Axial perturbations of black holes with primary scalar hair

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Motivations

- GW astronomy provides a new window to test GR, in particular in the strong field regime.

- Why Modify GR ? → GR seems compatible with all observations
  - ➡ Understand cosmological acceleration
  - ➡ Explore alternative gravitational theories→ understand what singles out GR as a theory
  - ➡ Step toward quantum gravity

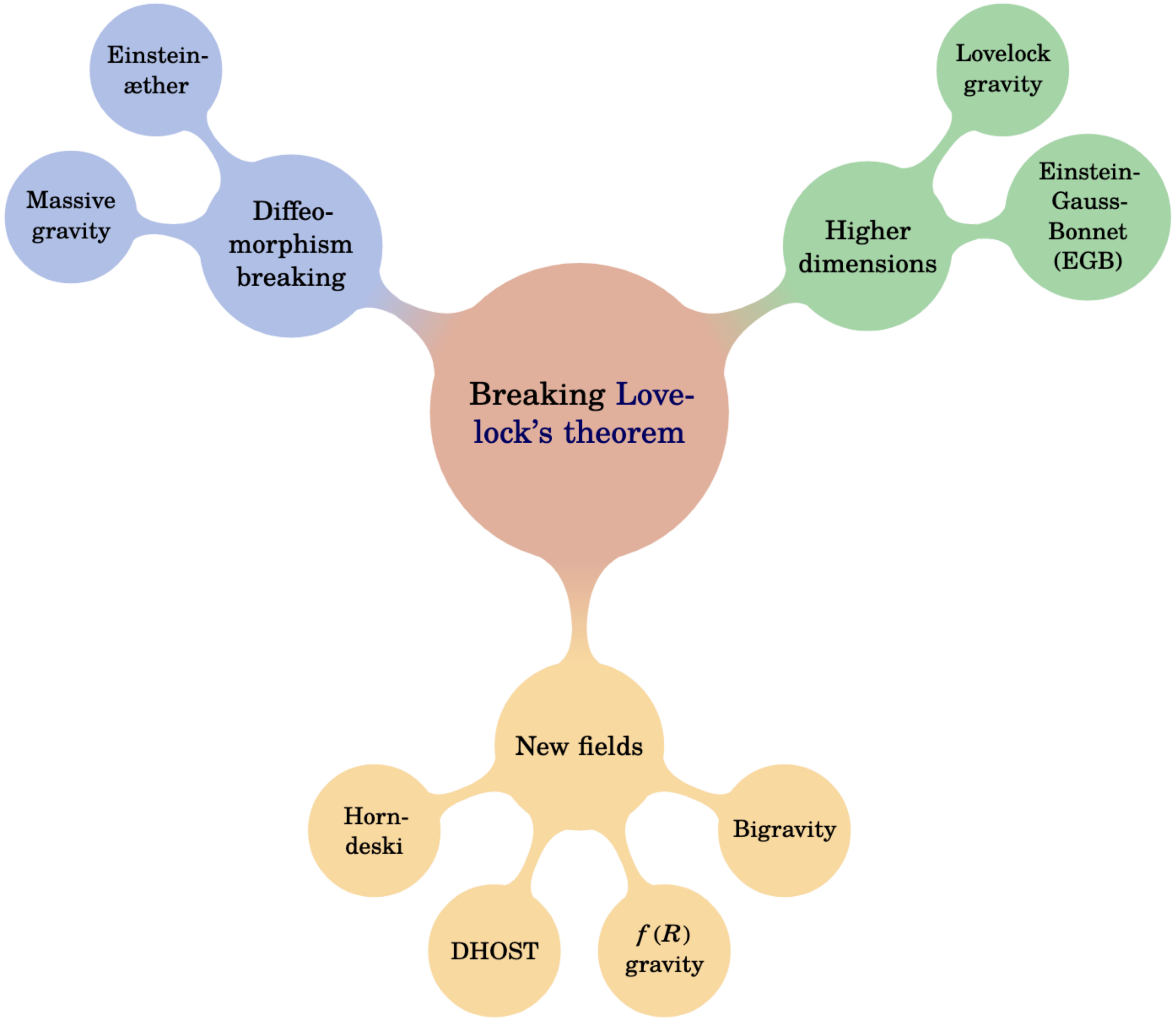
- In this talk: scalar-tensor theories (allowing for 2nd order derivatives in their Lagrangian)
  - ➡ Extract quantitative deviation of DHOST theories via QNMs

Modified Theories of gravity And DHOST theories

Modified Theories of Gravity And Scalar Tensors

Lovelock Theorem

Lovelock's theorem: In 4d, the only rank-2 tensor constructed solely from the metric $g_{\mu\nu}$ and its derivatives up to second differential order, preserving diffeomorphism invariance and arising from a least action principle is the Einstein tensor plus a cosmological term.



