Emergent cosmologocial spacetimes from Matrix Models

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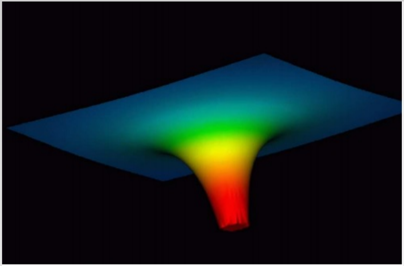

Credit: Pablo Laguna

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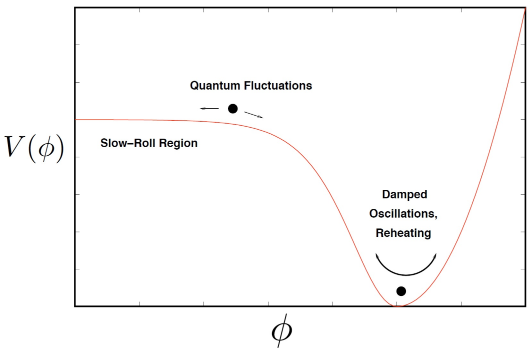

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Alternate description of the early-universe from *Matrix Theory*.

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- ✓ Expansion of the Universe: Hubble Law
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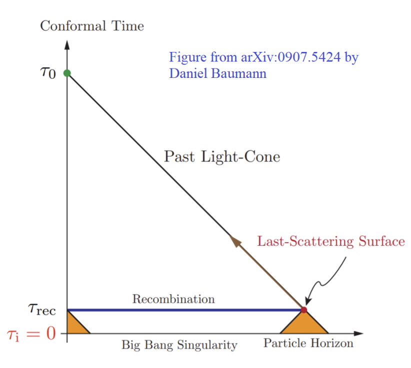

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- Solution to the standard horizon problem ⇐ Hubble radius smaller than causal horizon is all that is needed. [Brandenberger, 2011]
- Solution to the standard flatness problem ⇐ Spatially flat universe.
- Origin of structure ⇐ Scale invariant power spectrum of adiabatic perturbations. [Sunyaev & Zel'Dovich; Peebles & Yu, 1970]
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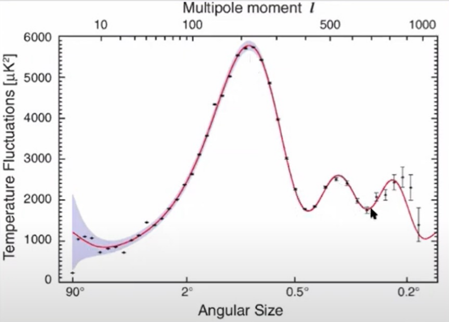

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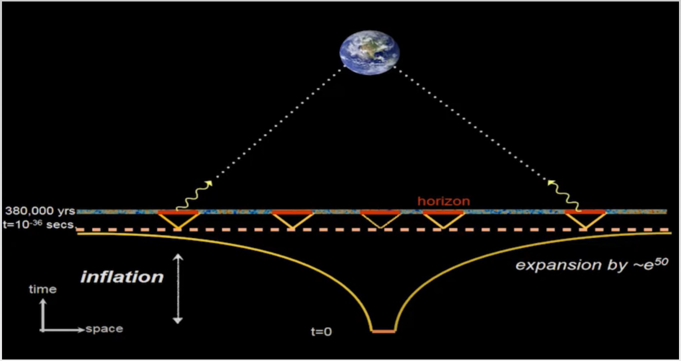

Photo credit: P. Adshead

Swampland implies short-lived meta-stable dS vacua can exist ⇒ Consistent with low-scale models of inflation.

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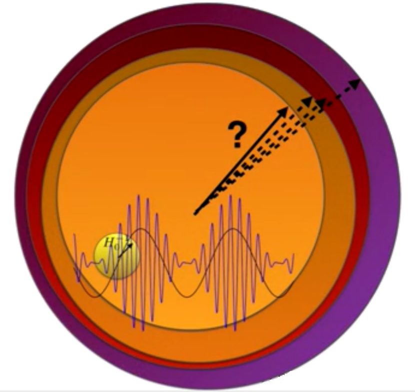

