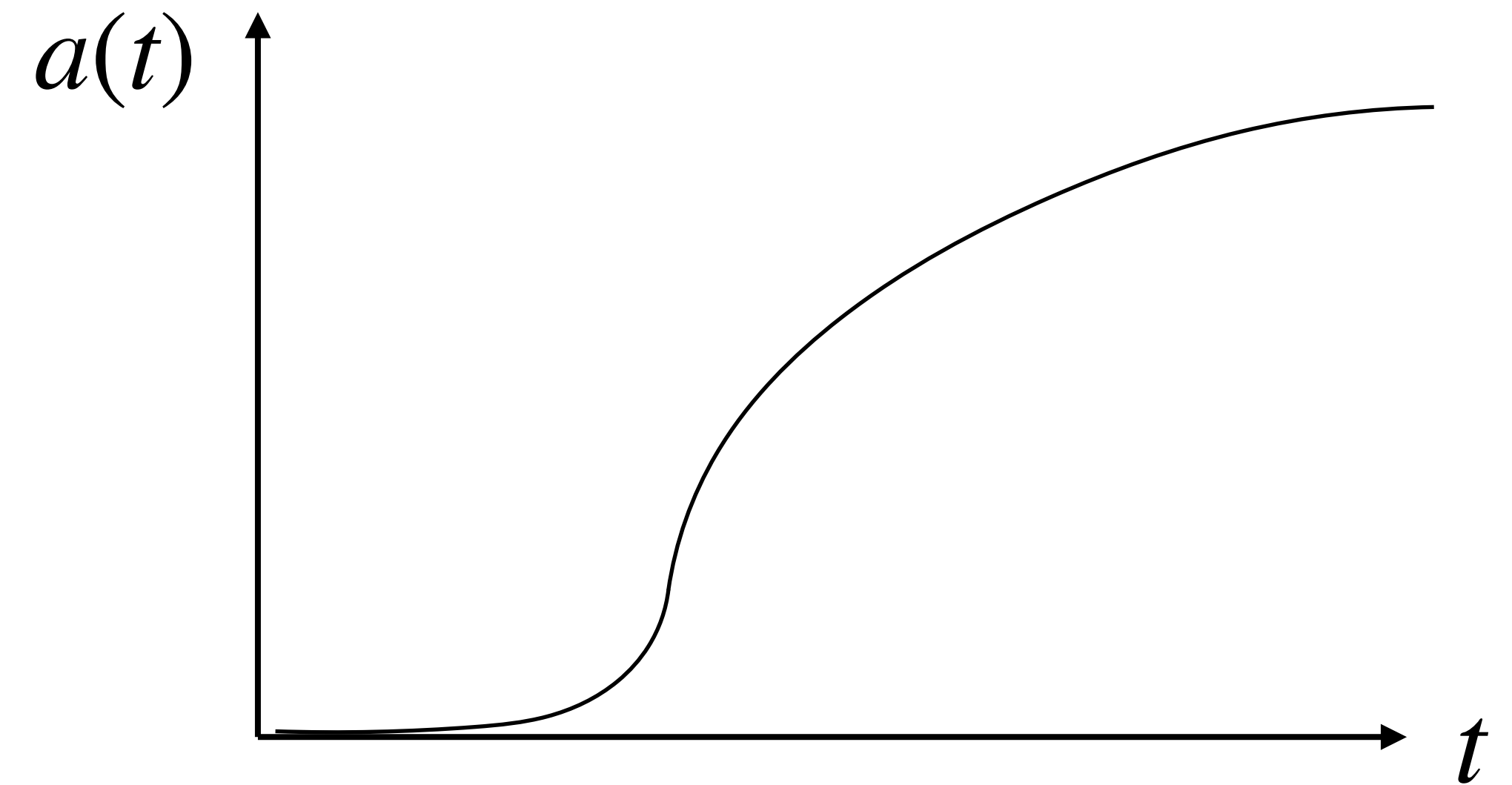


CMB power spectrum

GENERAL RELATIVITY ($K \neq 0$)

1. Matter described by a scalar field φ :

$$T^\mu{}_\nu = \nabla^\mu \varphi \nabla_\nu \varphi - \frac{1}{2} \nabla^\alpha \varphi \nabla_\alpha \varphi \delta^\mu_\nu - V \delta^\mu_\nu$$

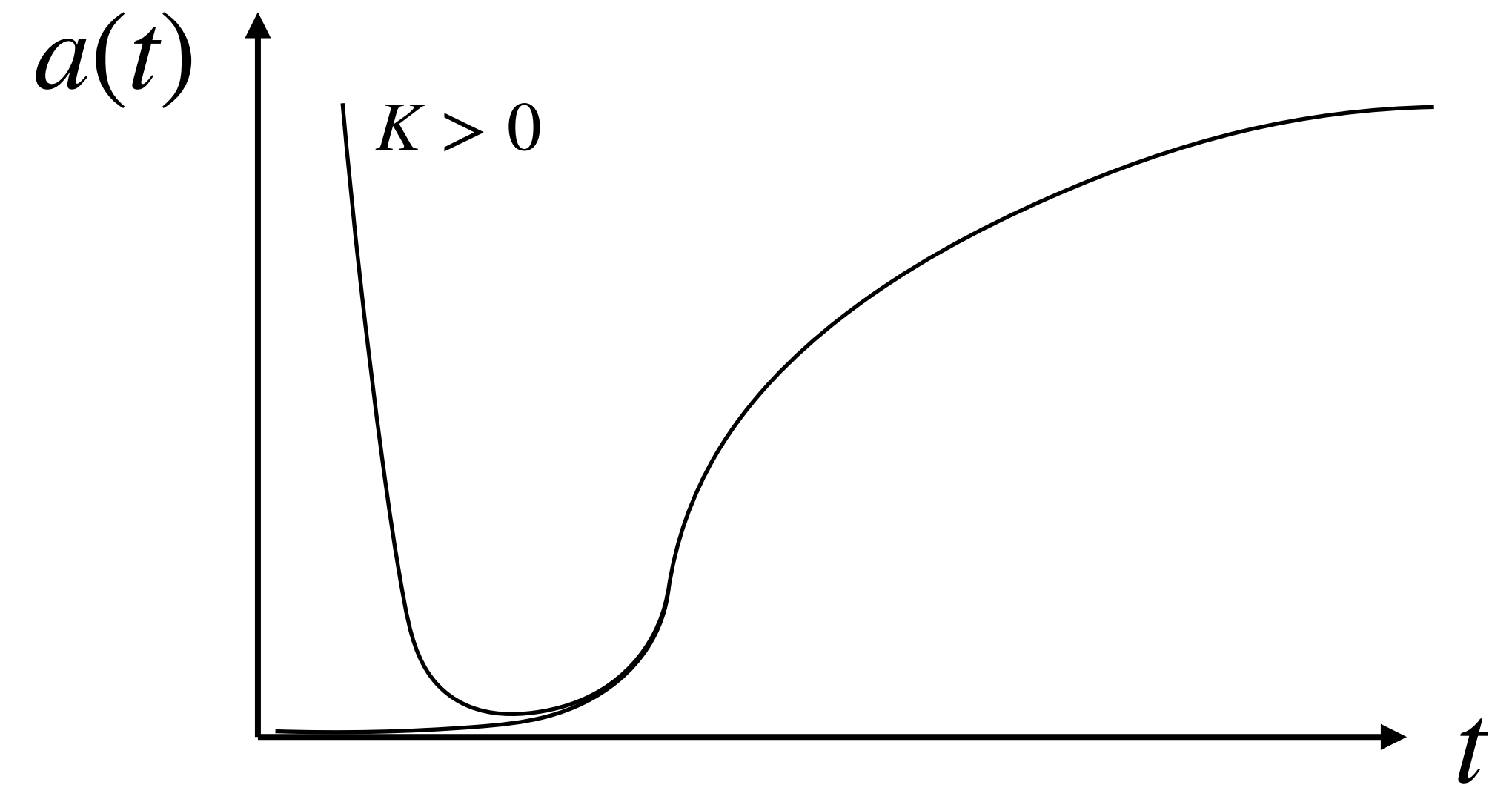


Provides an origin for the causality of the CMB

GENERAL RELATIVITY ($K \neq 0$)

1. Matter described by a scalar field φ :

$$T^{\mu}_{\nu} = \nabla^{\mu}\varphi \nabla_{\nu}\varphi - \frac{1}{2} \nabla^{\alpha}\varphi \nabla_{\alpha}\varphi \delta^{\mu}_{\nu} - V \delta^{\mu}_{\nu}$$



Provides an origin for the causality of the CMB

GENERAL RELATIVITY ($K \neq 0$)

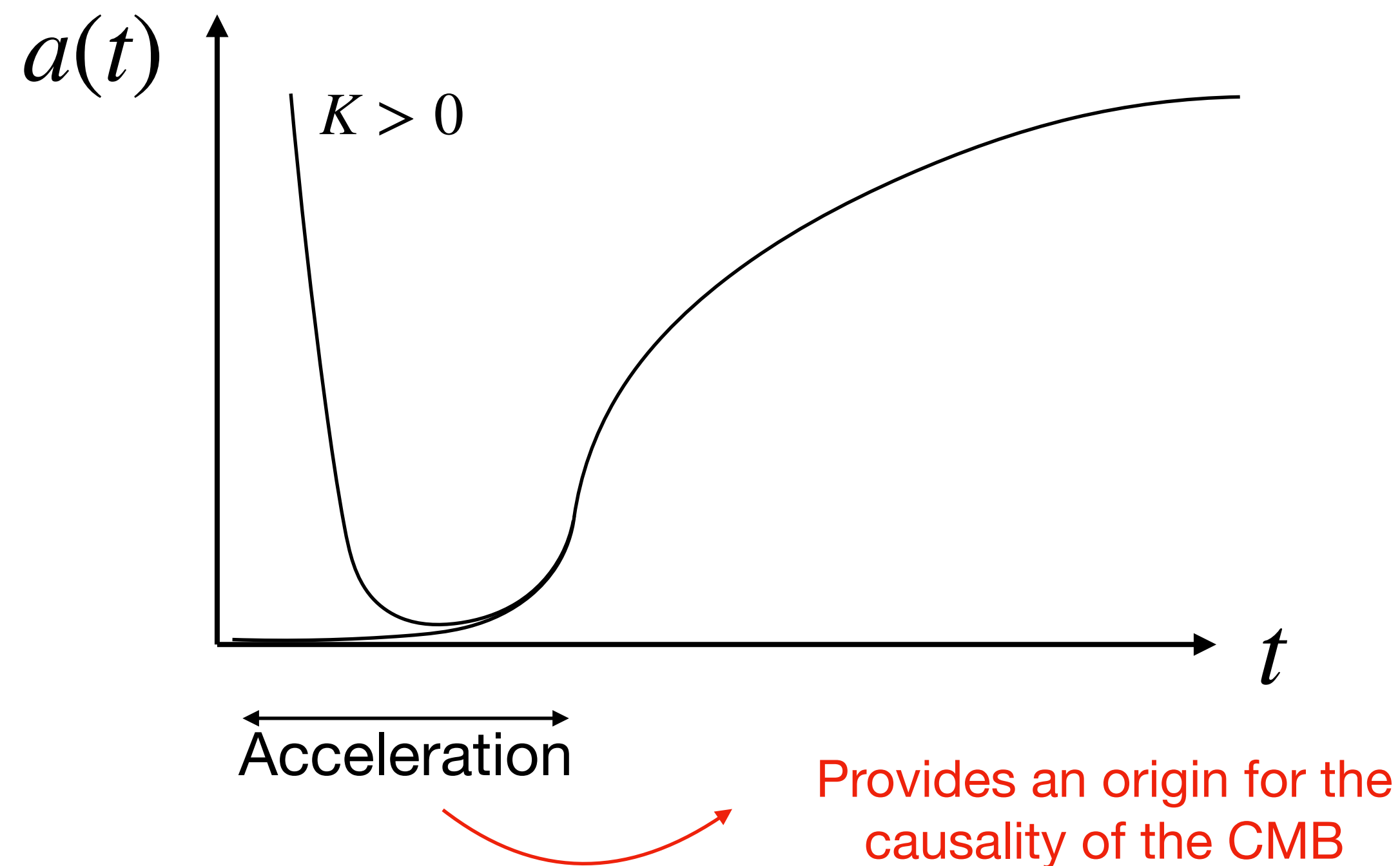
1. Matter described by a scalar field φ :

$$T^{\mu}_{\nu} = \nabla^{\mu}\varphi \nabla_{\nu}\varphi - \frac{1}{2}\nabla^{\alpha}\varphi \nabla_{\alpha}\varphi \delta^{\mu}_{\nu} - V\delta^{\mu}_{\nu}$$