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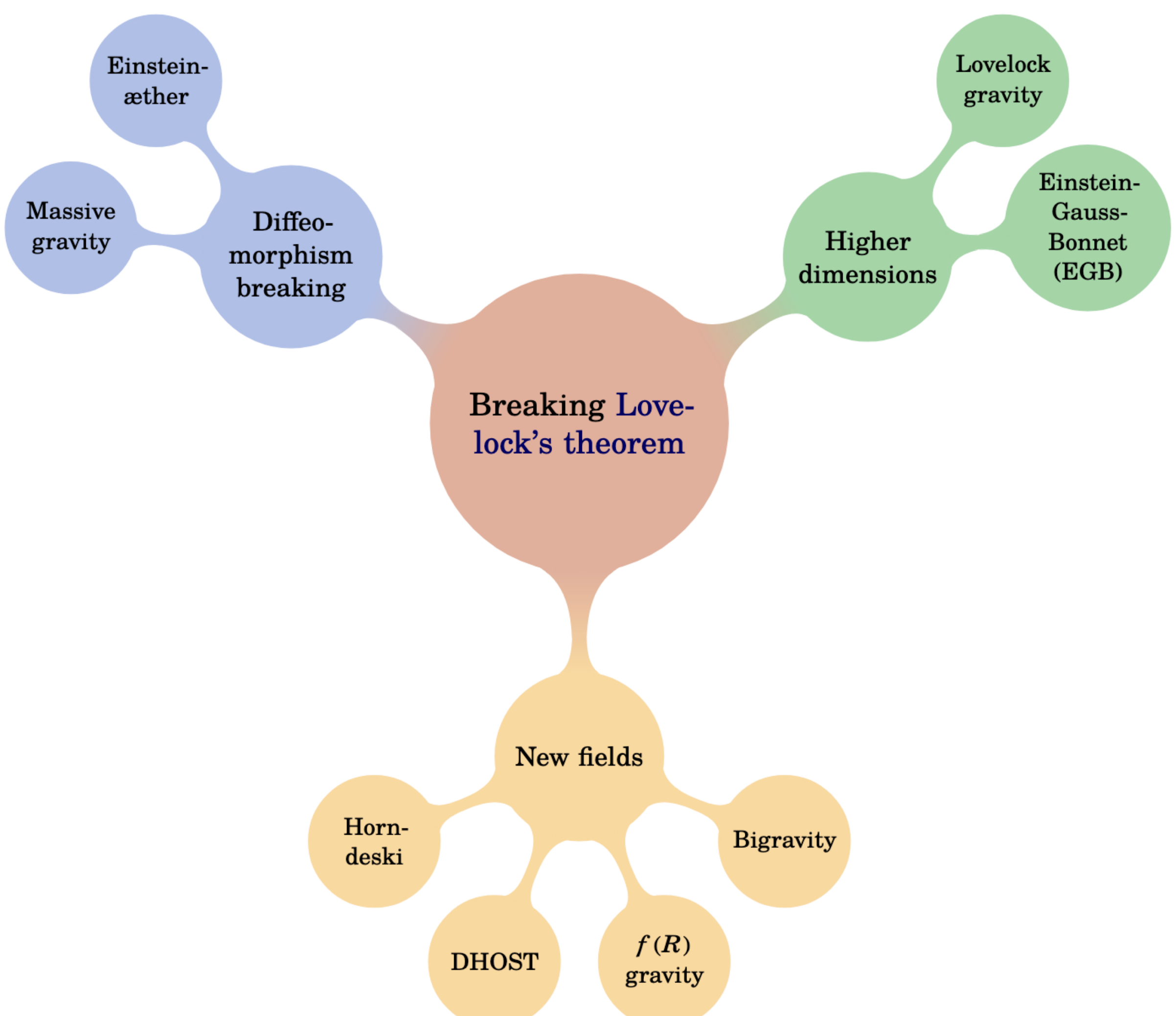

Scalar Tensor Theories

- Traditional scalar-tensor theories :
$$\mathscr{L}\left(\nabla_\lambda \phi, \phi\right)$$
- Generalized theories with second order derivatives
$$\mathscr{L}\left(\nabla_\mu \nabla_\nu \phi, \nabla_\lambda \phi, \phi\right)$$
- In general, they contain an extra degree of freedom, expected to lead to instabilities
$$L(\ddot{q}, \dot{q}, q)$$
- But this can be avoided if the theory is degenerate (2nd-order equations of motion not necessary !) →DHOST

DHOST theories

Quadratic DHOST

$$S = \int d^4x \sqrt{-g} \left[P(X,\phi) + Q(X,\phi)\Box\phi + F(X,\phi)R + \sum_{i=1}^{5} A_i(X,\phi) L_i^{(2)} \right]$$

Where $X \equiv -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$ $\quad \phi_\mu \equiv \nabla_\mu\phi$ $\quad \phi_{\mu\nu} \equiv \nabla_\nu\nabla_\mu\phi$

$$L_1^{(2)} = \phi_{\mu\nu}\phi^{\mu\nu}, \quad L_2^{(2)} = (\Box\phi)^2, \quad L_3^{(2)} = (\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu$$

$$L_4^{(2)} = \phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu, \quad L_5^{(2)} = \left(\phi^\mu\phi_{\mu\nu}\phi^\nu\right)^2$$