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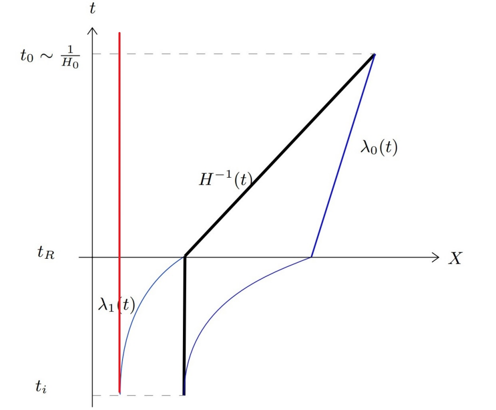

[Bedroya, Brandenberger, LoVerde, Vafa, 2019]

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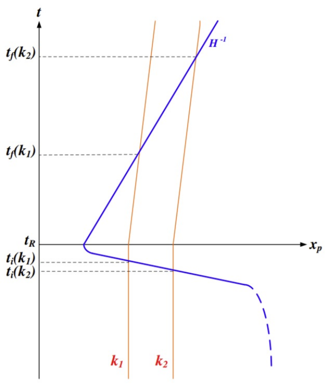
Symmetries of QG, *e.g.*, String Dualities, can play a pivotal role in this!
New symmetries (T-duality) ⇔ New states (Winding modes)

Alternatives

↪ Alternate descriptions of early-universe cosmology:

- Examples: *String Gas Cosmology* [Brandenberger & Vafa, 1989], *Ekpyrotic bounce* [Khoury, Ovrut, Steinhardt & Turok, 2001], *Early phase of topological gravity* [Agrawal, Gukov, Obied & Vafa, 2020], *CPT-symmetric universe* [Boyle & Turok, 2018-22], *Matter bounce* [Brandenberger, Wands, Wilson-Ewing, Cai & Wilson-Ewing, ...], , ...



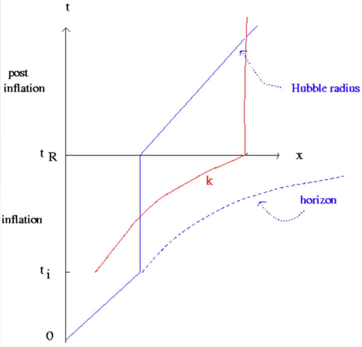


Figure Credit: R. Brandenberger

Matrix Models: Non-perturbative String Theory

↪ The BFSS matrix model [Banks, Fischler, Shenker & Susskind, 1997]:

$$S_{\rm BFSS} = \frac{1}{g^2}\int {\rm d}t {\rm Tr}\left\{\frac{1}{2}\left(D_t X_i\right)^2 - \frac{1}{4}\left[X_i, X_j\right]^2 + \frac{1}{2}\psi_\alpha D_t \psi_\alpha - \frac{1}{2}\psi_\alpha \gamma^i_{\alpha\beta}\left[X_i, \psi_\beta\right]\right\}$$

where $D_t \equiv \partial_t - i\left[A_t, \cdot\right]$ and $N \times N$ bosonic matrices $A(t), X_i(t)$ $(i = 1, \ldots, d)$ and $\psi_\alpha(t)$ $(\alpha = 1, \ldots, p)$.

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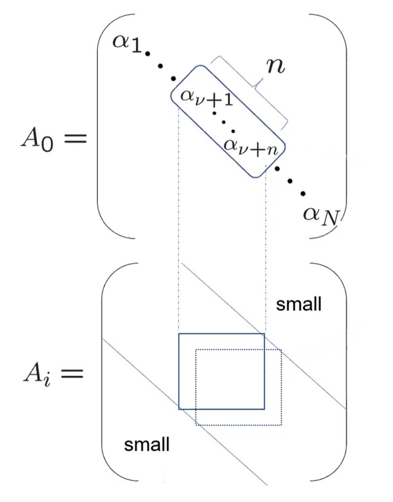
↪ A consistent, principled *top-down* approach to early-universe cosmology with the promise of rich phenomenology.

- **No** Fock-space EFT description: No **cosmological constant problem**!
- UV-complete: Eigenvalues never become trivial ⇒ **No singularities!**

Emergent $(3+1)$-d spacetime: Numerical evidence

[Aoki, Hirasawa, Ito, Kim, Nishimura, Tsuchiya, ...]

↪ Lorentzian IKKT model: $Z \sim \int \mathrm{d}A\,\mathrm{d}\Psi\, e^{iS_{\mathrm{IKKT}}}$



✓ Diagonalize $A_0 : \alpha_1 < \ldots < \alpha_N$.