2. Weak field equations

$$(\Delta + 3K)\Psi = 4\pi G a^2 \delta\rho_{\Delta},$$

...



GENERAL RELATIVITY ($K \neq 0$)

1. Matter described by a scalar field φ :

$$T^\mu{}_\nu = \nabla^\mu \varphi \nabla_\nu \varphi - \frac{1}{2} \nabla^\alpha \varphi \nabla_\alpha \varphi \delta^\mu_\nu - V \delta^\mu_\nu$$

2. Weak field equations

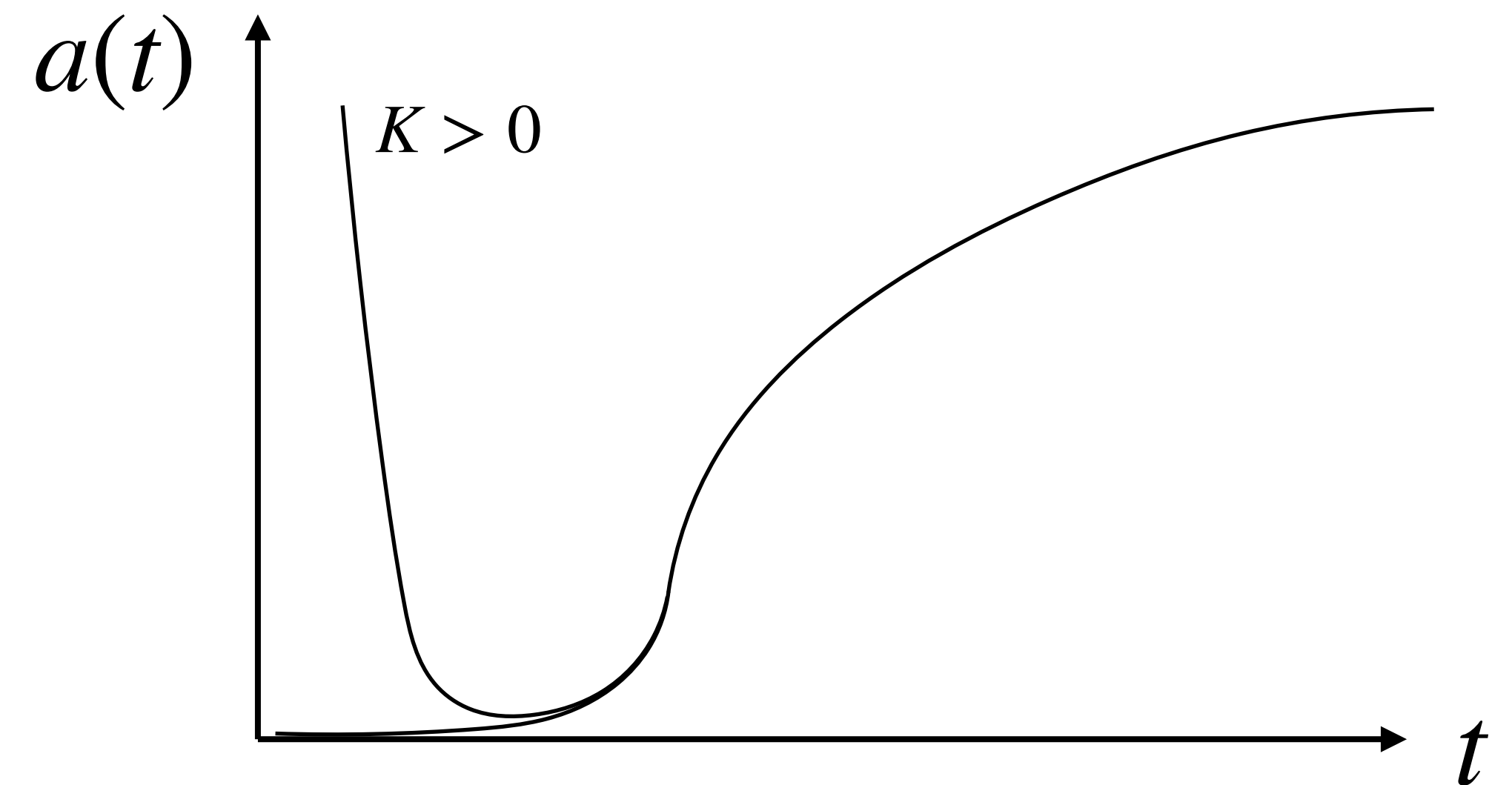
$$(\Delta + 3K) \Psi = 4\pi G a^2 \delta\rho_\Delta,$$

...

3. Curvature perturbation \mathcal{R} **NOT** solution of a wave equation:

$$(\mathcal{D}^2 - K\mathcal{E}) \mathcal{R}'' + \left(\frac{2z'}{z} \mathcal{D}^2 - 2K\mathcal{H}\mathcal{E} \right) \mathcal{R}' + \left[-\mathcal{D}^4 + K \left(1 - \frac{2z'}{\mathcal{H}z} + \mathcal{E} \right) \mathcal{D}^2 + K^2 \mathcal{E} \right] \mathcal{R} = 0$$

with $\mathcal{D}^2 = \Delta + 3K$.



Provides an origin for the causality of the CMB

GENERAL RELATIVITY ($K \neq 0$)

1. Matter described by a scalar field φ :

$$T^\mu{}_\nu = \nabla^\mu \varphi \nabla_\nu \varphi - \frac{1}{2} \nabla^\alpha \varphi \nabla_\alpha \varphi \delta^\mu_\nu - V \delta^\mu_\nu$$

2. Weak field equations

$$(\Delta + 3K) \Psi = 4\pi G a^2 \delta\rho_\Delta,$$

...

3. Curvature perturbation \mathcal{R} **NOT** solution of a wave equation:

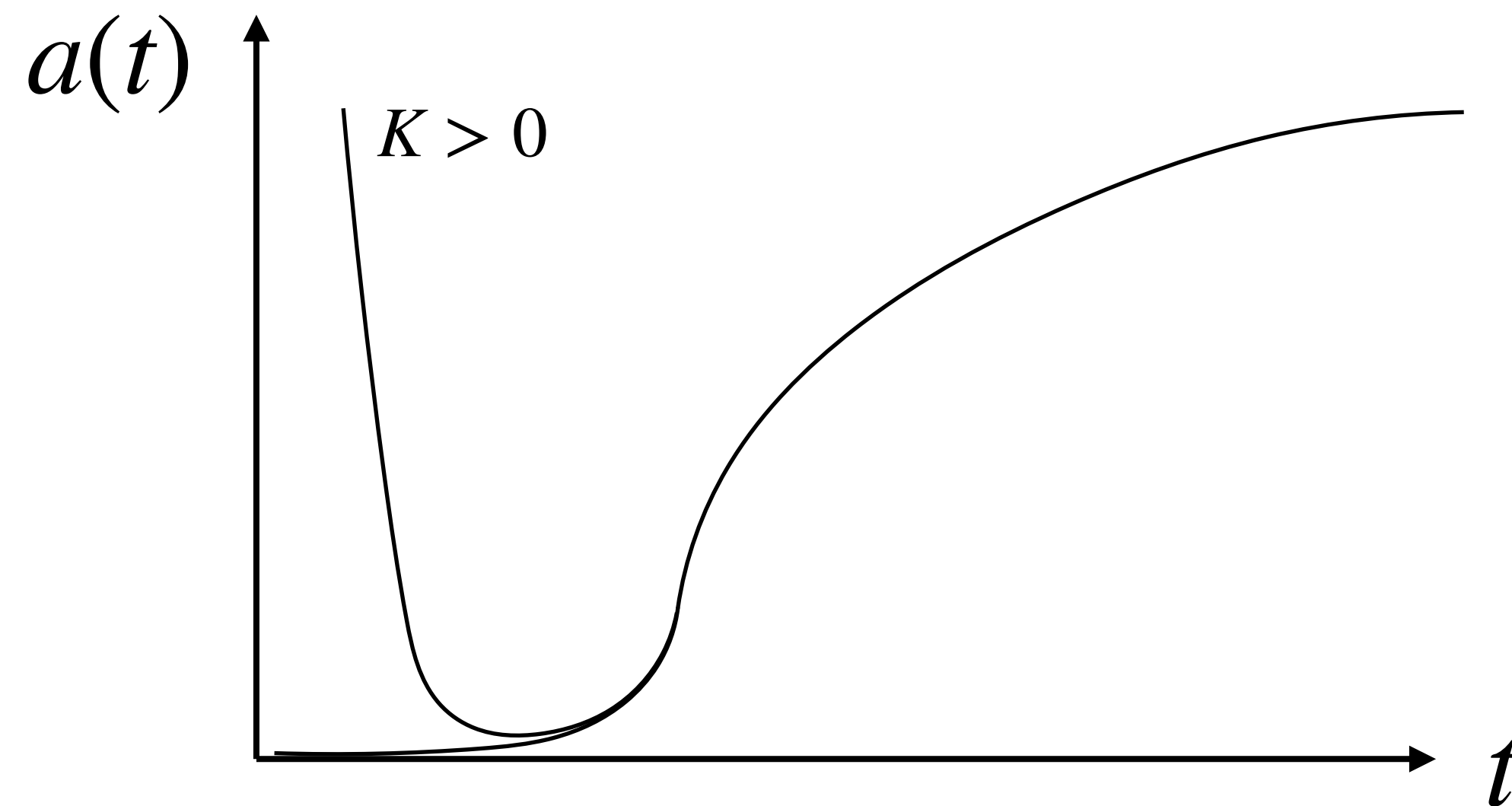
$$(\mathcal{D}^2 - K\mathcal{E}) \mathcal{R}'' + \left(\frac{2z'}{z} \mathcal{D}^2 - 2K\mathcal{H}\mathcal{E} \right) \mathcal{R}' + \left[-\mathcal{D}^4 + K \left(1 - \frac{2z'}{\mathcal{H}z} + \mathcal{E} \right) \mathcal{D}^2 + K^2 \mathcal{E} \right] \mathcal{R} = 0$$

with $\mathcal{D}^2 = \Delta + 3K$.