DHOST theories

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Possible extensions

- Possible extension to cubic order (in $\phi_{\mu\nu}$) [Ben Achour, Crisostomi, Koyama, Langlois, Noui & Tasinato '16]

$$L^{(3)} = F_3(X,\phi)G_{\mu\nu}\phi^{\mu\nu} + \sum_{i=1}^{10} B_i(X,\phi)L_i^{(3)}$$

- DHOST includes Horndeski, Beyond Horndeski, Einstein-scalar-Gauss-Bonnet

e.g. (quadratic) Horndeski: $A_2 = -A_1 = F_{,X}, \quad A_3 = A_4 = A_5 = 0$

Compact objects
In DHOST theories

DHOST theories

Non-Rotating BH

Consider a static spherically symmetric BH with a nontrivial scalar field

$$\mathrm{d}s^2 = -\mathscr{A}(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\mathscr{B}(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$$

- Scalar field: $\phi(t,r) = qt + \psi(r)$ [Babichev, Charmousis '13]

The scalar charge $q \neq 0$ possible in shift-symmetric theories

DHOST theories

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Some Examples

- «stealth» Schwarzschild: $\mathscr{A} = \mathscr{B} = 1 - \frac{\mu}{r}$
- «BCL» [Babichev, Charmousis, Lehébel '17]. $\mathscr{A} = \mathscr{B} = 1 - \frac{\mu}{r} - \xi\frac{\mu^2}{2r^2}$
- 4d Gauss-Bonnet [Glavan Lin '19, Lu Pang '2 0]. $\mathscr{A} = \mathscr{B} = 1 - \frac{2\mu/r}{1+\sqrt{1+4\alpha\mu/r^3}}$
- Scalar-Gauss-Bonnet [Julié Berti '19] $\mathscr{A} = \mathscr{B} = 1 - \frac{\mu}{r} + a_2(r)\varepsilon^2 + \dots$

Family of BH with Primary Hair

Quadratic DHOST theories considered

$$P(X) = -\frac{2\alpha}{\lambda^2}X^p, \qquad F(X) = 1 - 2XA_1, \qquad A_1(X) = \frac{\alpha}{2}X^{p-1}, \qquad A_3(X) = \frac{\alpha}{2}(2p-1)X^{p-2}$$

Then, one is able to find explicit solutions:

$$ds^2 = -\mathscr{A}(r)dt^2 + \frac{dr^2}{\mathscr{B}(r)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad \phi(t,r) = qt + \psi(r)$$

with
$$\mathscr{A}(r) = 1 - \frac{2\mu}{r} - \frac{2\xi_p}{r}\int_0^r \frac{u^2\, du}{\left(1+u^2\right)^p}$$

The scalar Hair

$$\xi_p \equiv \alpha(2p-1)\left(\frac{q^2}{2}\right)^p$$

and
$$X = \frac{q^2}{2\left(1+r^2\right)}, \qquad \psi'(r) = \frac{q}{\mathscr{A}(r)}\sqrt{1 - \frac{\mathscr{A}(r)}{1+r^2}}$$