✓ Define time via *coarse-graining*:

$$t(\nu) := \frac{1}{n}\sum_{i=1}^{n}\alpha_{\nu+i}, \quad \nu = 1, \ldots, N-n$$

✓ *Non-trivial* to obtain dynamical band-diagonal structure!

→ Time-dep $n \times n$ spatial matrices: $\left(\bar{A}_i\right)_{I,J}(t(\nu)) := \left(A_i\right)_{\nu+I,\nu+J}$

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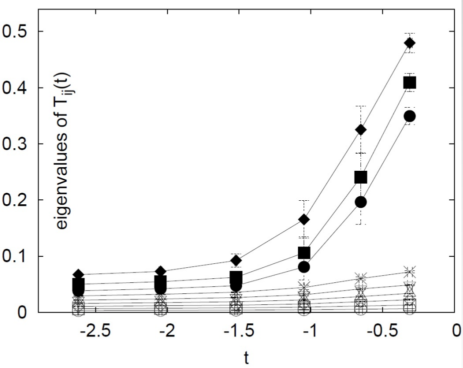
↪ *Emergence* of 3 large spatial dimensions:



✓ As order parameter, define moment of inertia tensor

$$T_{ij}(t) := \left\langle \frac{1}{n} \mathrm{Tr} \bar{A}_i(t) \bar{A}_j(t) \right\rangle$$

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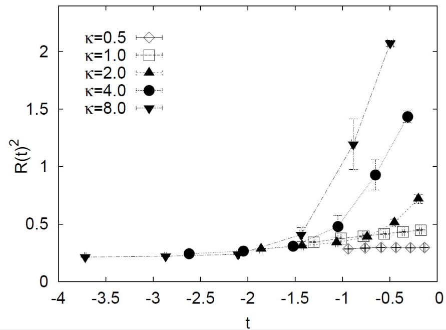

✓ Extent of a given spatial dimension parameter: $x_i^2(t) := \left\langle \frac{1}{n} \mathrm{Tr}\, \bar{A}_i(t)^2 \right\rangle$

→ Total extent of space parameter: $R^2(t) = \sum_{i=1}^{9} x_i^2(t)$

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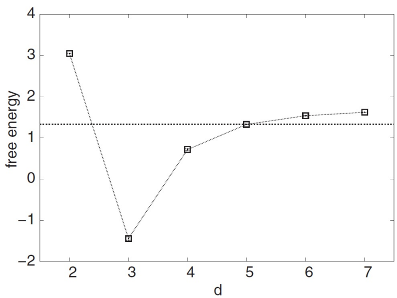

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Takeaway: A $(3+1)$-d universe emerges *dynamically* from non-perturbative (matrix model) description of superstring theory

Coarse-graining spacetime from abstract matrices

[S.B., Brandenberger & Laliberté, 2206.12468 (JHEP)]

↪ Emergent time identified from the diagonal elements of A_0 matrix, ordered as $A_0 = \mathrm{diag}(t_1, \ldots, t_N)$ with $t_i > t_j$ for $i > j$.

↪ In the diagonal A_0 basis, the eigenvalues of the A_i matrices **decay** when moving away from the diagonal ⇒ Band-diagonal structure.

- ✓ $t_{\max} \sim \sqrt{N}$, and discrete time eigenvalues scale as: $\Delta t \sim 1/\sqrt{N}$.
- ✓ In the $N \to \infty$ limit, *emergent* continuous and infinite time.
- ✓ Total **physical** extent of space: $\ell_{\mathrm{phys}} \sim \sqrt{N}$.
- ✓ Infinite extent of space when $N \to \infty$!

No Flatness Problem! Independent of isotropy assumption.

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Cosmology from a thermal state in BFSS

$\hookrightarrow$ Consider a thermal state in the BFSS model.

$\hookrightarrow$ Start with the Euclidean BFSS model and consider its compactification on a thermal circle: BFSS $\xrightarrow{T\to\infty}$ IKKT (natural to assume thermal state)

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↪ To confront with observations, need cosmological perturbations: go *beyond* IKKT model (include $\mathcal{O}(1/T)$ corrections for the thermal state expectation values).

