Supported by West University of Timisoara, Romania fellowship

Relativistic hydrodynamics with spin: motivation and recent progress

Int.J.M.Phys.A, 38 (20)

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Theoretical Physics Seminar

université de TOURS

June 12, 2025

Outline

- Relativistic hydrodynamics
- Heavy-ion collisions
- Spin polarization of hyperons
- Standard hydro efforts

1st part

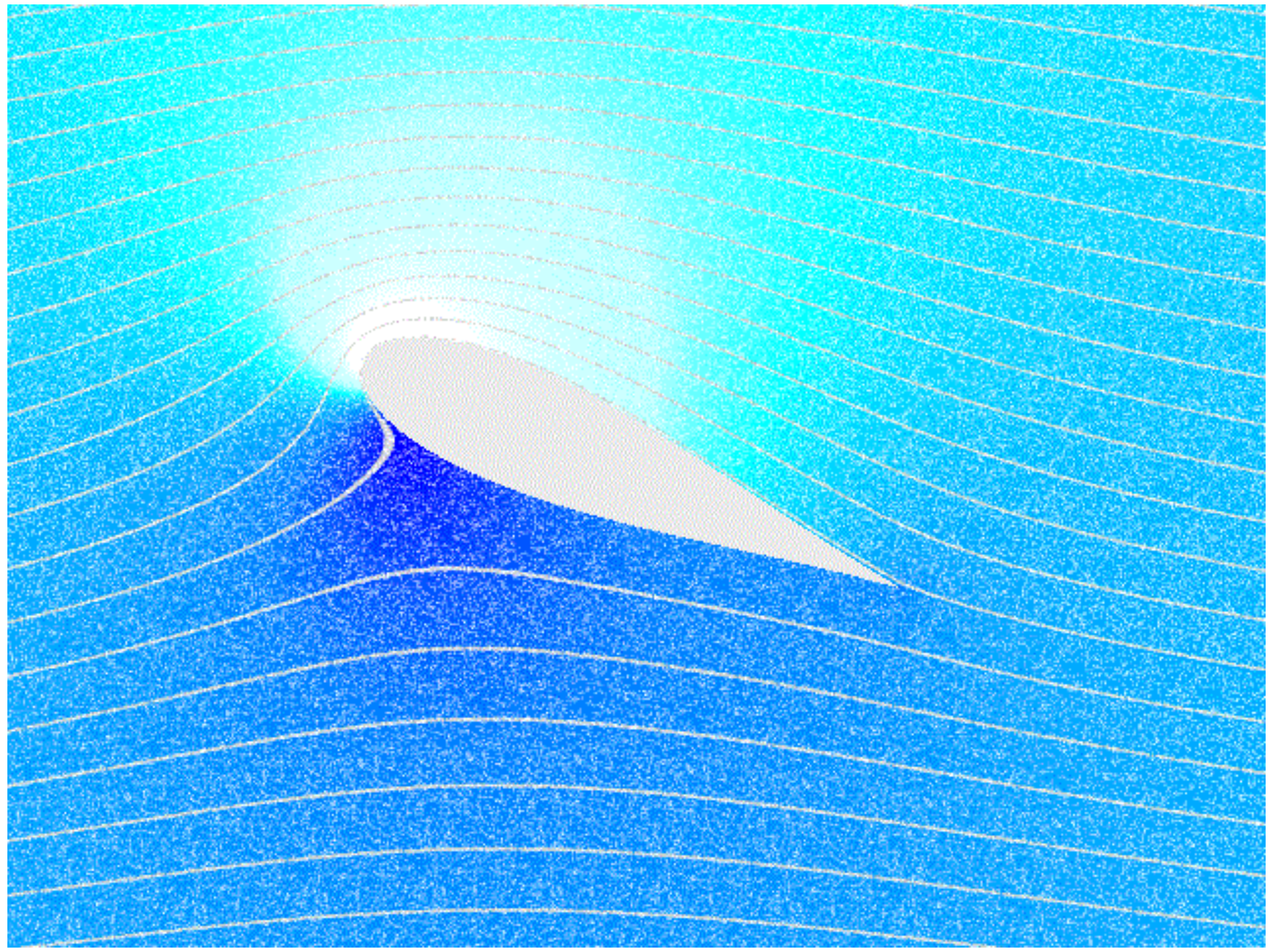
- Why spin hydro?
- GLW spin hydro

2nd part

First formulation of spin hydro

Groot, **L**eeuwen, **W**eert, Relativistic Kinetic Theory. Principles and Applications. North Holland, 1, 1980

Flow around a wing, satisfies the Euler equations—Wiki



for an ideal, neutral, uncharged, one-component fluid

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \boldsymbol{\partial})\, \mathbf{v} = -\frac{1}{\rho} \boldsymbol{\partial} p \,,$$

$$\partial_t \rho + \rho \boldsymbol{\partial} \cdot \mathbf{v} + \mathbf{v} \cdot \boldsymbol{\partial} \rho = 0 \,.$$