Examples : p=1/2 and p=2

p=1/2 Stealth Solution in Horndeski theory

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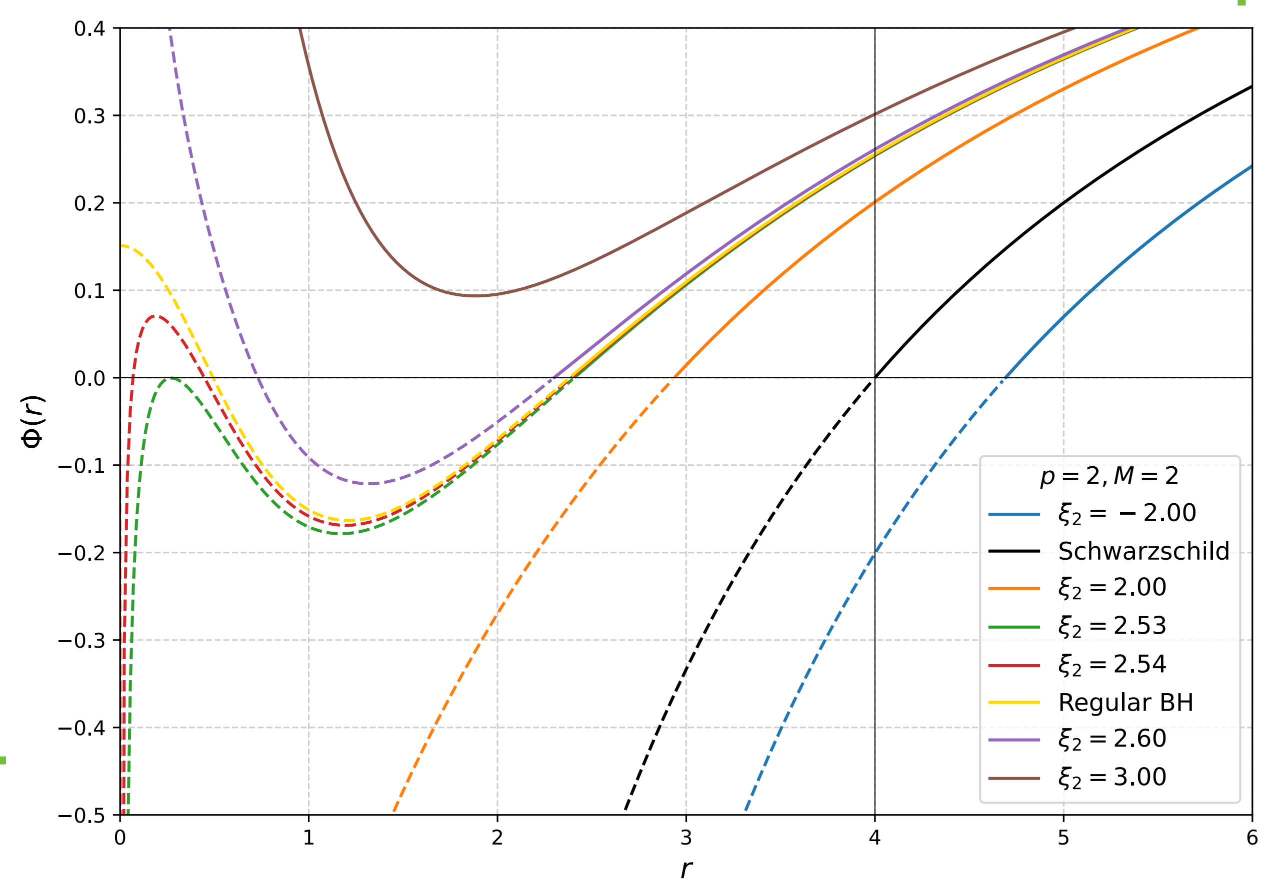
p=2 Deformation of Schwarzschild in a Beyond Horndeski theory

$$\mathscr{A}(r) = \mathscr{B}(r) = 1 - \frac{2M}{r} + \xi_2 \left(\frac{\pi/2 - \arctan r}{r} + \frac{1}{1+r^2} \right)$$

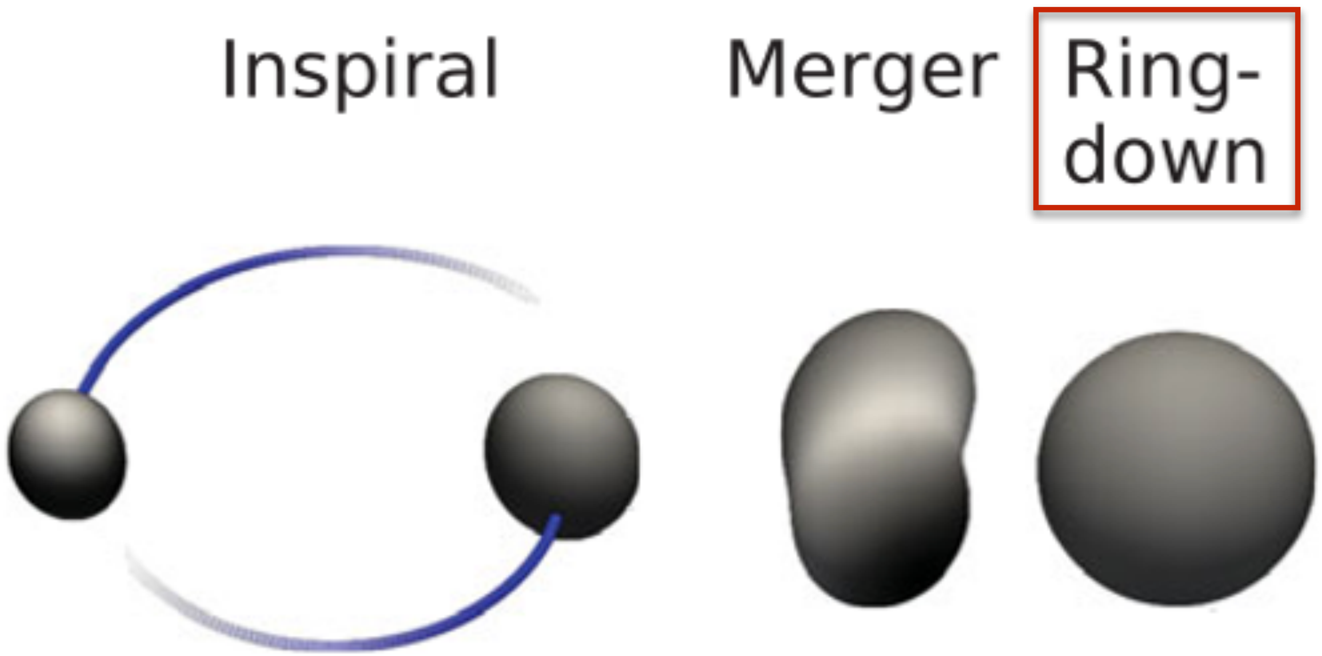
- No singularity at $r = 0$ if $M = \frac{\pi}{4}\xi_2$



BH Perturbation Theory

BH Perturbations

- Ringdown phase of a BH merger is interesting for modified gravity models, as it can be described by BH perturbations.