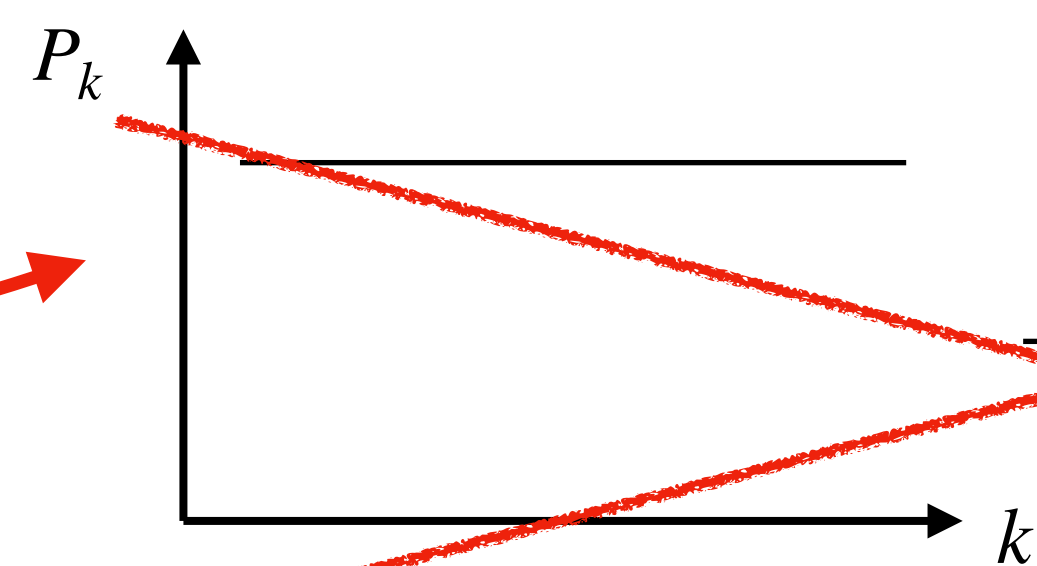
~~4. Canonical quantization in the limit $t \rightarrow 0$~~

~~→ Lowest energy state defines initial conditions~~

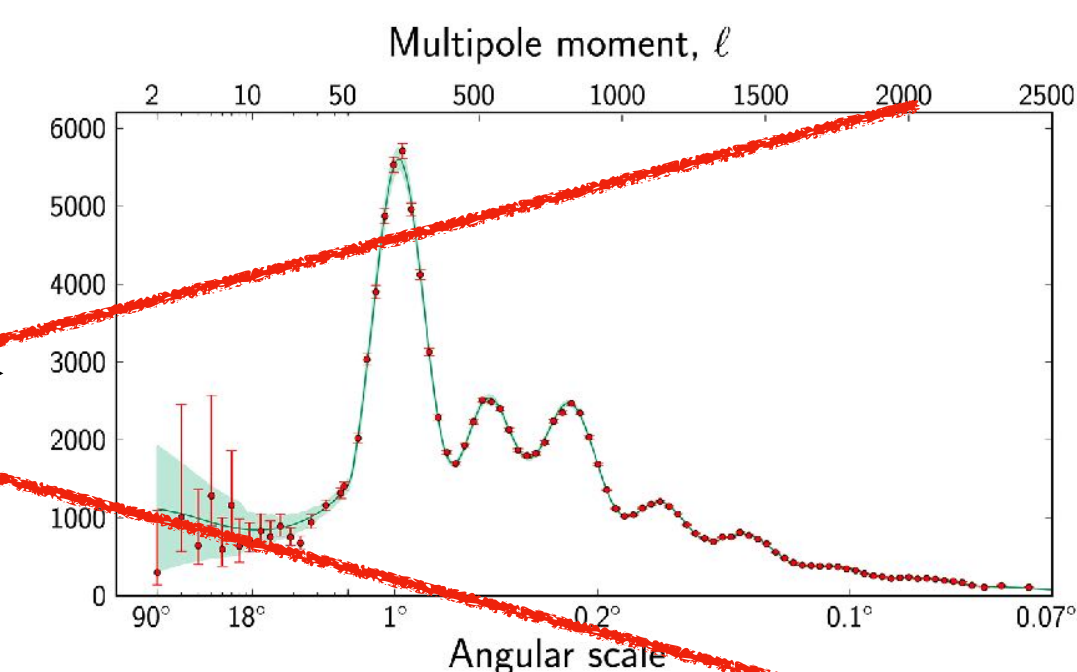
~~5. Scale-invariant primordial power spectrum as observed on the CMB.~~



Provides an origin for the causality of the CMB



Flat initial power spectrum



CMB power spectrum

GENERAL RELATIVITY ($K \neq 0$)

1. Matter described by a scalar field φ :

$$T^\mu{}_\nu = \nabla^\mu \varphi \nabla_\nu \varphi - \frac{1}{2} \nabla^\alpha \varphi \nabla_\alpha \varphi \delta^\mu_\nu - V \delta^\mu_\nu$$

2. Weak field equations

$$(\Delta + 3K) \Psi = 4\pi G a^2 \delta\rho_\Delta,$$

...

3. Curvature perturbation \mathcal{R} **NOT** solution of a wave equation:

$$(\mathcal{D}^2 - K\mathcal{E}) \mathcal{R}'' + \left(\frac{2z'}{z} \mathcal{D}^2 - 2K\mathcal{H}\mathcal{E} \right) \mathcal{R}' + \left[-\mathcal{D}^4 + K \left(1 - \frac{2z'}{\mathcal{H}z} + \mathcal{E} \right) \mathcal{D}^2 + K^2 \mathcal{E} \right] \mathcal{R} = 0$$

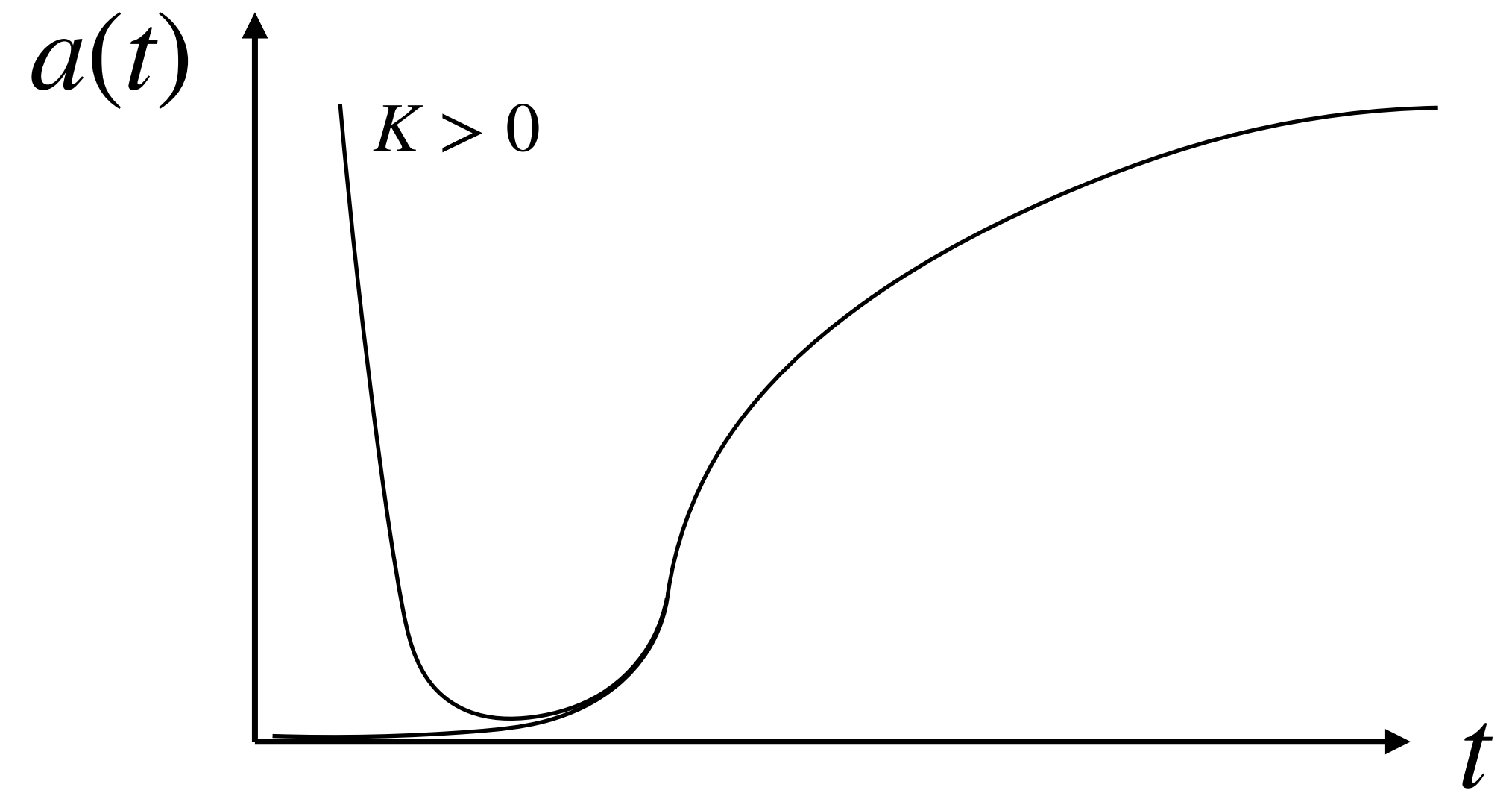
with $\mathcal{D}^2 = \Delta + 3K$.