Continuity equation

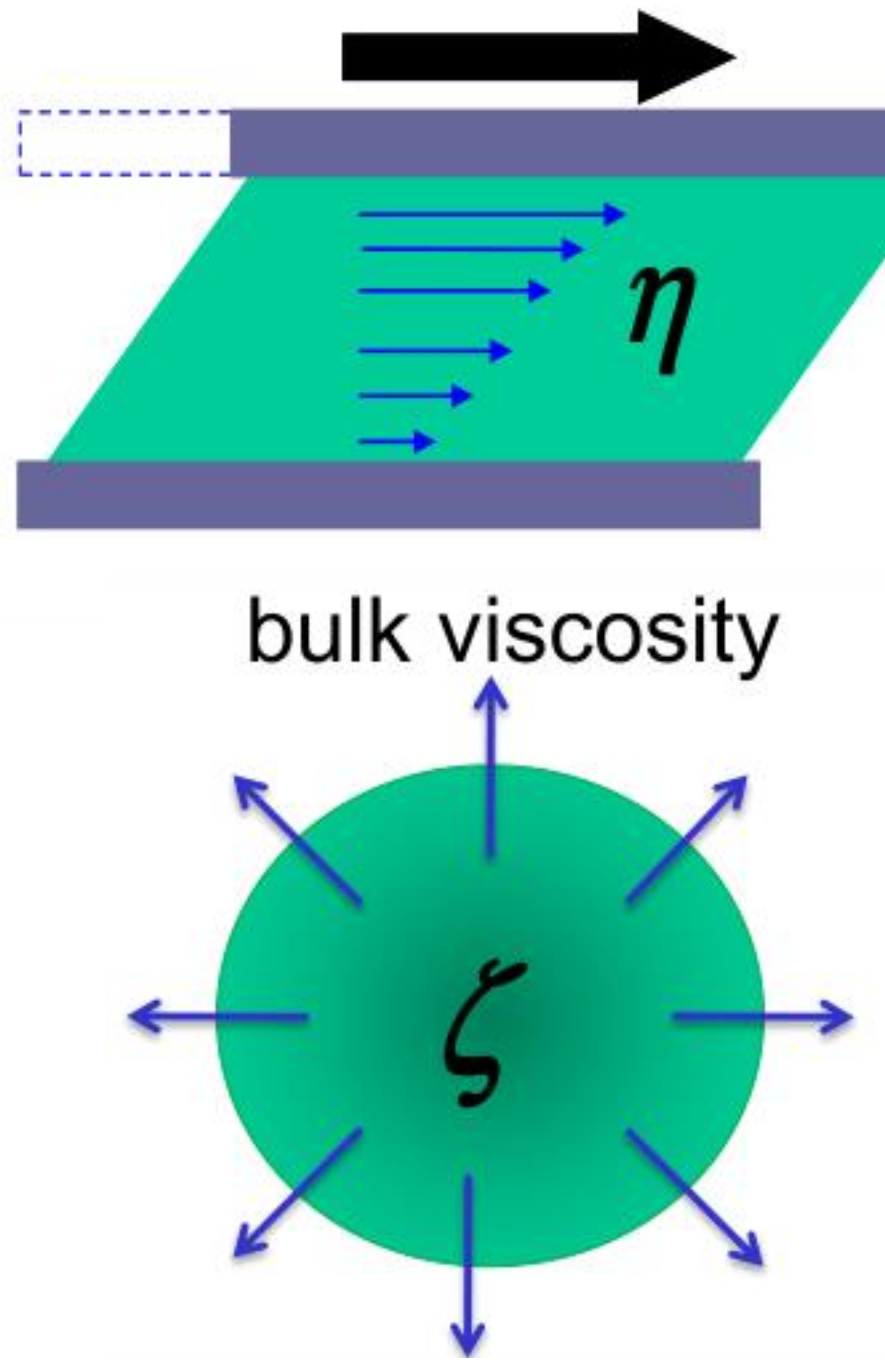
Euler equation

$v =$ fluid – velocity

$\rho =$ fluid – mass – density

$p =$ pressure

for an ideal, neutral, uncharged, one-component fluid



$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \boldsymbol{\partial})\, \mathbf{v} = -\frac{1}{\rho} \boldsymbol{\partial} p \,,$$

$$\partial_t \rho + \rho \boldsymbol{\partial} \cdot \mathbf{v} + \mathbf{v} \cdot \boldsymbol{\partial} \rho = 0 \,.$$

when dissipation is included

Euler equation generalizes to the "Navier–Stokes equation"

$$\frac{\partial v^i}{\partial t} + v^k \frac{\partial v^i}{\partial x^k} = -\frac{1}{\rho} \frac{\partial p}{\partial x^i} - \frac{1}{\rho} \frac{\partial \Pi^{ki}}{\partial x^k} \,,$$

$$\Pi^{ki} = -\eta \left(\frac{\partial v^i}{\partial x^k} + \frac{\partial v^k}{\partial x^i} - \frac{2}{3} \delta^{ki} \frac{\partial v^l}{\partial x^l} \right) - \zeta \delta^{ik} \frac{\partial v^l}{\partial x^l} \,,$$

shear viscosity

bulk viscosity

Relativistic ideal fluid dynamics

$$u^\mu \equiv \frac{dx^\mu}{d\mathcal{T}}$$

$$(d\mathcal{T})^2 = g_{\mu\nu}dx^\mu dx^\nu = (dt)^2 - (d\mathbf{x})^2 \, ,$$

$$= (dt)^2 \left[1 - \left(\frac{d\mathbf{x}}{dt}\right)^2\right] = (dt)^2 \left[1 - (\mathbf{v})^2\right]$$

$$u^\mu = \frac{dt}{d\mathcal{T}}\frac{dx^\mu}{dt} = \frac{1}{\sqrt{1-\mathbf{v}^{\,2}}}\begin{pmatrix}1\\ \mathbf{v}\end{pmatrix} = \gamma(\mathbf{v})\begin{pmatrix}1\\ \mathbf{v}\end{pmatrix}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Relativistic ideal fluid dynamics