- Deviations to GR in the context of DHOST theories ?

BH perturbations in GR

In General Relativity

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{bgd}} + h_{\mu\nu}$$

- Perturbation parametrized by the 10 components of $h_{\mu\nu}$
- Expand on Fourier modes $f(t,r) = f(r)e^{-i\omega t}$ and on spherical harmonics $Y_{\ell m}(\theta, \varphi)$

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Axial Sector

$h_0(r), h_1(r)$ [Regge-Wheeler gauge]

$$h_{\mu\nu} = \sum_{\ell,m} \begin{pmatrix} 0 & 0 & \frac{1}{\sin\theta} h_0^{\ell m}\partial_\varphi & -\sin\theta h_0^{\ell m}\partial_\theta \\ 0 & 0 & \frac{1}{\sin\theta} h_1^{\ell m}\partial_\varphi & -\sin\theta h_1^{\ell m}\partial_\theta \\ \text{sym} & \text{sym} & 0 & 0 \\ \text{sym} & \text{sym} & 0 & 0 \end{pmatrix} Y_{\ell m}(\theta,\varphi)$$

Polar Sector

H_0, H_1, H_2, K [Zerilli gauge]

$$h_{\mu\nu} = \sum_{\ell,m} \begin{pmatrix} A(r)H_0^{\ell m}(r) & H_1^{\ell m}(r) & 0 & 0 \\ H_1^{\ell m}(r) & A^{-1}(r)H_2^{\ell m}(r) & 0 & 0 \\ 0 & 0 & K^{\ell m}(r)r^2 & 0 \\ 0 & 0 & 0 & K^{\ell m}(r)r^2\sin^2\theta \end{pmatrix} Y_{\ell m}(\theta,\varphi)$$

(And $\delta\phi$ in DHOST)

Axial perturbations

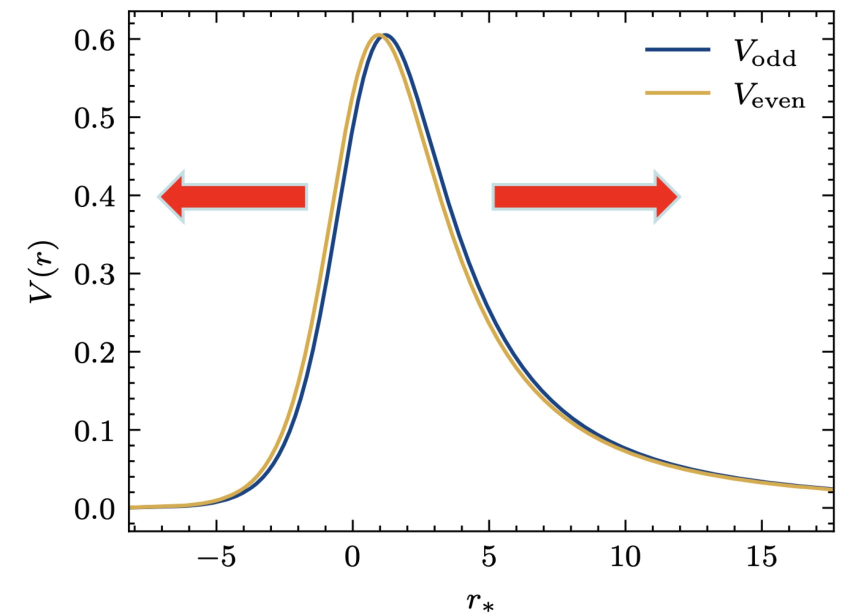
In General Relativity

The linearised metric eqs yield only 2 independent eqs

$$\frac{dY}{dr} = M(r)Y(r), \quad Y = \begin{pmatrix} h_0 \\ h_1/\omega \end{pmatrix}$$

or, in a Schrödinger form, [Regge & Wheeler '57]

$$\frac{d^2\hat{Y}}{dr_*^2} + \left(\omega^2 - V(r)\right)\hat{Y} = 0$$ with r_* tortoise coordinate



- We impose that at spatial infinity: $r_* \to -\infty, +\infty$

$$e^{-i\omega t}\hat{Y}(r) \approx a_+ e^{-i\omega(t - r_*)} + a_- e^{-i\omega(t + r_*)}$$

Outgoing Ingoing

- Quasi-normal modes: $a_+^{\text{hor}} = 0$ and $a_-^{\infty} = 0$

BH Perturbation Theory Framework

In DHOST

- The equations have a similar structure with GR [Langlois, Noui & Roussille '22]

$$\frac{dY}{dr} = MY \qquad\qquad M \equiv \begin{pmatrix} 2/r + i\omega\Psi & -i\omega^2 + 2i\lambda\Phi/r^2 \\ -i\Gamma & \Delta + i\omega\Psi \end{pmatrix}$$

where Ψ, Φ, Γ and Δ depend on the Lagrangian's functions and on the background.