~~4. Canonical quantization in the limit $t \rightarrow 0$~~

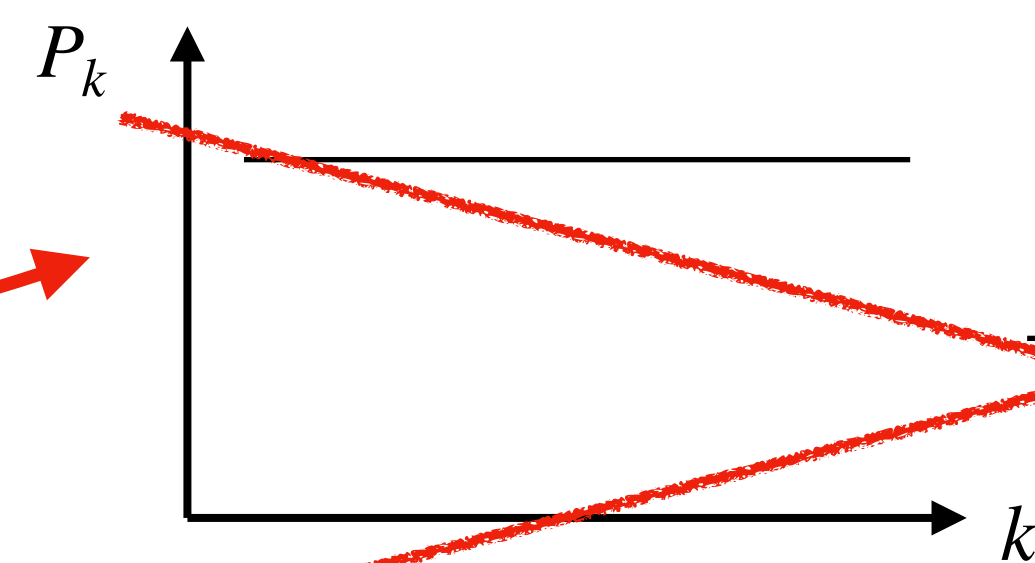
~~→ Lowest energy state defines initial conditions~~

~~5. Scale-invariant primordial power spectrum as observed on the CMB.~~

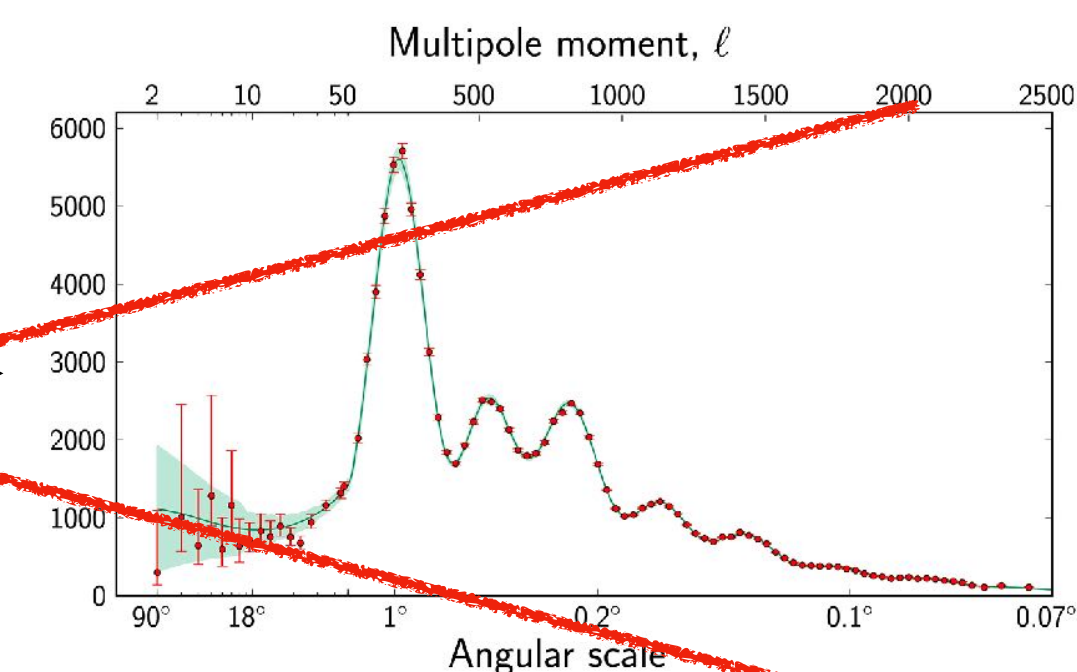
⇒ No more well-defined procedure to predict the power spectrum (e.g. [Handley \(2019\)](#), [Renevey et al. \(2022\)](#)).



Provides an origin for the causality of the CMB



Flat initial power spectrum

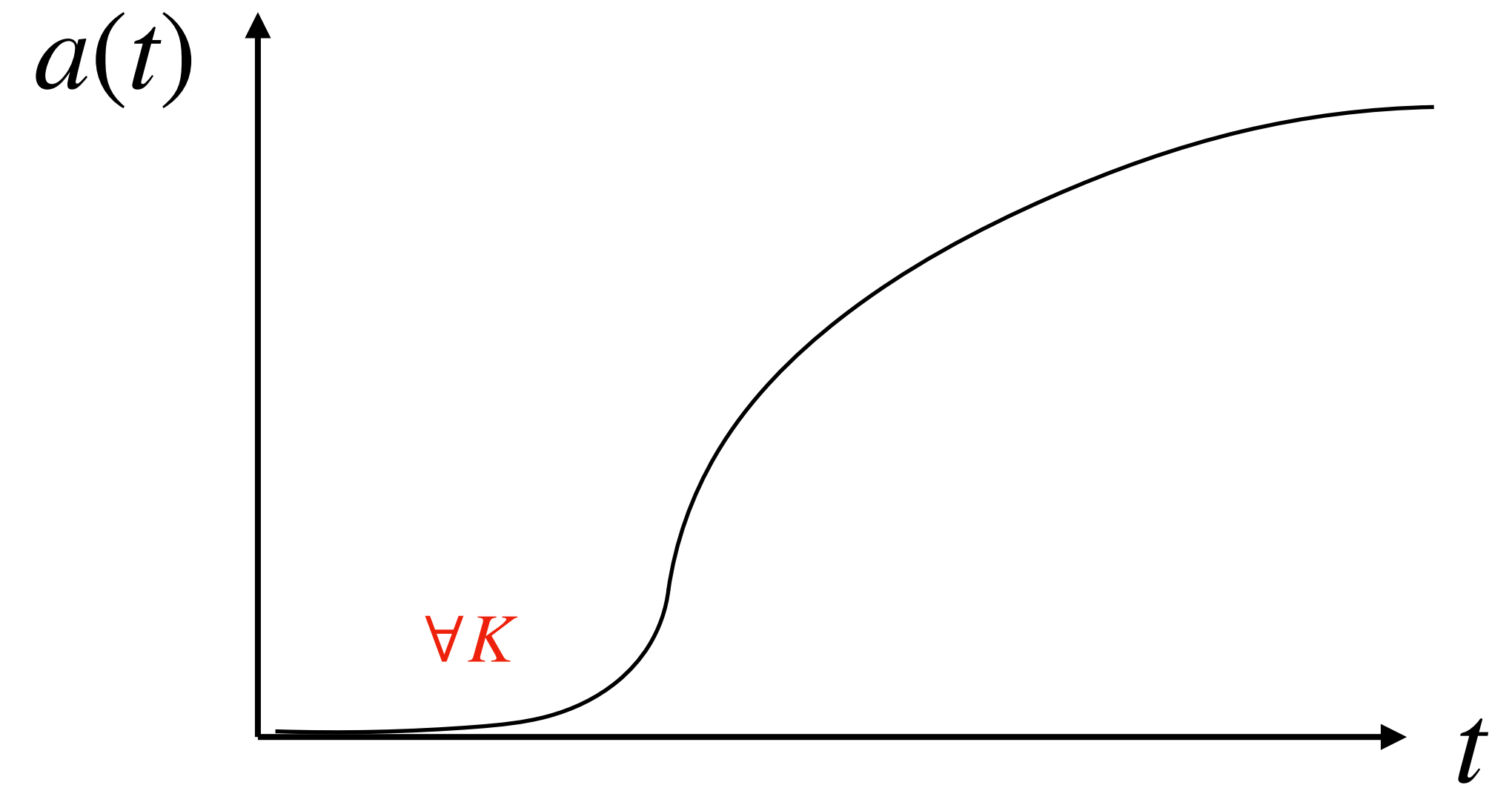


CMB power spectrum

Topo-GR ($\forall K$)

1. Matter described by a scalar field φ :

$$T^\mu{}_\nu = \nabla^\mu \varphi \nabla_\nu \varphi - \frac{1}{2} \nabla^\alpha \varphi \nabla_\alpha \varphi \delta^\mu{}_\nu - V \delta^\mu{}_\nu$$