Ideal relativistic Euler equations are strictly hyperbolic because their eigenvalues are real and distinct

$$u^\mu \equiv \frac{dx^\mu}{d\mathcal{T}}$$

$$(d\mathcal{T})^2 = g_{\mu\nu} dx^\mu dx^\nu = (dt)^2 - (d\mathbf{x})^2 \,,$$

$$= (dt)^2 \left[1 - \left(\frac{d\mathbf{x}}{dt}\right)^2\right] = (dt)^2 \left[1 - (\mathbf{v})^2\right]$$

$$u^\mu = \frac{dt}{d\mathcal{T}} \frac{dx^\mu}{dt} = \frac{1}{\sqrt{1 - \mathbf{v}^2}} \begin{pmatrix} 1 \\ \mathbf{v} \end{pmatrix} = \gamma(\mathbf{v}) \begin{pmatrix} 1 \\ \mathbf{v} \end{pmatrix}$$

$$T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - p \Delta^{\mu\nu}$$

$$D\epsilon + (\epsilon + p)\partial_\mu u^\mu = 0$$

$$(\epsilon + p) D u^\alpha - \nabla^\alpha p = 0$$

$$\partial_\mu T^{\mu\nu}_{(0)} = 0$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Relativistic hydrodynamics

→ Conservation of Baryon Number $\partial_\alpha N^\alpha = 0$

$$N^\alpha(x) = \underbrace{\mathcal{N}\, U^\alpha}_{\text{ideal}} + \cancel{n}^\alpha$$

$\mathcal{N}$ = number density
$\mathcal{E}$ = energy density
$\mathcal{P}$ = pressure
U^α = fluid four – velocity = $\gamma(1,\vec{v})$

5 eqns. 5 unknowns

$T(x), \mu(x), \vec{v}$

→ Conservation of Energy-momentum tensor $\partial_\alpha T^{\alpha\beta} = 0$

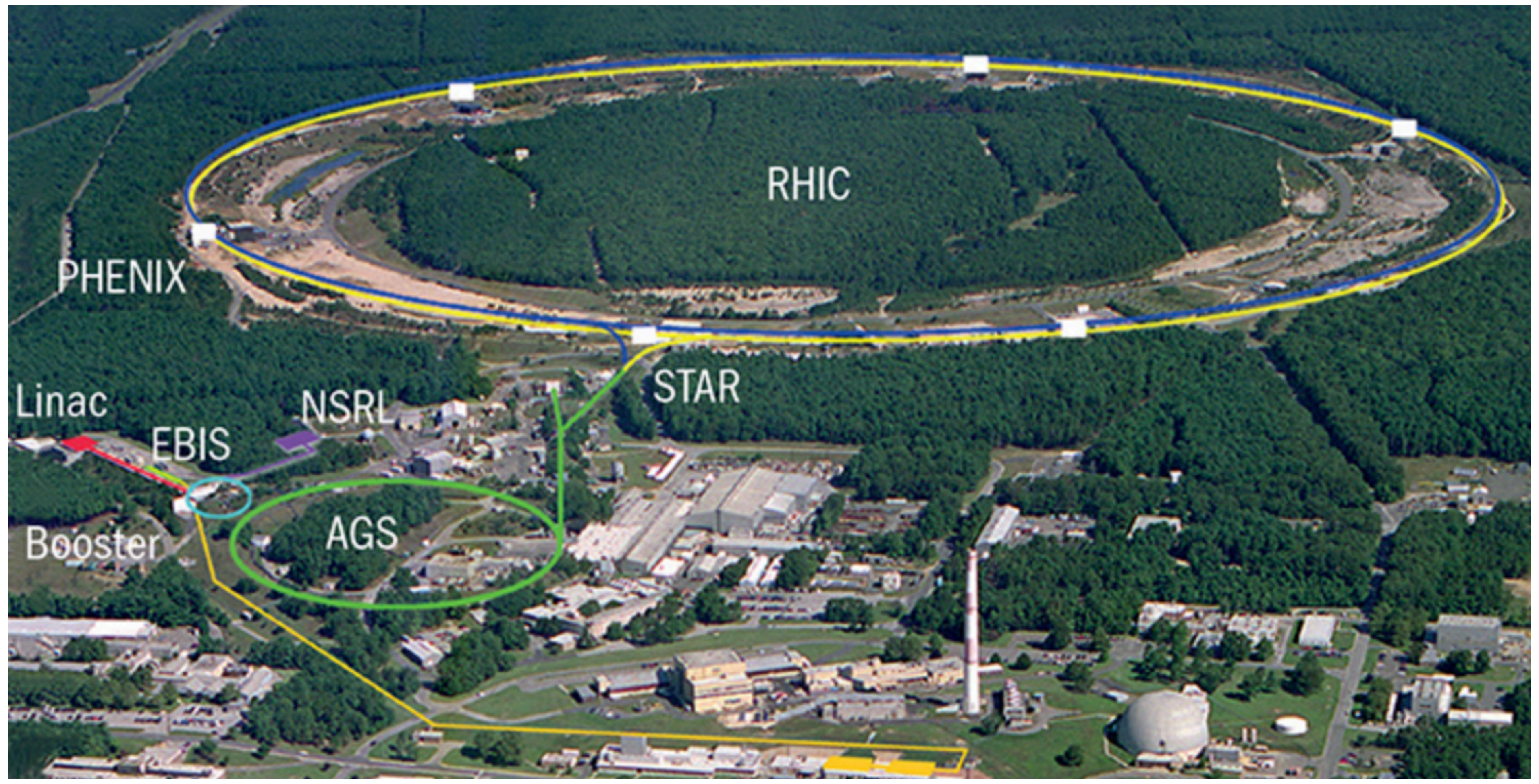
$$T^{\mu\nu}(x) = \underbrace{(\mathcal{E} + \mathcal{P})\, U^\mu U^\nu - \mathcal{P}(\mathcal{E})\, \eta^{\mu\nu}}_{\text{ideal}} + \cancel{\Pi}^{\mu\nu}$$