- **Correspondence** (in quadratic DHOST theories)

DHOST axial modes in $g_{\mu\nu}$ $\Longleftrightarrow$ GR axial modes in $\hat{g}_{\mu\nu}$

with the effective metric

$$d\hat{s}^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu = |\mathcal{F}|\sqrt{\frac{\Gamma\mathcal{B}}{\mathcal{A}}}\left(-\Phi(dt - \Psi dr)^2 + \Gamma\Phi dr^2 + r^2 d\Omega^2\right)$$

Effective metric for axial modes

BHs with primary hair

The effective metric reduces to

$$d\hat{s}^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu = \sqrt{F}\left(-\Phi dt_*^2 + \frac{F}{\Phi}dr^2 + r^2 d\Omega^2\right) \quad \text{with} \quad \Phi = \mathscr{A} - q^2 A_1$$

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- Photons & gravitons «see» different geometries: $g_{\mu\nu}$ and $\hat{g}_{\mu\nu}$, and therefore different horizons (for BHs), determined by

$$\mathscr{A}\left(r_\ell\right) = 0, \quad \Phi\left(r_g\right) = 0$$

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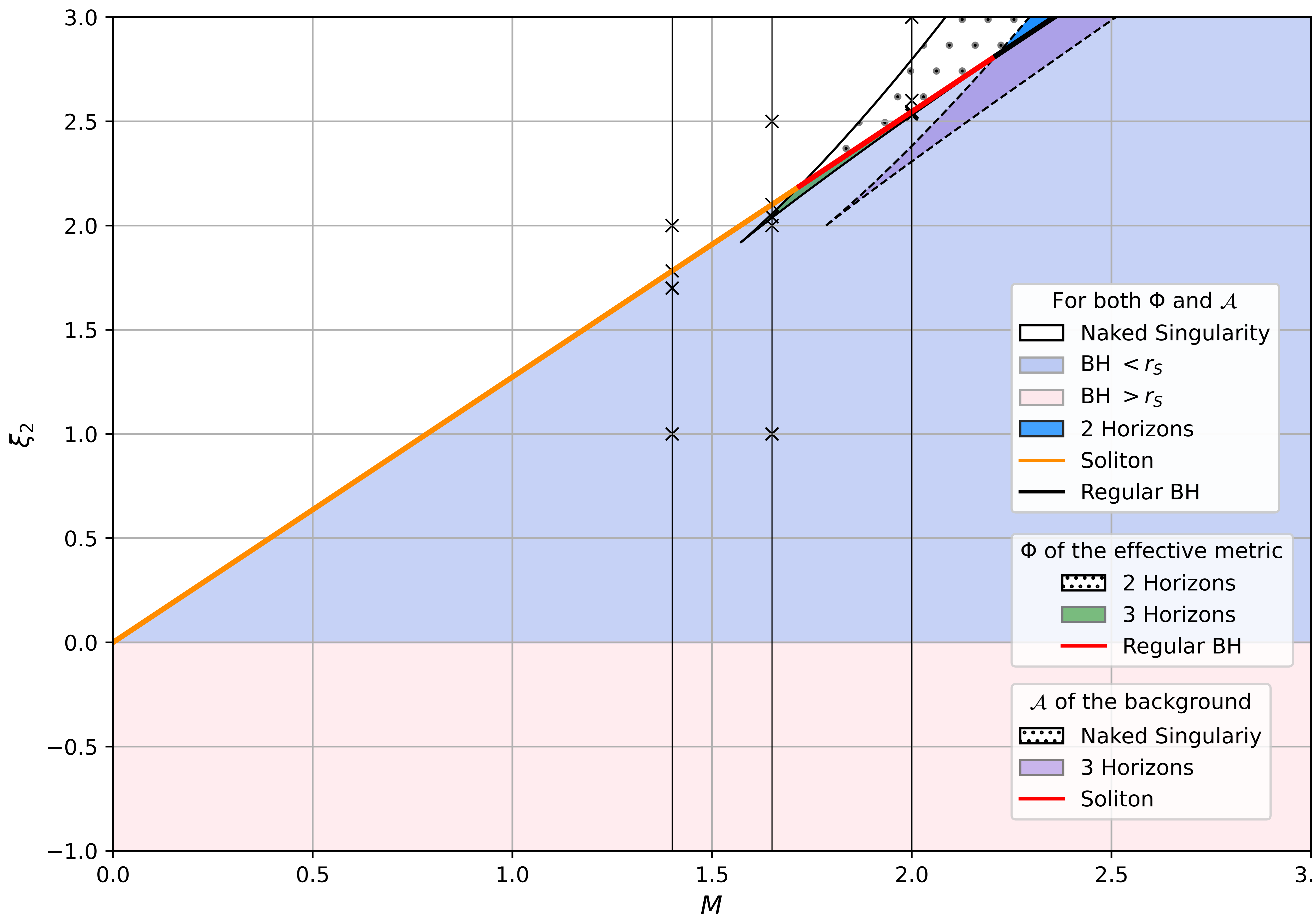
- In quadratic DHOST theories, the effective metric corresponds to a disformal transformation such that $\hat{F} = 1 \quad \hat{A}_1 = 0$ in a frame where $c_g = c_l$. In our case:

$$\hat{g}_{\mu\nu} = \sqrt{F}\left(g_{\mu\nu} + A_1 \phi_\mu \phi_\nu\right)$$

Effective metric for axial modes

« Phase Diagram » for the compact objects with primary hair, p=2



Effective metric for axial modes

BHs with primary hair

As in GR, one can write a Schrödinger-like equation

$$-\frac{d^2\mathcal{Y}}{d\hat{r}_*^2} + V_\ell \mathcal{Y} = \omega^2 \mathcal{Y} \qquad \mathcal{Y} = \frac{\Phi}{rF^{1/4}\omega}\left(h_1 + \Psi h_0\right)$$

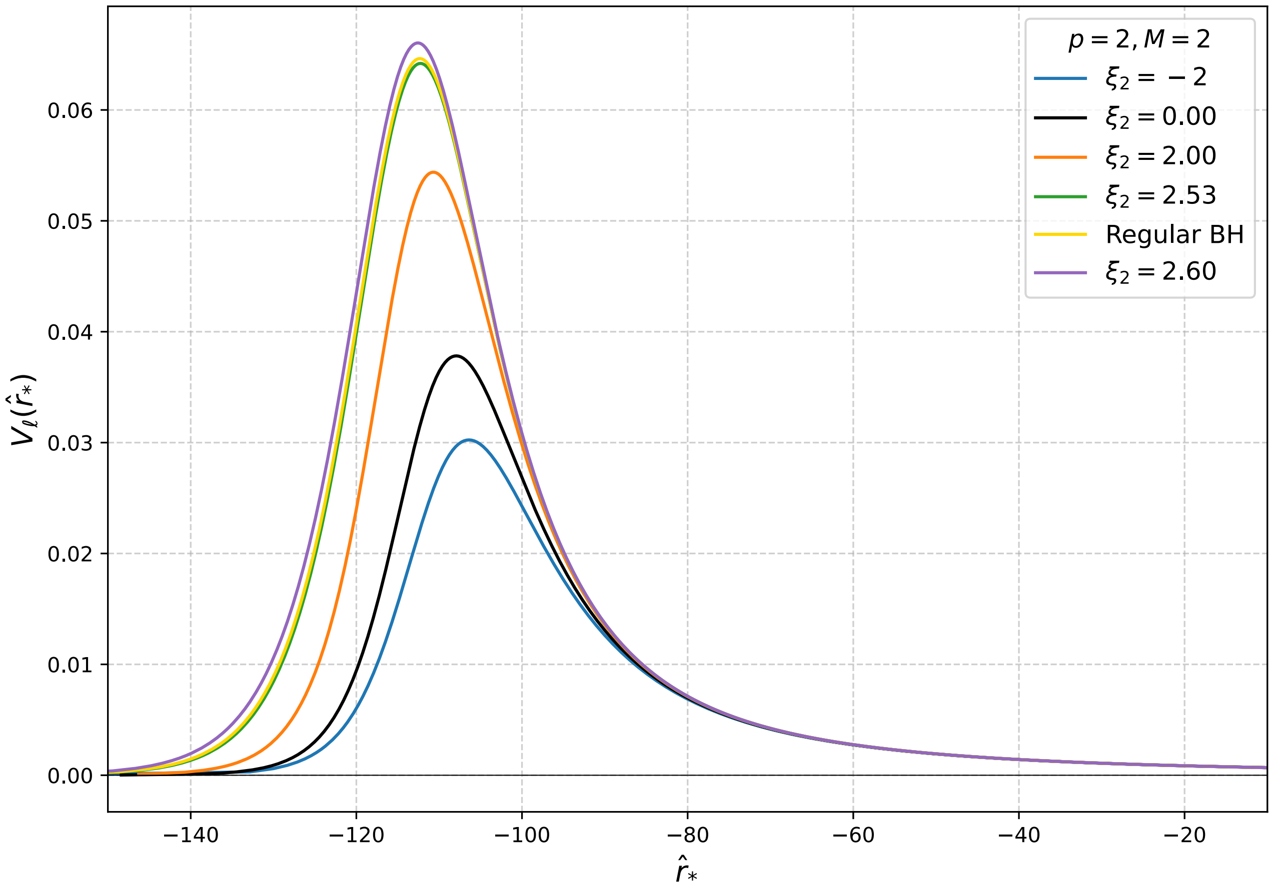
where

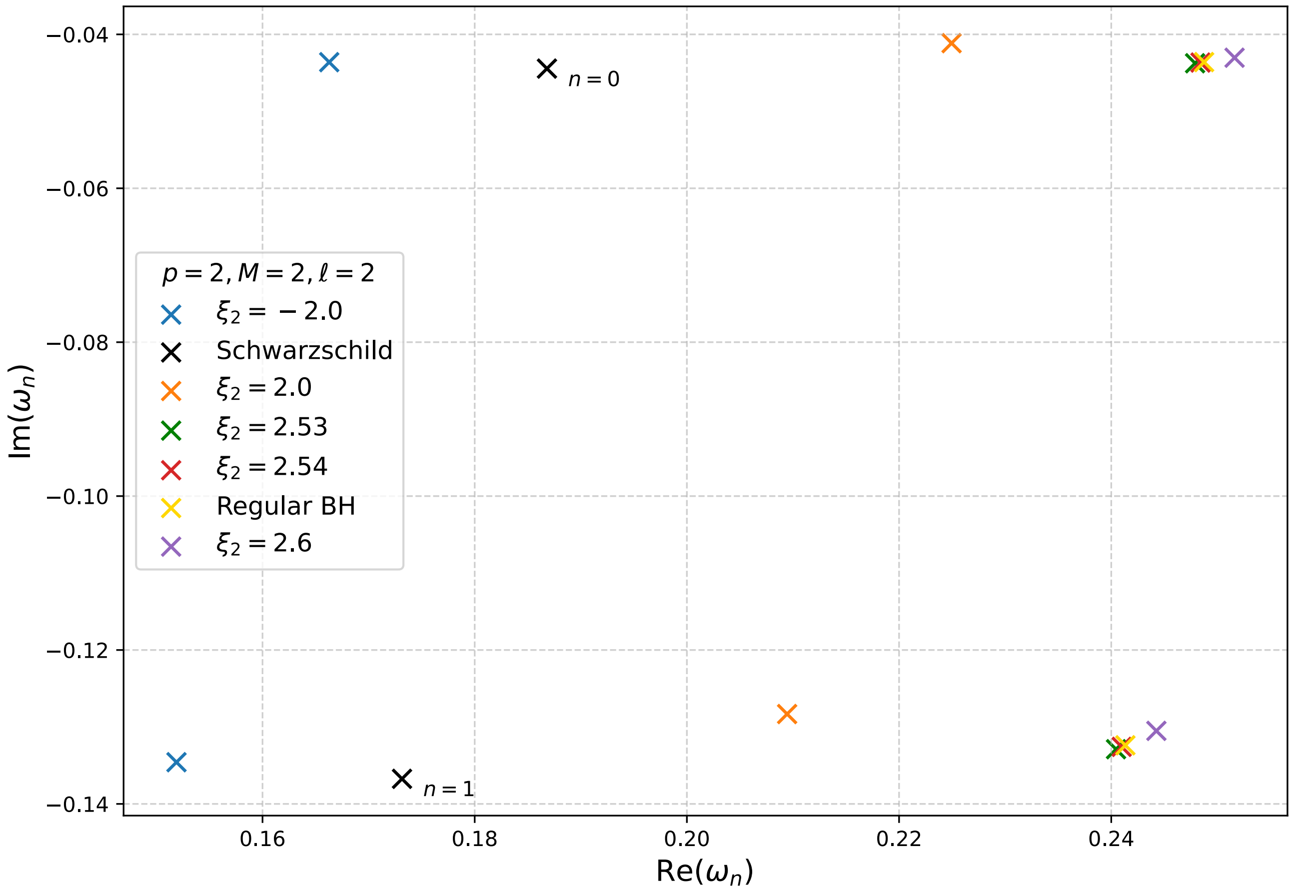
$$V_\ell(r) = \Phi \left[\frac{\ell^2 + \ell - 2}{r^2} - \frac{1}{r}\kappa_1\left(F, F', r\right)\Phi' + \frac{2}{r^2}\kappa_2\left(F, F', F'', r\right)\Phi \right]$$

Effective metric for axial modes

QNMs for Hairy BH with p=2

Using the WKB method





Conclusions

Conclusion

- DHOST theories give a very rich phenomenology, which can describe deviations from General Relativity, both for compact objects and in cosmology, and be tested in future observations.

- Interesting subfamily of DHOST theories allowing for exact BHs solutions with primary hair:
  - Axial modes: effective metric & QNMs
  - Polar modes: for future work

Additional Slides

Disformal Transformation

Disformal Transformation

Transformation [Bekenstein '92]

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X,\phi)g_{\mu\nu} + D(X,\phi)\partial_\mu\phi\partial_\nu\phi$$

Generating new theories

From $\tilde{S}\left[\phi, \tilde{g}_{\mu\nu}\right]$, one gets the new action

$$S\left[\phi, g_{\mu\nu}\right] \equiv \tilde{S}\left[\phi, \tilde{g}_{\mu\nu} = Cg_{\mu\nu} + D\phi_\mu\phi_\nu\right]$$

Disformal Transformation

Physically relevant DHOST families are closed under these transformations

Generating new theories

When standard fields are minimally coupled, two disformally related theories are physically inequivalent!

$$S\left[g_{\mu\nu},\phi\right]+S_m\left[\Psi_m,g_{\mu\nu}\right]\neq\tilde{S}\left[\tilde{g}_{\mu\nu},\phi\right]+S_m\left[\Psi_m,\tilde{g}_{\mu\nu}\right]$$