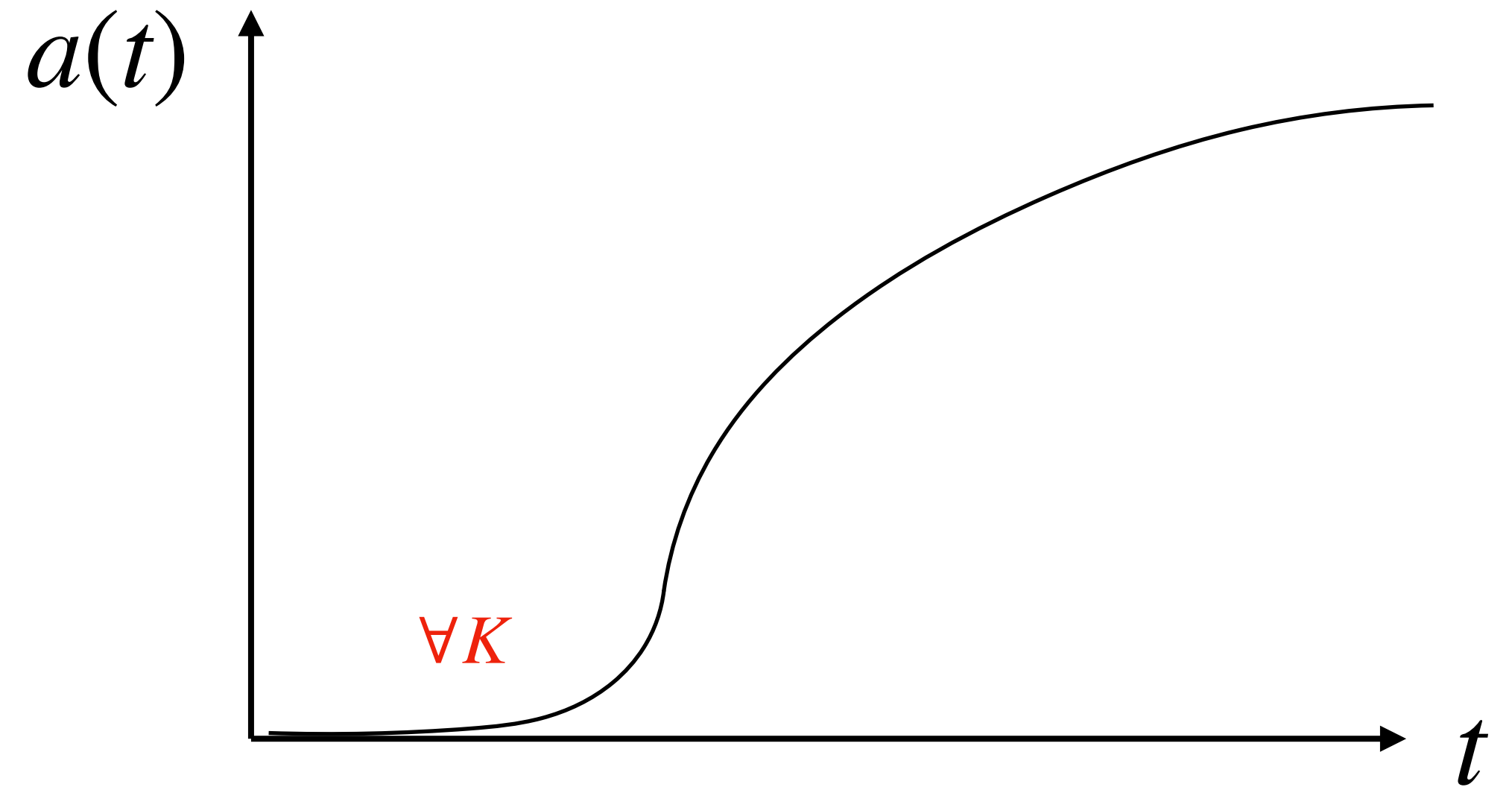


Provides an origin for the causality of the CMB

1. N

2. V

General relativity	topo-GR
FIELD EQUATIONS	
$R_{\mu\nu} = \kappa \left(T_{\mu\nu} - \frac{T_{\alpha\beta} g^{\alpha\beta}}{2} g_{\mu\nu} \right)$	$R_{\mu\nu} - \bar{R}_{\mu\nu} = \kappa \left(T_{\mu\nu} - \frac{T_{\alpha\beta} g^{\alpha\beta}}{2} g_{\mu\nu} \right)$ $\nabla^\nu \left(\bar{R}_{\mu\nu} - \frac{\bar{R}_{\alpha\beta} g^{\alpha\beta}}{2} g_{\mu\nu} \right) = 0$
HOMOGENEOUS AND ISOTROPIC SOLUTION	
$\mathcal{H}^2 = a^2 \frac{\kappa}{3} \rho - \mathbf{K}$ $\mathcal{H}' = -a^2 \frac{\kappa}{6} (\rho + 3p)$	$\mathcal{H}^2 = a^2 \frac{\kappa}{3} \rho$ $\mathcal{H}' = -a^2 \frac{\kappa}{6} (\rho + 3p)$ $\forall \mathbf{K}$
FIRST ORDER PERTURBATIONS	
Scalar modes	
$(\Delta + \mathbf{3K})\Psi = a^2 \frac{\kappa}{2} \delta\rho_\sigma + 3\mathcal{H}(\Psi' + \mathcal{H}\Phi)$ $\Psi' + \mathcal{H}\Phi = -a^2 \frac{\kappa}{2} (\rho + p)\mathcal{V}$ $\Psi - \Phi = a^2 \kappa \Pi$ $\Psi'' + 2\mathcal{H}\Psi' + \mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi - \mathbf{K}\Psi = a^2 \frac{\kappa}{2} \left(c_s^2 \delta\rho_\sigma + \delta p_{\neq \text{ad}} + \frac{2}{3} \Delta \Pi \right)$	$\Delta\Psi = a^2 \frac{\kappa}{2} \delta\rho_\sigma + 3\mathcal{H}(\Psi' + \mathcal{H}\Phi) + \mathbf{K}\Delta\mathcal{C}$ $\Psi' + \mathcal{H}\Phi = -a^2 \frac{\kappa}{2} (\rho + p)\mathcal{V} + \mathbf{K}\mathcal{C}'$ $\Psi - \Phi = a^2 \kappa \Pi + \mathbf{4K}\mathcal{C}$ $\Psi'' + 2\mathcal{H}\Psi' + \mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi = a^2 \frac{\kappa}{2} \left(c_s^2 \delta\rho_\sigma + \delta p_{\neq \text{ad}} + \frac{2}{3} \Delta \Pi \right) + \mathbf{K}\Delta\mathcal{C}$
Vector modes	
$(\Delta + \mathbf{2K})Q_i = -a^2 2\kappa(\rho + p)(\mathcal{V}_i - Q_i)$ $Q_i' + 2\mathcal{H}Q_i = a^2 \kappa \Pi_i$	$(\Delta - \mathbf{2K})Q_i = -a^2 2\kappa(\rho + p)(\mathcal{V}_i - Q_i) + \mathbf{4K}\mathcal{C}'_i$ $Q_i' + 2\mathcal{H}Q_i = a^2 \kappa \Pi_i + \mathbf{4K}\mathcal{C}_i$
Tensor mode	
$f_{ij}'' + 2\mathcal{H}f_{ij}' + (\mathbf{2K} - \Delta)f_{ij} = a^2 \kappa \Pi_{ij}$	$f_{ij}'' + 2\mathcal{H}f_{ij}' + (\mathbf{6K} - \Delta)f_{ij} = a^2 \kappa \Pi_{ij}$
Conservation of $\bar{R}_{\alpha\beta} - \frac{\bar{R}_{\mu\nu} g^{\mu\nu}}{2} g_{\alpha\beta}$	
\emptyset	$\mathcal{C}'' + 2\mathcal{H}\mathcal{C}' - \Delta\mathcal{C} = -a^2 \kappa \Pi$ $\mathcal{C}_i'' + 2\mathcal{H}\mathcal{C}_i' + (\mathbf{2K} - \Delta)\mathcal{C}_i = -a^2 \kappa \Pi_i$



Provides an origin for the causality of the CMB

Topo-GR ($\forall K$)

1. Matter described by a scalar field φ :

$$T^\mu{}_\nu = \nabla^\mu \varphi \nabla_\nu \varphi - \frac{1}{2} \nabla^\alpha \varphi \nabla_\alpha \varphi \delta^\mu_\nu - V \delta^\mu_\nu$$

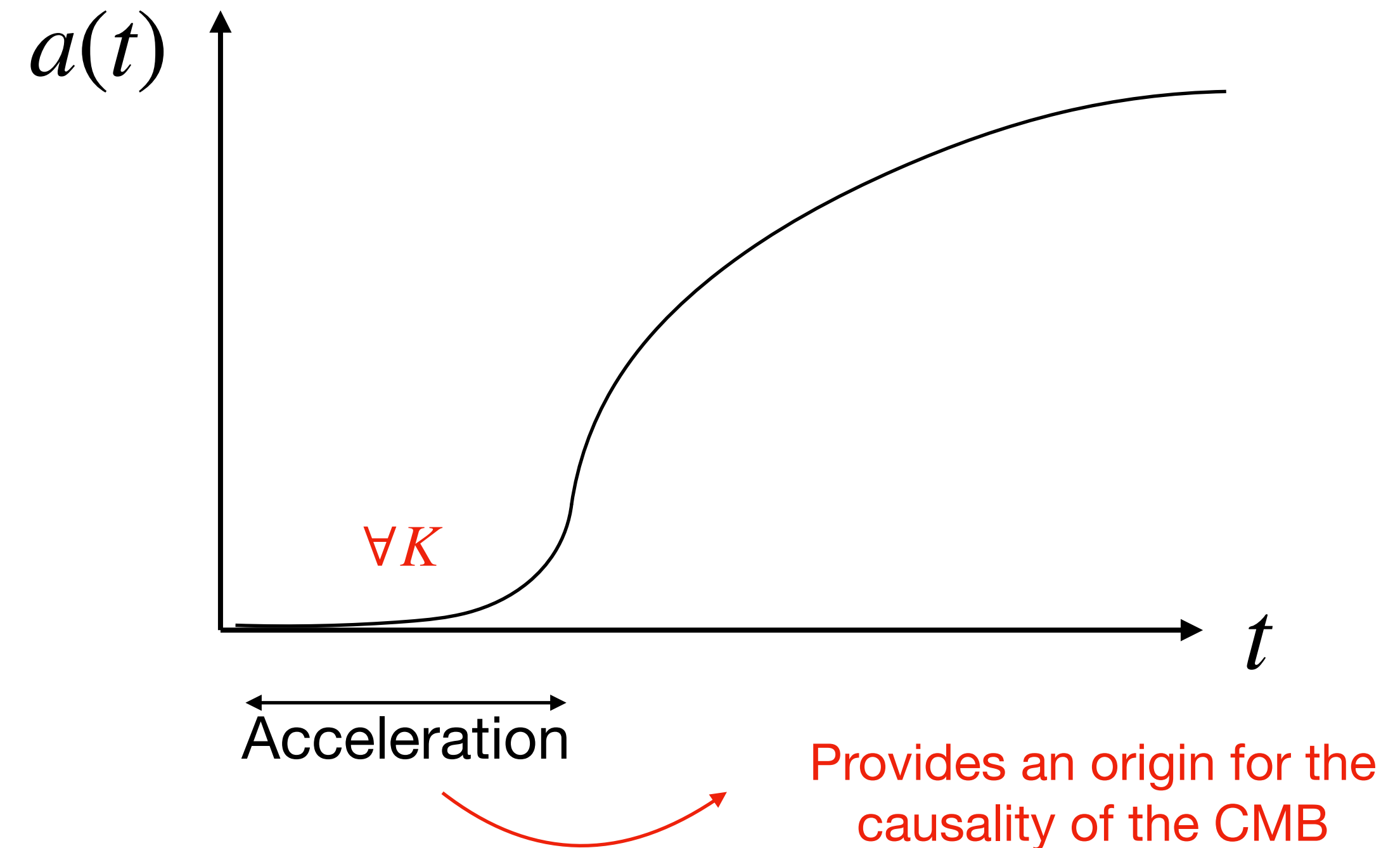
2. Weak field equations

$$\Delta \Psi = 4\pi G a^2 \delta \rho_\Delta, \quad \forall K$$

...

3. Curvature perturbation \mathcal{R} solution of a wave equation:

$$\mathcal{R}'' + \frac{2z'}{z} \mathcal{R}' - \Delta \mathcal{R} = 0, \quad \forall K$$



Topo-GR ($\forall K$)

1. Matter described by a scalar field φ :

$$T^\mu{}_\nu = \nabla^\mu \varphi \nabla_\nu \varphi - \frac{1}{2} \nabla^\alpha \varphi \nabla_\alpha \varphi \delta^\mu{}_\nu - V \delta^\mu{}_\nu$$

2. Weak field equations

$$\Delta \Psi = 4\pi G a^2 \delta \rho_\Delta, \quad \forall K$$

...