For this talk, I will work only with idea hydro (no dissipation)

+ - - - Minkowski flat space

Relativistic heavy-ion collisions - a tool to study QGP

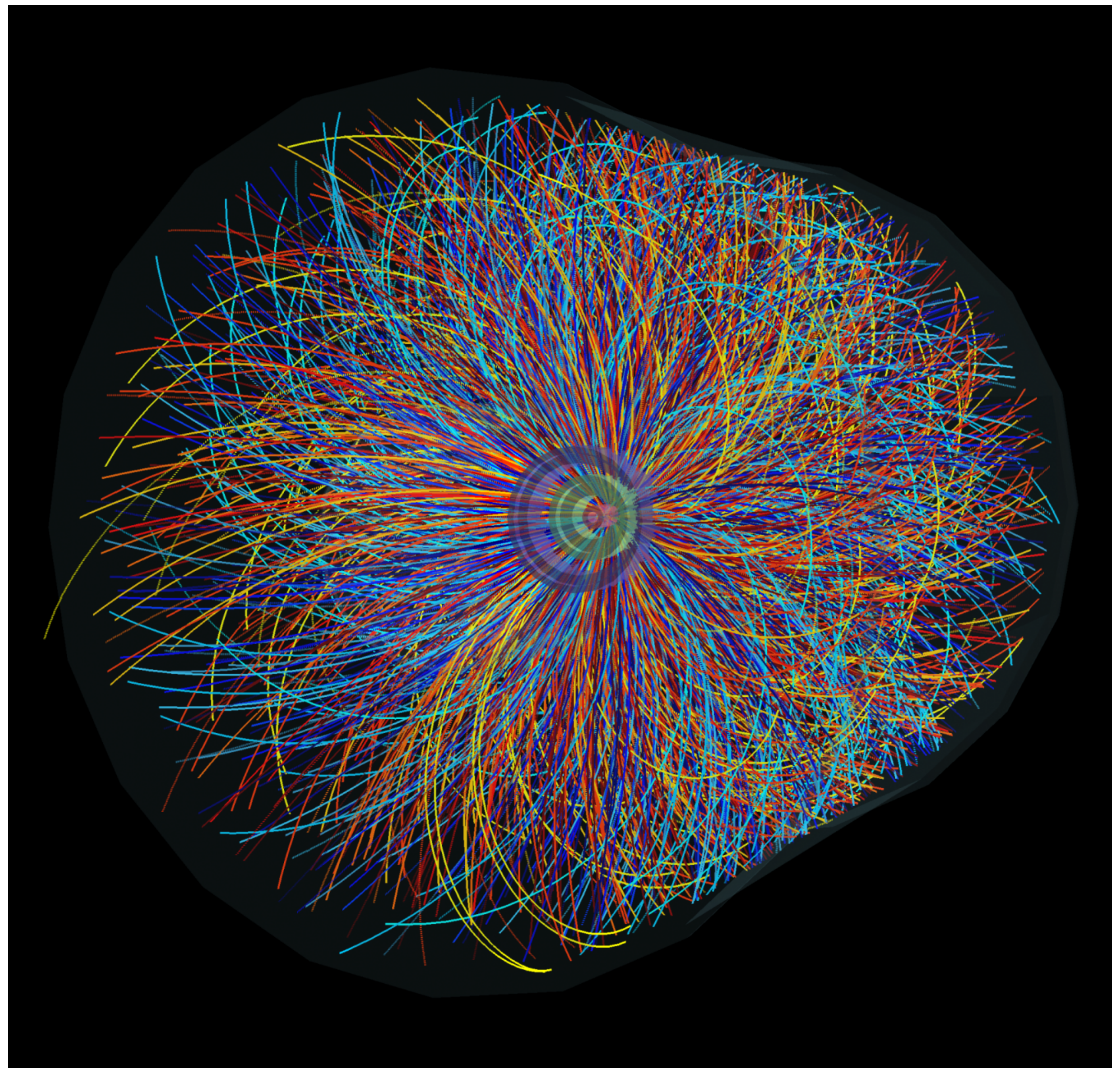
Figure: BNL



At high beam energies we observe many particles being produced

Figure: CERN

The study of QGP possible only indirectly through the energy and momenta of emitted particles