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High temperature thermodynamics of BFSS model

↪ Calculate thermodynamic quantities in the *Euclidean BFSS model* and expand in the high T limit (dimensionless expansion parameter: $\sqrt{g^2 N / T^3}$)

↪ $S_{\rm BFSS} = S_0 + S_{\rm kin} + S_{\rm int}$, where $S_0 =: S_{\rm IKKT}$ (zero-mode action).

↪ Free energy up to *next-to-leading* order (IKKT + corrections):

$$\mathcal{F}(R,\beta) = \frac{3N^2}{4\beta}\left[\chi_2 \ln\beta - \frac{2}{3}\left(\frac{d-1}{12} - \frac{p}{8}\right)\left(N^2\chi_2 - \frac{N^2}{d}\chi_2 - 4\right)\chi_1\,\beta^{3/2}\right]$$

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Cosmological observables from the BFSS theory

We find scale-invariant spectrum for *IR modes* of observational interest
[S.B., Brandenberger & Laliberté, 2107.11512 (JHEP)]

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$\hookrightarrow$ UV-modes for density perturbations have a Poisson spectrum ($\propto k^2$); *distinct* from inflation but not of (direct) observable consequence. Tensor spectrum only has a scale-invariant part.

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✓ Free from swampland constraints $\rightarrow$ No landscape of tunable parameters coming from curling up extra dimensions. *Not anything goes!*

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How does space emerge from matrices?

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- The collective field formalism:

$$\phi_k = \mathrm{Tr}\left(e^{ikM}\right) = \sum_{i=1}^{N} e^{ik\lambda_i}$$

$$\phi(x) = \int \frac{\mathrm{d}k}{2\pi} e^{-ikx}\,\mathrm{Tr}\left(e^{ikM}\right) = \mathrm{Tr}\left(\delta(x-M)\right) = \sum_{i=1}^{N} \delta\left(x-\lambda_i\right)$$

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✓ Change of variables $M \to \phi$ leads to $(1+1) - d$ field theory:

$$H_\phi = \int \mathrm{d}x \left[\frac{1}{2}\partial_x\pi(x)\;\phi(x)\;\partial_x\pi(x) + \frac{\pi^2}{6}\phi^3(x) - (\mu_F - V(x))\,\phi(x)\right]$$

The collective field has a natural interpretation as a field in a higher dimensional theory [Das & Jevicki, ...]

Collective field formalism for multi-matrix models

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$$\phi_C = \mathrm{Tr}\left(M_1^a M_2^b M_3^c \ldots\right)$$

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$$S_{\mathrm{eff}} = \int \mathrm{d}\tau \left[\frac{1}{2l_s}\left(\sum_i^N \dot{\lambda}_i^2 + \sum_i^N \dot{\rho}_i^2\right)\right] - \frac{1}{2}\sum_{i<j,i=1}^N \mathrm{Tr}\log\left(1 - \frac{1}{2l_s^4}\int \mathrm{d}s\, |s-\tau| \left(\lambda_i(s) - \lambda_j(s)\right)^2\right)$$

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To get a time-local effective action, we need to add a mass term Tr $\left(m^2 Y^2\right)$ to the action. Then, to leading order:

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Possible to get a $(2+1)$-d collective field action! Starting point for connection to String Theory $\rightarrow$ *Geometry from Entanglement*

[S.B., Brandenberger, Dasgupta, Lei & Pasiecznik, 2411.10880 (JHEP)]

Collective field: Integrating out off-diagonal elements

$$
\begin{aligned}
H^{\phi} \;=\; & \frac{l_s}{2}\int \mathrm{d}x\,\mathrm{d}y\,\left[\partial_x\pi(x,y)\phi(x,y)\partial_x\pi(x,y)+\partial_y\pi(x,y)\phi(x,y)\partial_y\pi(x,y)\right] \\
& +\frac{l_s\pi^2}{6}\int \mathrm{d}x\,\mathrm{d}y\,\phi^3(x,y)+\frac{l_s}{8}\int \mathrm{d}x\,\mathrm{d}y\,\frac{(\partial_y\phi(x,y))^2}{\phi(x,y)} \\
& -\frac{1}{16ml_s^4}\int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}x'\,\mathrm{d}y'\,\phi(x,y)(x-x')^2\phi(x',y')+\frac{l_sm^2}{2}\int \mathrm{d}x\,\mathrm{d}y\,y^2\phi(x,y) \\
& -\lambda\left[\int \mathrm{d}x\,\mathrm{d}y\,\phi(x,y)-N\right]
\end{aligned}
$$

- Has the right expression when we turn off the second matrix (compactifying y-direction to zero)!
- Crucial for the matrices to have mass to result in a time-local theory!
- Mass-deformed BMN model better starting point for well-defined Stringy model.
- Seemingly well-defined path towards generalization to higher dimensions!