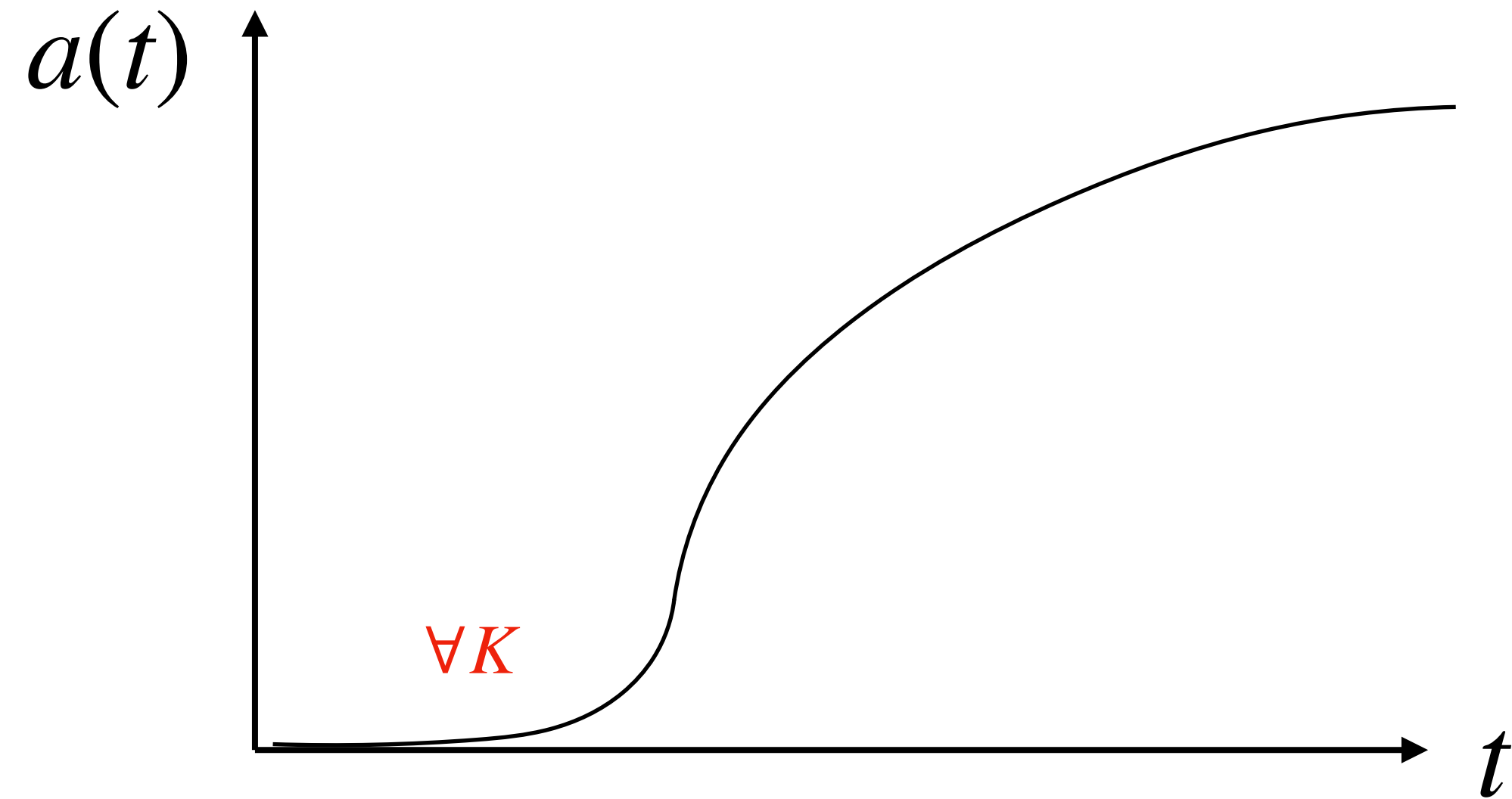
3. Curvature perturbation \mathcal{R} solution of a wave equation:

$$\mathcal{R}'' + \frac{2z'}{z} \mathcal{R}' - \Delta \mathcal{R} = 0, \quad \forall K$$

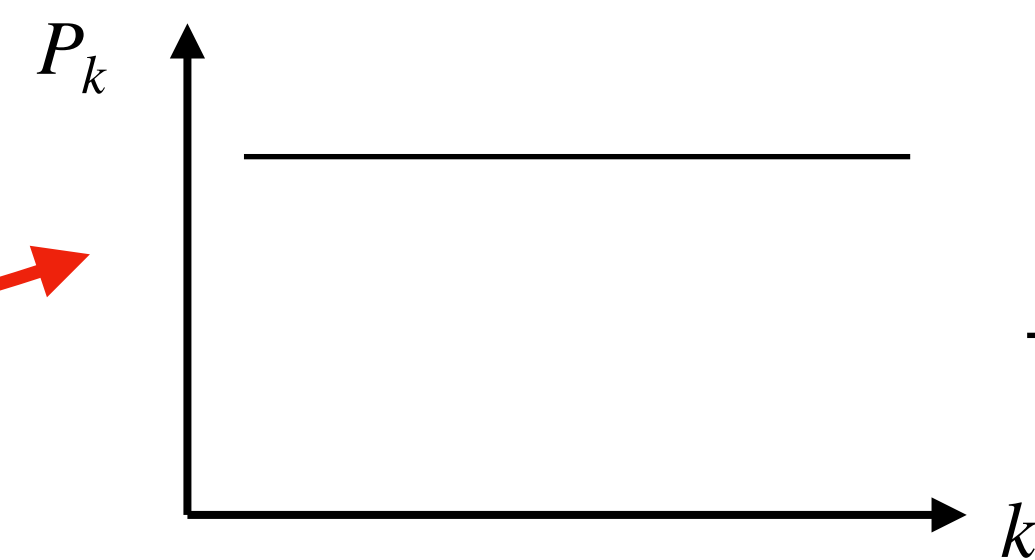
4. Canonical quantization in the limit $t \rightarrow 0$

→ Lowest energy state defines initial conditions

5. Scale-invariant primordial power spectrum as observed on the CMB

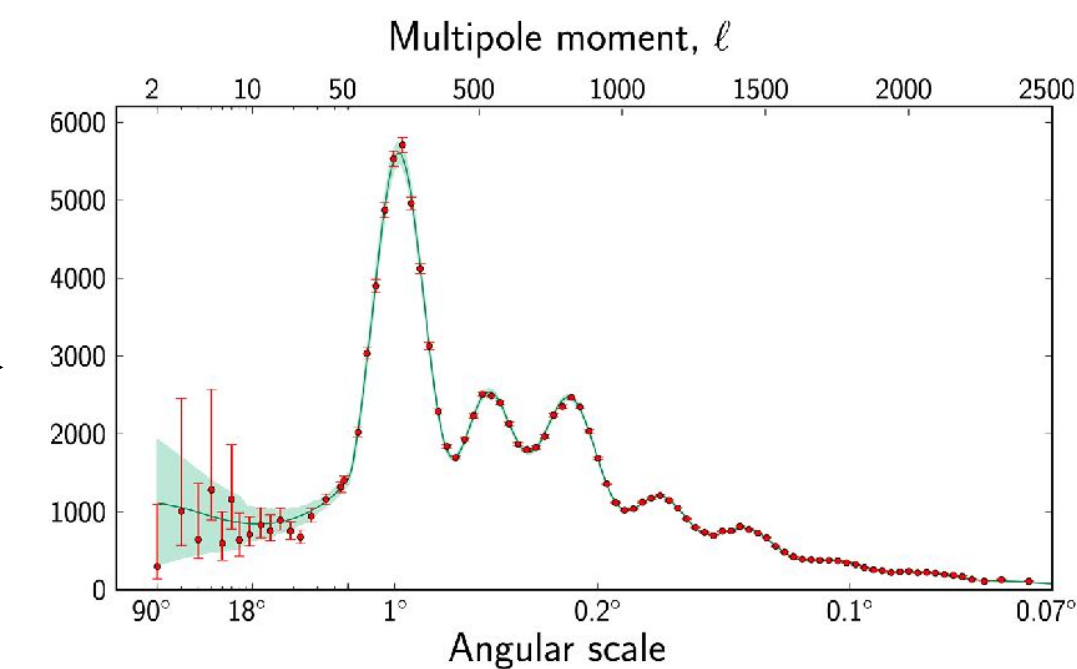


Provides an origin for the causality of the CMB



Flat initial power spectrum

Evolution



CMB power spectrum

Topo-GR ($\forall K$)

1. Matter described by a scalar field φ :

$$T^\mu{}_\nu = \nabla^\mu \varphi \nabla_\nu \varphi - \frac{1}{2} \nabla^\alpha \varphi \nabla_\alpha \varphi \delta^\mu{}_\nu - V \delta^\mu{}_\nu$$

2. Weak field equations

$$\Delta \Psi = 4\pi G a^2 \delta \rho_\Delta, \quad \forall K$$

...

3. Curvature perturbation \mathcal{R} solution of a wave equation:

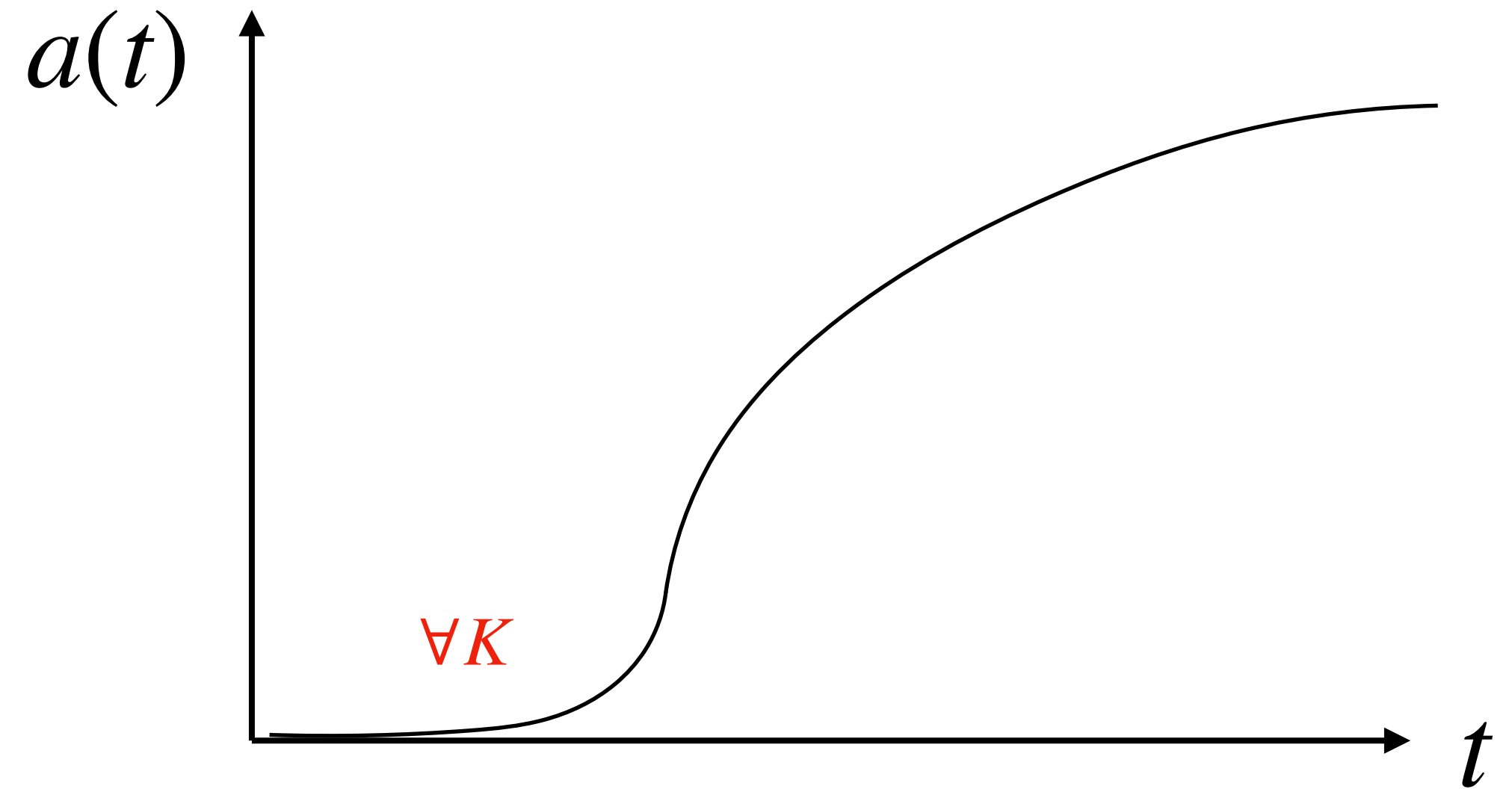
$$\mathcal{R}'' + \frac{2z'}{z} \mathcal{R}' - \Delta \mathcal{R} = 0, \quad \forall K$$