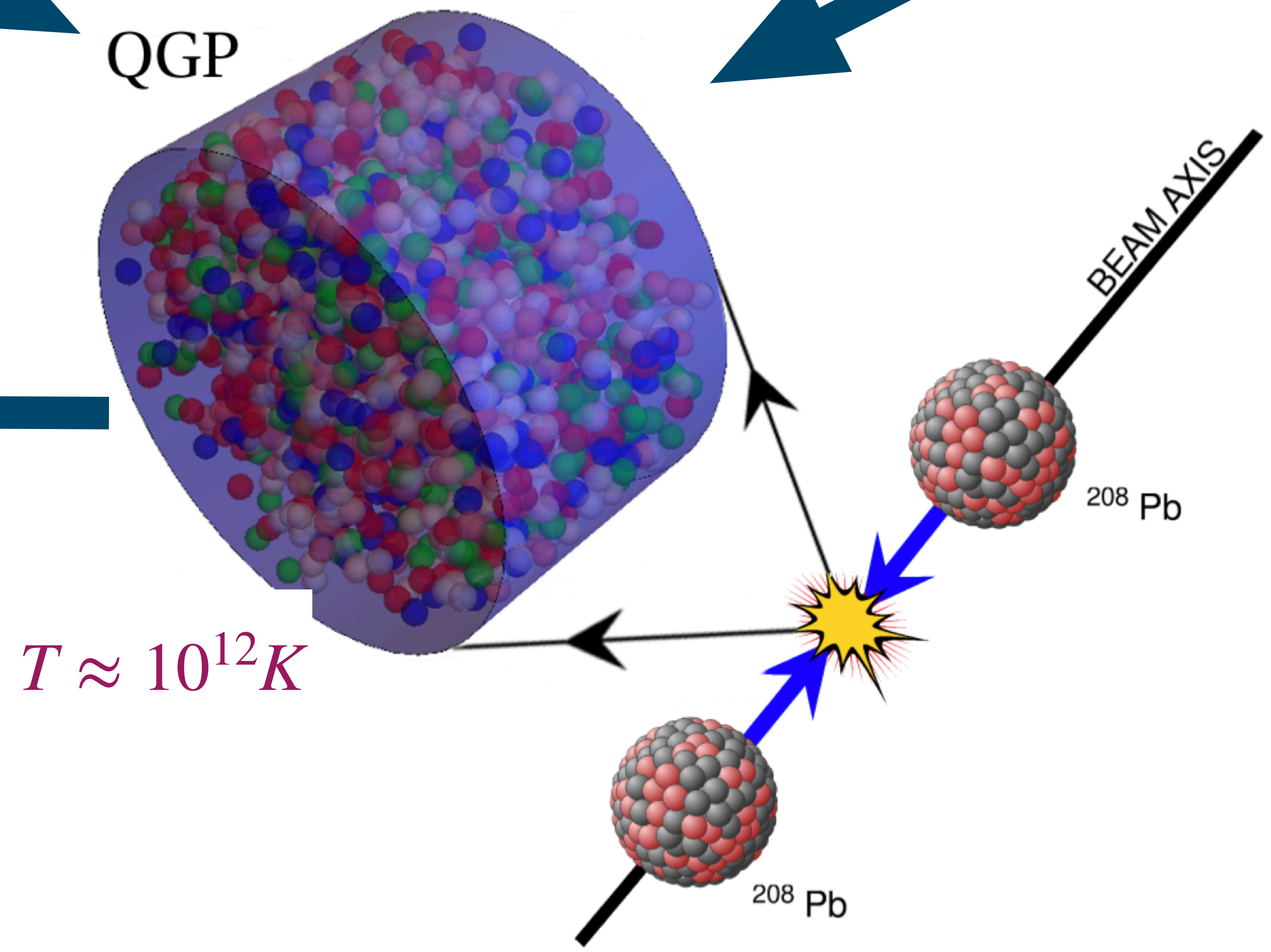


Figure: Nature Physics 16, 615–619(2020)

Heavy-ion collisions and QGP

→ As per thermal history of our Universe, it is expected that Early Universe was in a state of quark-gluon plasma (QGP).

Annual Review of Nuclear and Particle Science 2006 56:1, 441-500

→ Then phase transition happened, when $T_{\text{universe}} \approx 200\,\text{MeV}$ & $\text{Age}_{\text{universe}} \approx 10^{-6}\,\text{sec}$, from QGP to confined hadrons.

→ QGP properties and phase transition can be studied using relativistic HI collisions. This is crucial to understand the existence of nuclear matter and confinement.

Prog.Part.Nucl.Phys. 72 (2013) 99-154

→ Experimental evidences of QGP showed that it behaves as a strongly-coupled system as perfect fluid.

nucl-th/0002042, Nucl.Phys.A757:184-283,2005

→ Due to very small kinematic shear viscosity obtained from the transverse momentum spectra of charged particles.