The toy BMN model: Vacuum state

[with Brandenberger, Dasgupta & Lei, *forthcoming*]

$$H = \left[\frac{1}{2}\left(\vec{\Pi}\cdot\vec{\Pi}\right) + \frac{1}{2}\left((\nu X + i\,[Y,Z])^2 + (\nu Y + i\,[Z,X])^2 + (\nu Z + i\,[X,Y])^2\right)\right]$$

↪ Diagonalize X, integrate out the off-diagonal entries of Y, Z, and keep leading terms.

↪ The density constraint: $\int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\phi(x,y,z) = N$.

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The $(3+1)$-d collective field Hamiltonian:

$$\begin{aligned}
H^{\phi} \quad = \quad & \frac{1}{2}\iiint \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\partial_x\pi\,\phi(x,y,z)\partial_x\pi + \partial_y\pi\,\phi(x,y,z)\partial_y\pi + \partial_z\pi\,\phi(x,y,z)\partial_z\pi \\
& + \frac{\pi^2}{6}\iiint \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\phi^3(x,y,z) + \frac{1}{8}\iiint \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\left(\frac{(\partial_y\phi(x,y,z))^2}{\phi(x,y,z)} + \frac{(\partial_z\phi(x,y,z))^2}{\phi(x,y,z)}\right) \\
& + \frac{1}{8\nu}\iiint \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\iiint \mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'\,\phi(x,y,z)\left[(x-x')^2 + \tfrac{1}{2}(y-y')^2 + \tfrac{1}{2}(z-z')^2\right]\phi(x',y',z') \\
& + \frac{\nu^2}{2}\iiint \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,(x^2+y^2+z^2)\phi(x,y,z) \\
& - \mu\left[\iiint \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\phi(x,y,z) - N\right].
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• The ground state solved by (generalization of Wigner semi-circle law): $\phi_0(x,y,z) = \sqrt{\alpha - \beta x^2 - \gamma y^2 - \delta z^2}$ with vanishing boundary conditions for the ellipsoid: $\phi_0(\pm 2a, 0, 0) = \phi_0(0, \pm 2b, 0) = \phi_0(0, 0, \pm 2c) = 0$.

• The extent of space depends on the mass ratios of the matrices:

$$\frac{a^2}{b^2} \approx \frac{\nu_y^2}{\nu_x^2}, \; \frac{a^2}{c^2} \approx \frac{\nu_z^2}{\nu_x^2}, \; \frac{b^2}{c^2} \approx \frac{\nu_z^2}{\nu_y^2}$$

• Find the spectra of fluctuations: $\phi_0(x,y,z) + \eta(x,y,z,t)$

Conclusions

✓ It has been notoriously difficult to find accelerating solutions in string theory → Inflation as a coherent state over warped Minskowski? dS *does* have such an interpretation!

[S.B., Dasgupta & Tatar, JHEP, 2020; S.B., Dasgupta, Guo & Kulinich, JHEP, 2024]

✓ M-theory consistency rules out large classes of bounce models.

[Bernardo, S.B., Dasgupta Mir & Tatar, Phys. Rev. Lett., 2021]

⋆ A new path towards a **UV-complete paradigm** for the early universe:

✓ Numerical evidence for the emergence of 3 large spatial dimensions from full String Theory.

✓ Analytically extract a coarse-grained time, space and metric.

✓ Thermal fluctuations ⇒ Scale invariant primordial perturbations.

✓ Horizon problem, Flatness problem and formation of structure from first-principles in a fundamental quantum gravity theory.

✓ No vacuum energy problem → Transition from non-geometric emergent phase to radiation dominated era. No cosmological constant.

✓ Possible to work with the full IKKT model **alone** and consider a thermal state in it. [Laliberté & S.B., JHEP, 2023]

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Looking ahead

⋆ A new path towards a UV-complete paradigm for the early-universe.
Ambitious, promising but a lot remains to be done:

- Details of non-geometric phase → Connection to non-commutative geometry emergent in the UV? [H. Steinacker; A. Chaney & A. Stern; . . .]
- Physical explanation of the SSB phase? Connection with String Gas? Interestingly, non-geometric phase has $p = 0$ (quasi-static phase).