4. Canonical quantization in the limit $t \rightarrow 0$

→ Lowest energy state defines initial conditions

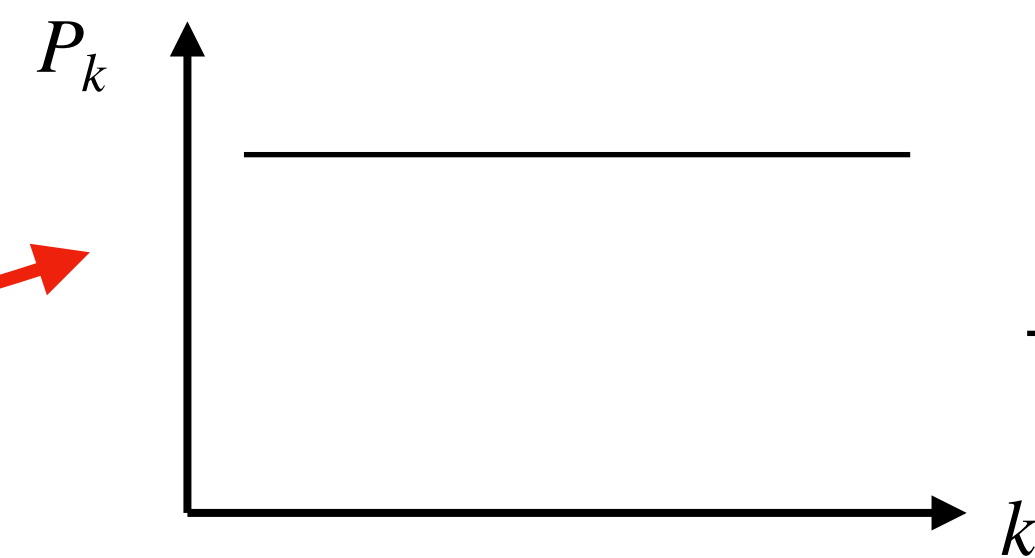
5. Scale-invariant primordial power spectrum as observed on the CMB

⇒ Simple, natural, model explaining CMB anisotropies for any curvature. [Vigneron & Larena \(2024\)](#)



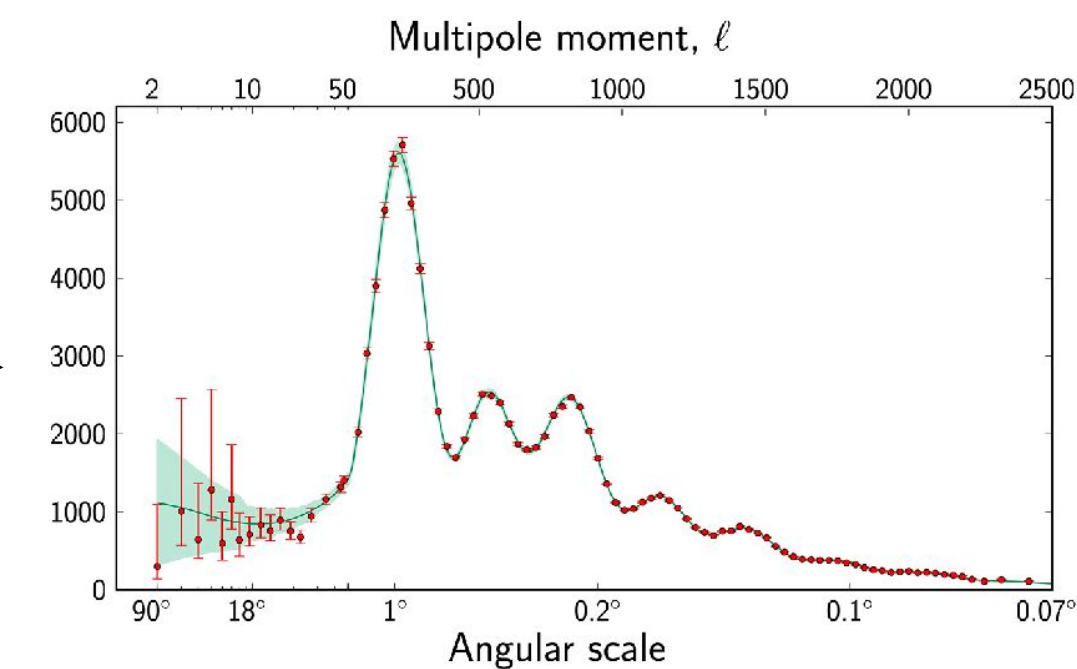
Acceleration

Provides an origin for the causality of the CMB



Flat initial power spectrum

Evolution



CMB power spectrum

$$R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$

Motivations:

- * Existence of a Newtonian limit in any topology.
 - *Q.V. (PRD 2021; PRD 2022; CQG 2022; FoP 2024); Q.V. & B. Roukema (PRD 2023)*
- * From the Carrollian limit? (In prog.)
 - *Q.V. & H. Barzegar & J. Read (2025, preprint)*
- * Well-posed variational principle in any topology? (In prog.)

Cosmology: a universal dynamics

- * Expansion does not depend on Ω_K .
 - *Q.V. & V. Poulin (PRD 2023)*
- * Simple model for curved inflation.
 - *Q.V. & J. Larena (CQG 2024)*
- * **Homogeneous but anisotropic solutions. (in prog.)**
 - *Q.V. & H. Barzegar (in prep.)*

Why homogeneous but anisotropic models?

Why homogeneous but anisotropic models?

→ Anisotropy of the expansion: $\sigma_{ij} = \Theta_{ij} - Hh_{ij} \neq 0$

→ Anisotropy of the curvature: $\mathcal{R}_{ij} - \frac{\mathcal{R}}{3}h_{ij} \neq 0$

Why homogeneous but anisotropic models?

* Is imposed by the topology if $\tilde{\Sigma} \neq \mathbb{E}^3, \mathbb{S}^3$ or \mathbb{H}^3 .

→ Anisotropy of the expansion: $\sigma_{ij} = \Theta_{ij} - Hh_{ij} \neq 0$

→ Anisotropy of the curvature: $\mathcal{R}_{ij} - \frac{\mathcal{R}}{3}h_{ij} \neq 0$

Why homogeneous but anisotropic models?

* Is imposed by the topology if $\tilde{\Sigma} \neq \mathbb{E}^3, \mathbb{S}^3$ or \mathbb{H}^3 .

* Dominates the energy budget in the early Universe:

$$6H^2 = 16\pi G\rho + \sigma^2 + (-\mathcal{R}) \quad ; \quad \partial_t(\sigma^2) = -6H\sigma^2 - 2\sigma^{ij}\mathcal{R}_{ij}$$

with $\rho > 0$, $\sigma^2 \geq 0$, $-\mathcal{R} \longrightarrow$ the sign depends on the topological class.

Flat anisotropic model (i.e. Bianchi I): $\sigma^2 \propto 1/a^6$.

→ Anisotropy of the expansion: $\sigma_{ij} = \Theta_{ij} - Hh_{ij} \neq 0$

→ Anisotropy of the curvature: $\mathcal{R}_{ij} - \frac{\mathcal{R}}{3}h_{ij} \neq 0$

Why homogeneous but anisotropic models?

* Is imposed by the topology if $\tilde{\Sigma} \neq \mathbb{E}^3, \mathbb{S}^3$ or \mathbb{H}^3 .

* Dominates the energy budget in the early Universe:

$$6H^2 = 16\pi G\rho + \sigma^2 + (-\mathcal{R}) \quad ; \quad \partial_t(\sigma^2) = -6H\sigma^2 - 2\sigma^{ij}\mathcal{R}_{ij}$$

with $\rho > 0$, $\sigma^2 \geq 0$, $-\mathcal{R} \longrightarrow$ the sign depends on the topological class.

Flat anisotropic model (i.e. Bianchi I): $\sigma^2 \propto 1/a^6$.

→ Anisotropy of the expansion: $\sigma_{ij} = \Theta_{ij} - Hh_{ij} \neq 0$

→ Anisotropy of the curvature: $\mathcal{R}_{ij} - \frac{\mathcal{R}}{3}h_{ij} \neq 0$

Is isotropisation of the Universe a generic behaviour?

Why homogeneous but anisotropic models?

→ Anisotropy of the expansion: $\sigma_{ij} = \Theta_{ij} - Hh_{ij} \neq 0$

→ Anisotropy of the curvature: $\mathcal{R}_{ij} - \frac{\mathcal{R}}{3}h_{ij} \neq 0$

* Is imposed by the topology if $\tilde{\Sigma} \neq \mathbb{E}^3, \mathbb{S}^3$ or \mathbb{H}^3 .

* Dominates the energy budget in the early Universe:

$$6H^2 = 16\pi G\rho + \sigma^2 + (-\mathcal{R}) \quad ; \quad \partial_t(\sigma^2) = -6H\sigma^2 - 2\sigma^{ij}\mathcal{R}_{ij}$$

with $\rho > 0$, $\sigma^2 \geq 0$, $-\mathcal{R} \longrightarrow$ the sign depends on the topological class.

Flat anisotropic model (i.e. Bianchi I): $\sigma^2 \propto 1/a^6$.

Is isotropisation of the Universe a generic behaviour?

$\tilde{\Sigma}$	\mathbb{E}^3	\mathbb{S}^3	\mathbb{H}^3	$\mathbb{R} \times \mathbb{S}^2$	$\mathbb{R} \times \mathbb{H}^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Bianchi type	I	IX	V	K.-S.	III	VIII	II	VI ₀
GR <i>R. Wald (1986)</i>	✓	✗	✓	✗	✓	✓	✓	✓

Why homogeneous but anisotropic models?

→ Anisotropy of the expansion: $\sigma_{ij} = \Theta_{ij} - Hh_{ij} \neq 0$

→ Anisotropy of the curvature: $\mathcal{R}_{ij} - \frac{\mathcal{R}}{3}h_{ij} \neq 0$

* Is imposed by the topology if $\tilde{\Sigma} \neq \mathbb{E}^3, \mathbb{S}^3$ or \mathbb{H}^3 .

* Dominates the energy budget in the early Universe:

$$6H^2 = 16\pi G\rho + \sigma^2 + (-\mathcal{R} + \bar{\mathcal{R}}) \quad ; \quad \partial_t(\sigma^2) = -6H\sigma^2 - 2\sigma^{ij} \left(\mathcal{R}_{ij} - \bar{\mathcal{R}}_{ij} \right)$$

with $\rho > 0$, $\sigma^2 \geq 0$, $-\mathcal{R} \longrightarrow$ the sign depends on the topological class.