Nucl.Phys.A750:30-63,2005
Prog.Part.Nucl.Phys.62:48-101,2009

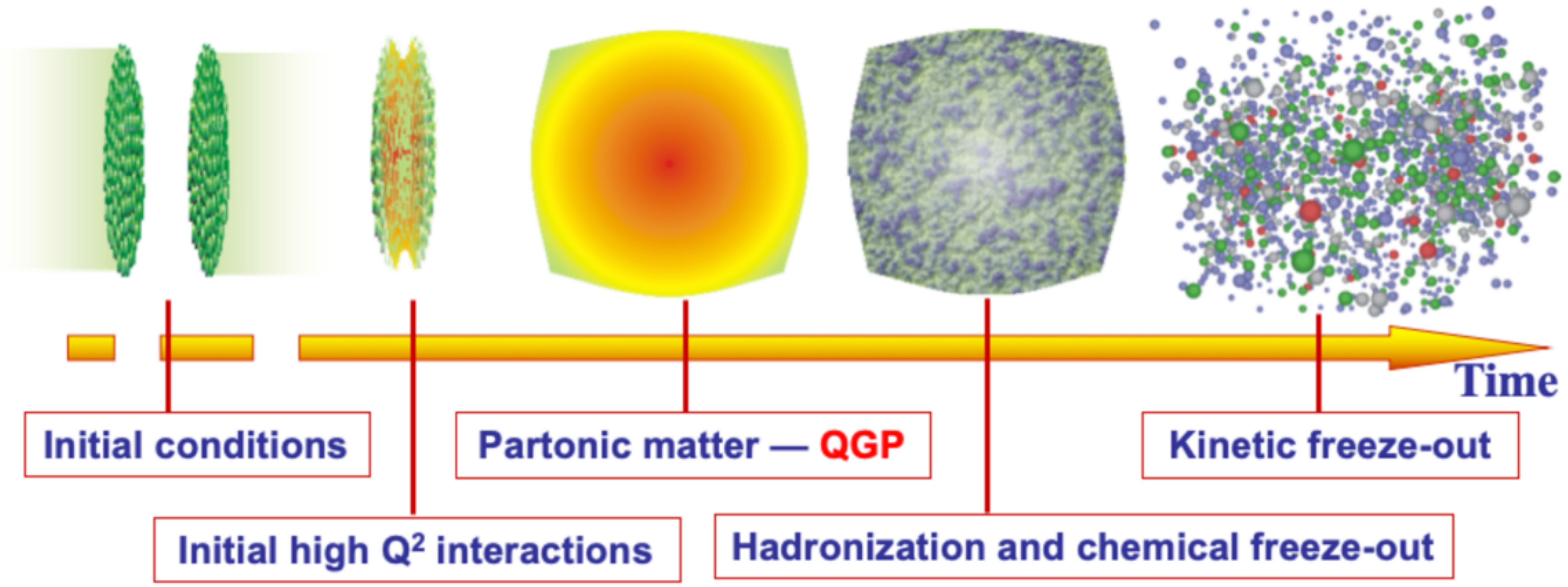
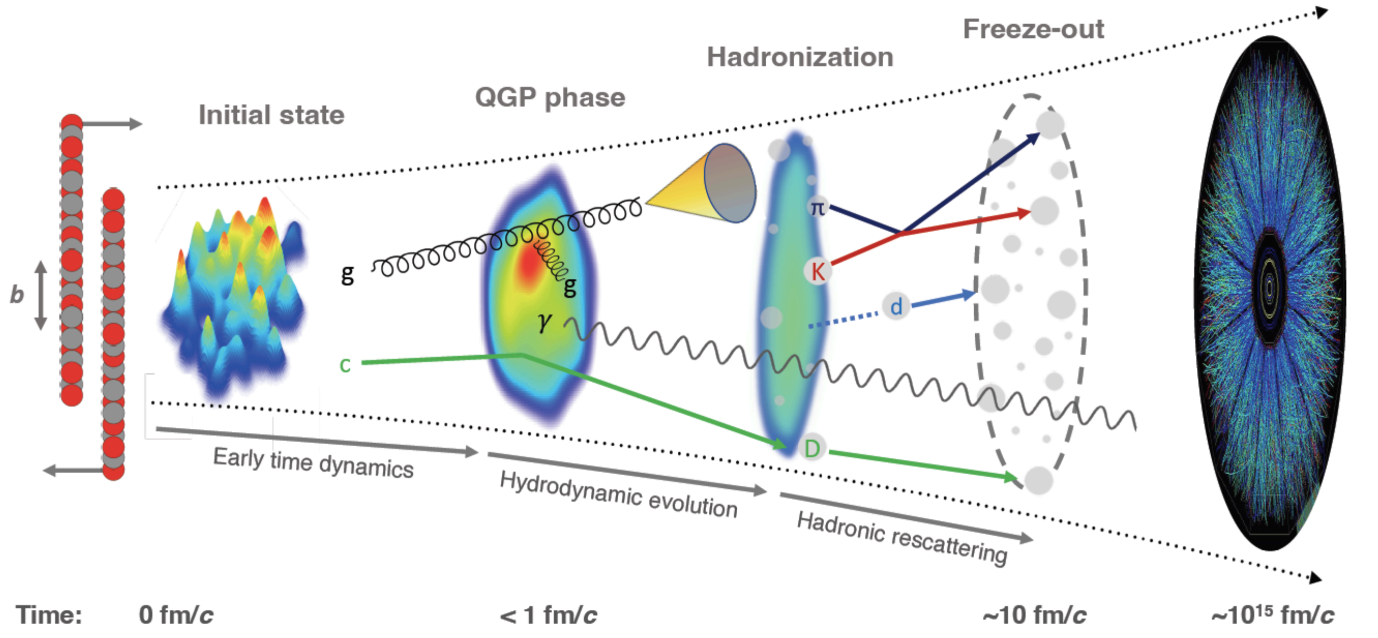


Figure: Evolution stages of ultra-relativistic heavy-ion collision. Time in the horizontal axis has the unit of fm/c and Q^2 denotes four-momentum transfer squared.
(Nucl.Phys.News 30 (2020) 2, 10-16)

Spin polarization

Hot QCD White Paper (arXiv:2303.17254)



Phys.Rev.Lett. 94 (2005) 102301, Phys. Rev. C 77, 024906

- Non-central UR-HIC, due to spatial inhomogeneity, create large OAM, $\boldsymbol{L}_{\text{initial}} \approx 10^5\,\hbar$.