↪ Strings as solitonic states in Matrix models → Annihilation of string loops into radiation. Collective field formalism!

- Connect the spectrum of collective-field fluctuations in toy models to (3+1)-d String Theory.
- A gauge-invariant notion of entanglement for matrices using the Collective Field → Area-law for toy models. [Frenkel & Hartnoll; Das, Kaushal, Mandal, Liu, Trivedi; Hampapura, Harper & Lawrence, . . .]
- Observable consequences: NG, PBHs from the Poisson part of the UV spectrum, Primordial B fields, . . .

⋮

Emergence of continuous time

[S.B., Brandenberger & Laliberté, 2206.12468]

→ Emergent time identified from the diagonal elements of A_0 matrix, ordered as $A_0 = \mathrm{diag}(t_1, \ldots, t_N)$ with $t_i > t_j$ for $i > j$.

↪ Using the numerical scaling from [Aoki, Kim, Nishimura, Tsuchiya, 2012]: $\frac{1}{N}\langle \mathrm{Tr} A_0^2 \rangle \sim \kappa N$, for a small constant κ.

- ✓ $t_{\mathrm{max}} \sim \sqrt{N}$, assuming they are equally spaced.
- ✓ The discrete time eignevalues scale as: $\Delta t \sim 1/\sqrt{N}$.
- ✓ In the $N \to \infty$ limit, emergent continuous and infinite time.

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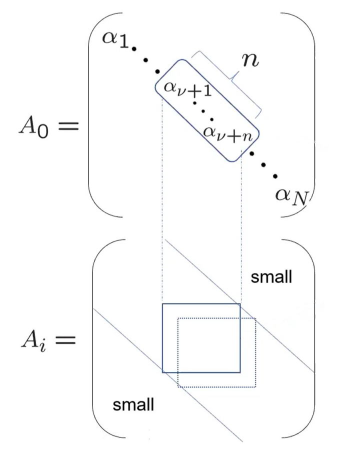

Emergence of continuous space

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→ In the diagonal A_0 basis, the eigenvalues of the A_i matrices (which define the emergent spatial directions) **decay** when moving away from the diagonal.

- ✓ $\sum_i \langle |A_i|^2_{ab} \rangle \to 0$, for $n \in |a-b| > n_c$.
- ✓ Using Riemann-Lebesgue lemma, we get $n_c \sim \sqrt{N}$, confirming numerical results.
- ✓ First approx: sub-matrix elements **constant** for $n < n_c$.

Proposal :

- $n_i \times n_i$ submatrix: $A_i^{n_i}(t)$ centered a distance t down diagonal.
- n_i ranges from 0 to n_c, **propose** to view n_i as a comoving spatial coordinate in direction i.
- Physical distance between $n = 0$ and n (*emergent space*):

$$\ell^2_{\mathrm{phys,i}}(n,t) := \langle \mathrm{Tr} \bar{A}_i(t)^2 \rangle$$

Emergence of continuous space

[S.B., Brandenberger & Laliberté, 2206.12468]

→ In the diagonal A_0 basis, the eigenvalues of the A_i matrices (which define the emergent spatial directions) **decay** when moving away from the diagonal.

- ✓ $\sum_i \langle |A_i|^2_{ab} \rangle \to 0$, for $n \in |a-b| > n_c$.
- ✓ Using Riemann-Lebesgue lemma, we get $n_c \sim \sqrt{N}$, confirming numerical results.
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Emergence of spacetime metric

[S.B., Brandenberger & Laliberté, 2206.12468]

↪ Assuming constant sub-matrix elements for $n < n_c$, one finds

$$\ell_{\text{phys,i}}(n) \sim n$$

↪ Total **physical** extent of space: $\ell_{\text{phys}} \sim n_c \sim \sqrt{N}$.

✓ Infinite extent of space when $N \to \infty$!

↪ Emergent metric: $g_{ii}(n) := \frac{d}{dn}\ell_{\text{phys,i}}(n)$ since $\ell_{\text{phys,i}}(x) = \int_0^x \sqrt{g_{ii}(y)}\, dy$.

✓ Result:

$$g_{ij}(n,t) = \mathcal{A}(t)\,\delta_{ij}$$

↪ Assumption: $SO(3)$ symmetry of the emerging system.

No Flatness Problem! Independent of isotropy assumption.

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