Flat anisotropic model (i.e. Bianchi I): $\sigma^2 \propto 1/a^6$.

Is isotropisation of the Universe a generic behaviour?

$\tilde{\Sigma}$	\mathbb{E}^3	\mathbb{S}^3	\mathbb{H}^3	$\mathbb{R} \times \mathbb{S}^2$	$\mathbb{R} \times \mathbb{H}^2$	$\tilde{SL}(2, \mathbb{R})$	<i>Nil</i>	<i>Sol</i>
Bianchi type	I	IX	V	K.-S.	III	VIII	II	VI ₀
GR <i>R. Wald (1986)</i>	✓	✗	✓	✗	✓	✓	✓	✓

$$R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$

Motivations:

- * Existence of a Newtonian limit in any topology.
 - *Q.V. (PRD 2021; PRD 2022; CQG 2022; FoP 2024); Q.V. & B. Roukema (PRD 2023)*
- * From the Carrollian limit? (In prog.)
 - *Q.V. & H. Barzegar & J. Read (2025, preprint)*
- * Well-posed variational principle in any topology? (In prog.)

Cosmology: a universal dynamics

- * Expansion does not depend on Ω_K .
 - *Q.V. & V. Poulin (PRD 2023)*
- * Simple model for curved inflation.
 - *Q.V. & J. Larena (CQG 2024)*
- * Homogeneous but anisotropic solutions. (in prog.)
 - *Q.V. & H. Barzegar (in prep.)*

$$R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$

Motivations:

- * Existence of a Newtonian limit in any topology.
 - *Q.V. (PRD 2021; PRD 2022; CQG 2022; FoP 2024); Q.V. & B. Roukema (PRD 2023)*
- * From the Carrollian limit? (In prog.)
 - *Q.V. & H. Barzegar & J. Read (2025, preprint)*
- * Well-posed variational principle in any topology? (In prog.)

Cosmology: a universal dynamics

- * Expansion does not depend on Ω_K .
 - *Q.V. & V. Poulin (PRD 2023)*
- * Simple model for curved inflation.
 - *Q.V. & J. Larena (CQG 2024)*
- * Homogeneous but anisotropic solutions. (in prog.)
 - *Q.V. & H. Barzegar (in prep.)*

Between GR and topo-GR, it is rather GR which is a gravity theory that depends on topology.

$$R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$

Motivations:

- * Existence of a Newtonian limit in any topology.
 - *Q.V. (PRD 2021; PRD 2022; CQG 2022; FoP 2024); Q.V. & B. Rouke*
- * From the Carrollian limit? (In prog.)
 - *Q.V. & H. Barzegar & J. Read (2025, preprint)*
- * Well-posed variational principle in any topology? (In prog.)

Cosmology: a universal dynamics

- * Expansion does not depend on Ω_K .
 - *Q.V. & V. Poulin (PRD 2023)*
- * Simple model for curved inflation.
 - *Q.V. & J. Larena (CQG 2024)*
- * Homogeneous but anisotropic solutions. (in prog.)
 - *Q.V. & H. Barzegar (in prep.)*

Some perspectives

Black holes:

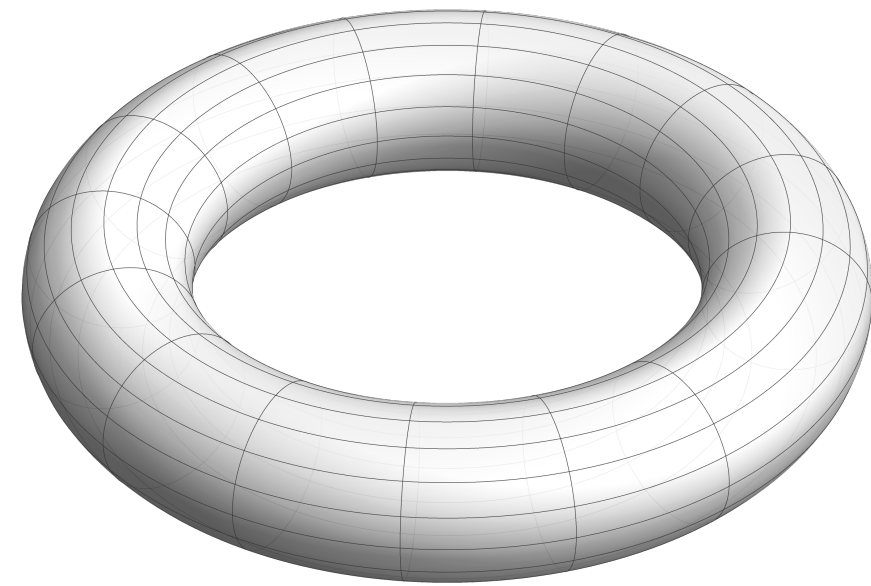
- * Propagating massless scalar mode in vacuum.
- * Exact solutions (e.g. Q.V. & Szabo & Mourier, EPJC 2024).
- * Definition of mass in non-trivial topologies.

Cosmology:

- * Inflation in general topologies
- * BKL singularity
- * Waiting for a measurement of spatial curvature...

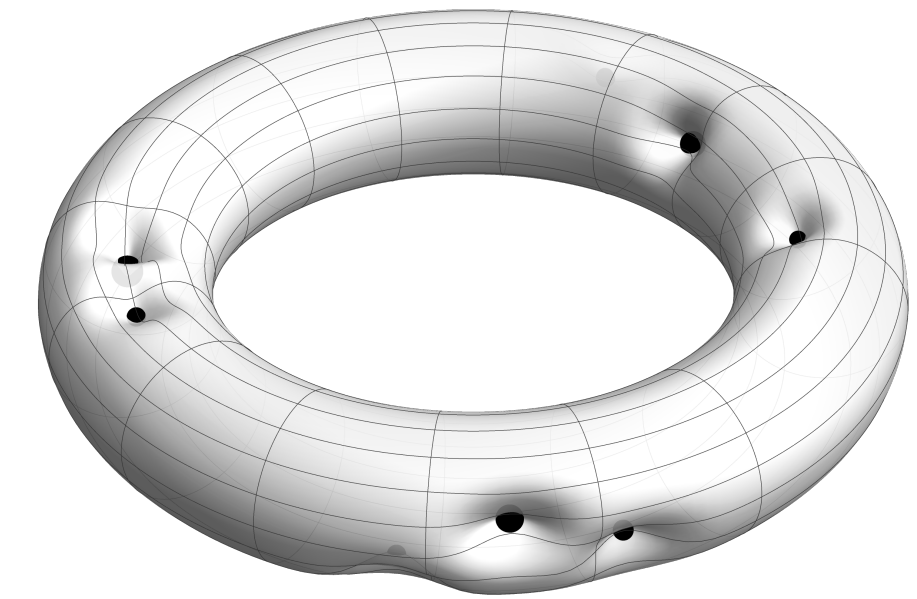
Quantum gravity:

- * e.g., $\bar{R}_{\mu\nu}$ arising from topological change in quantum gravity?



Matter

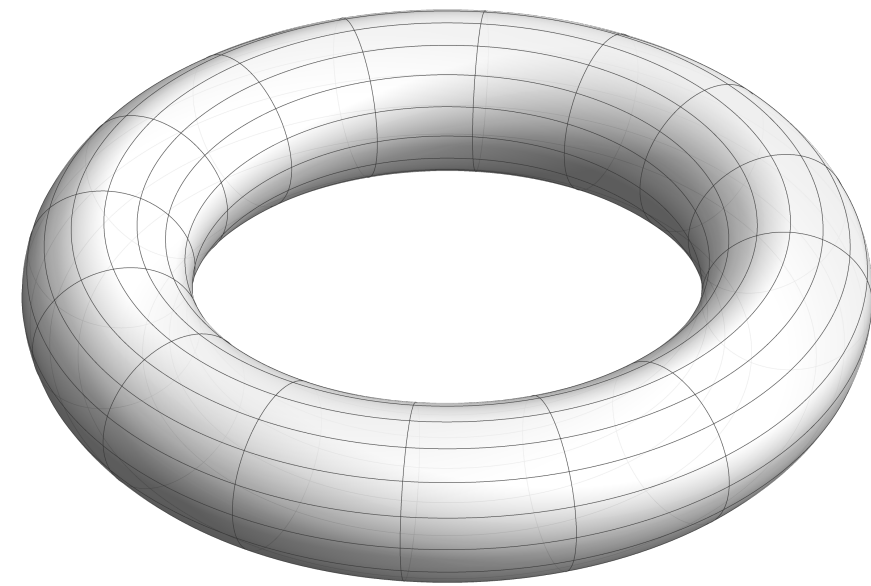
$$R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$



(1) Matter does not curve spacetime, it only induces a deviation from the curvature "imposed" by the spacetime topology.

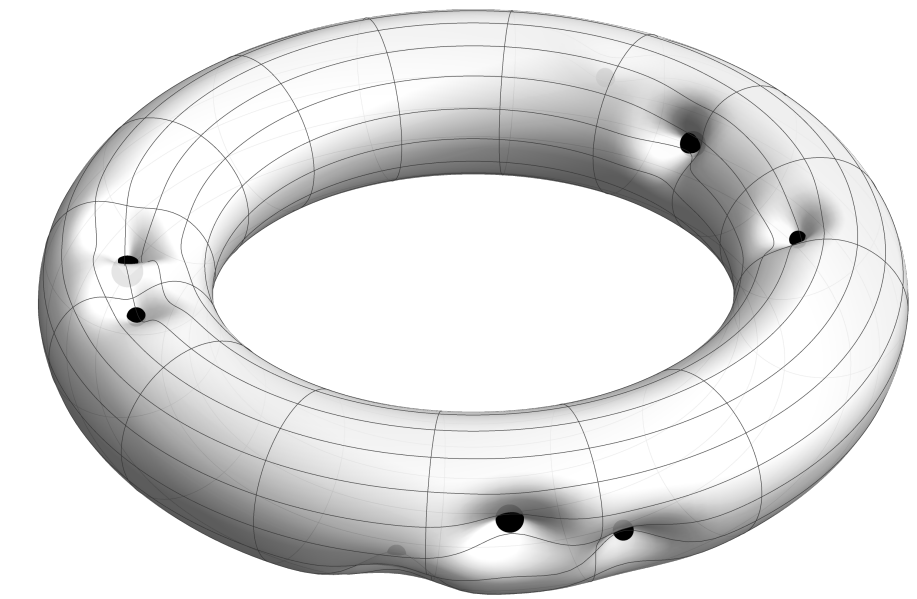
(2) Gravity in topo-GR is more universal than in GR.

Thank you for your attention!



Matter

$$R_{\mu\nu} - \bar{R}_{\mu\nu}^{\text{topo}} = 8\pi G \left(T_{\mu\nu} - T/2 g_{\mu\nu} \right)$$



(1) *Matter does not curve spacetime, it only induces a deviation from the curvature "imposed" by the spacetime topology.*

(2) *Gravity in topo-GR is more universal than in GR.*

(3) *I am searching for a postdoc ;)*

Thank you for your attention!