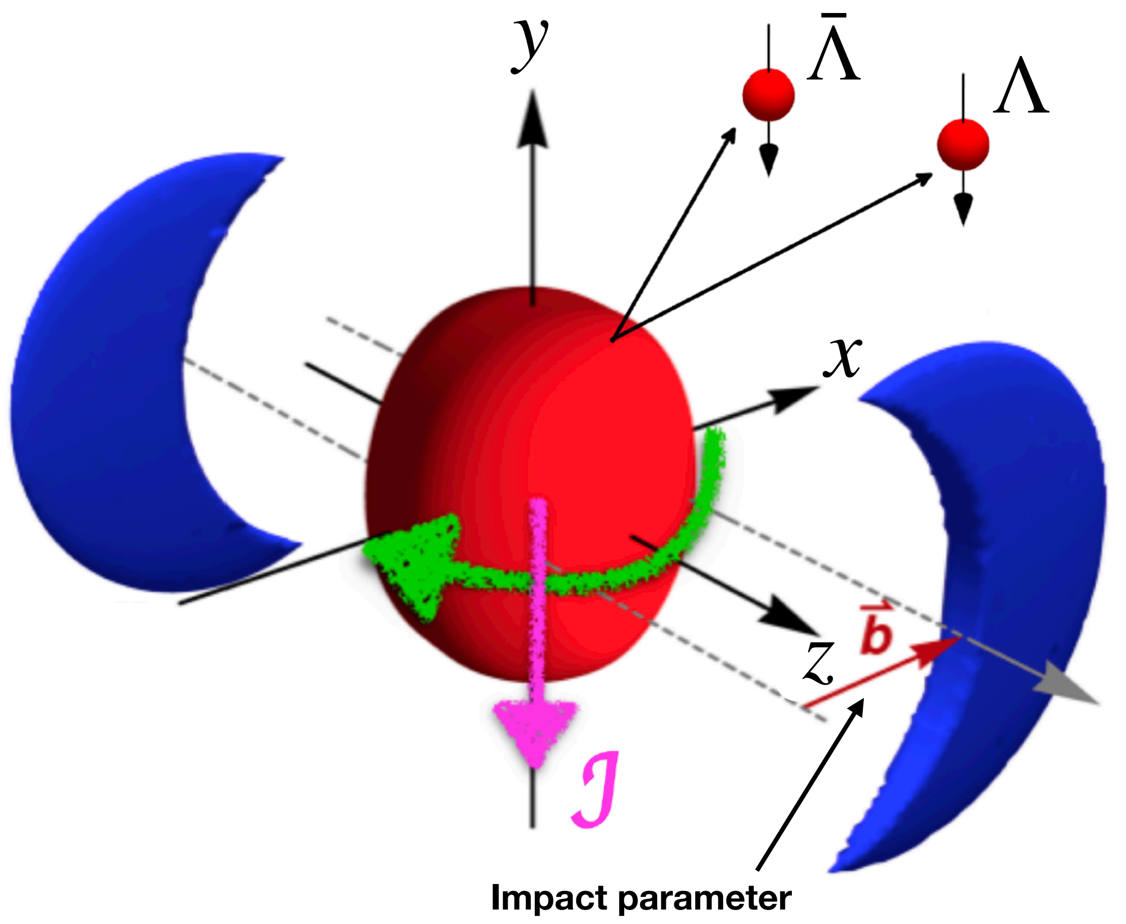
- This OAM is along y-axis (orthogonal to reaction plane) & may polarize spin of the QGP constituents.

Phys. Rev. Lett. 94 (2005) 102301

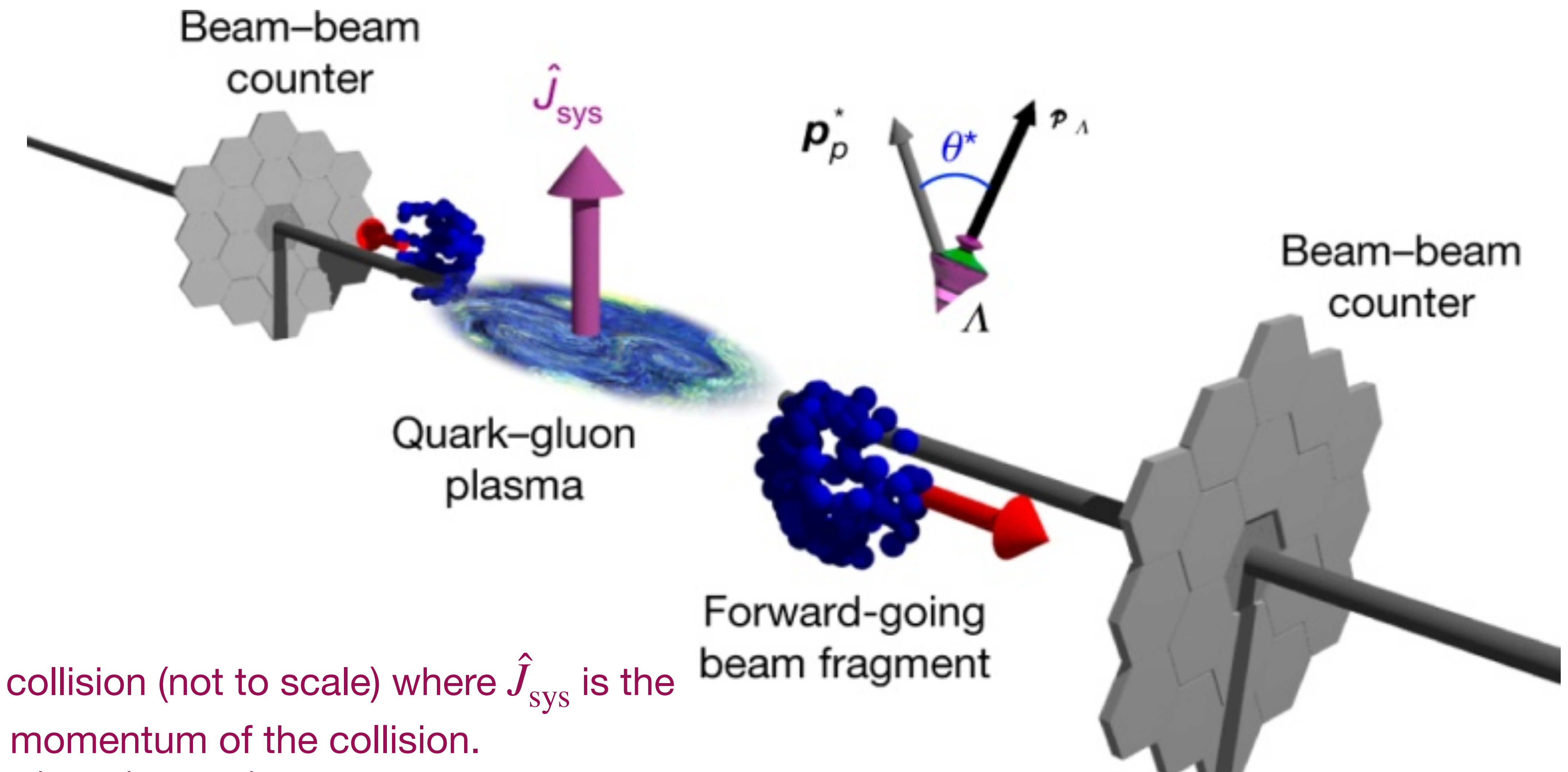
Predicted in 2005

- Spin polarization is expected to be transferred to the hadrons leading to their global spin polarization.

- Among various spin-polarizable hadrons, $\Lambda(\bar{\Lambda})$ hyperons are special as they are self-analyzing.



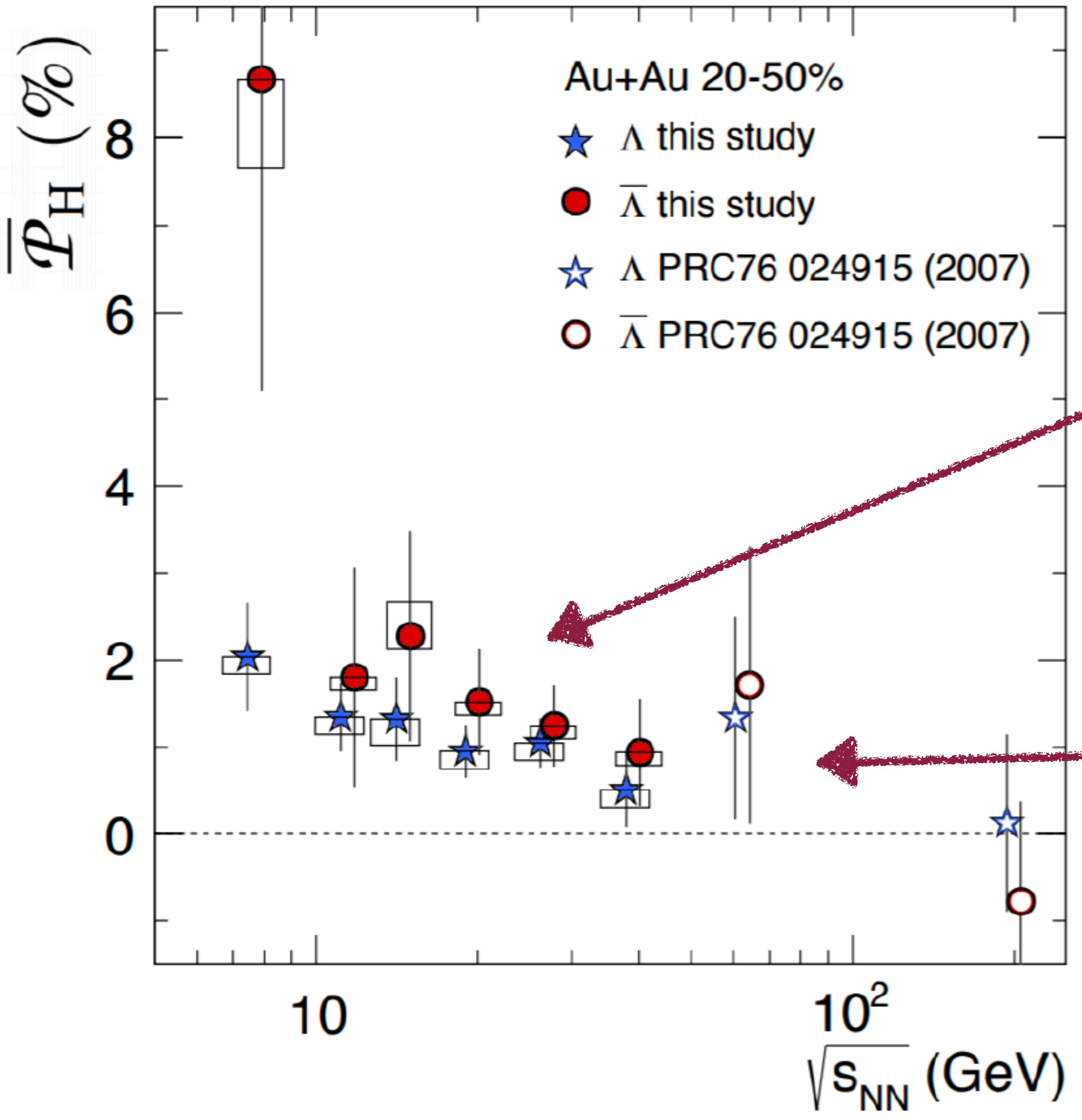
Schematic diagram of the initial angular momentum orientation in non-central heavy-ion collision.
(Prog.Part.Nucl.Phys. 108 (2019) 103709)



Schematic diagram of a $\mathrm{Au}+\mathrm{Au}$ collision (not to scale) where $\hat{J}_{\text{sys}}$ is the direction of the angular momentum of the collision.
(Nature 548 (2017) 62-65)

Spin polarization

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65



~2% - small but measurable effect

Self-analysing parity-violating hyperon weak decay allows to measure polarization of Λ

Small difference in the magnitude of Λ & $\bar{\Lambda}$ possibly due to initial magnetic field

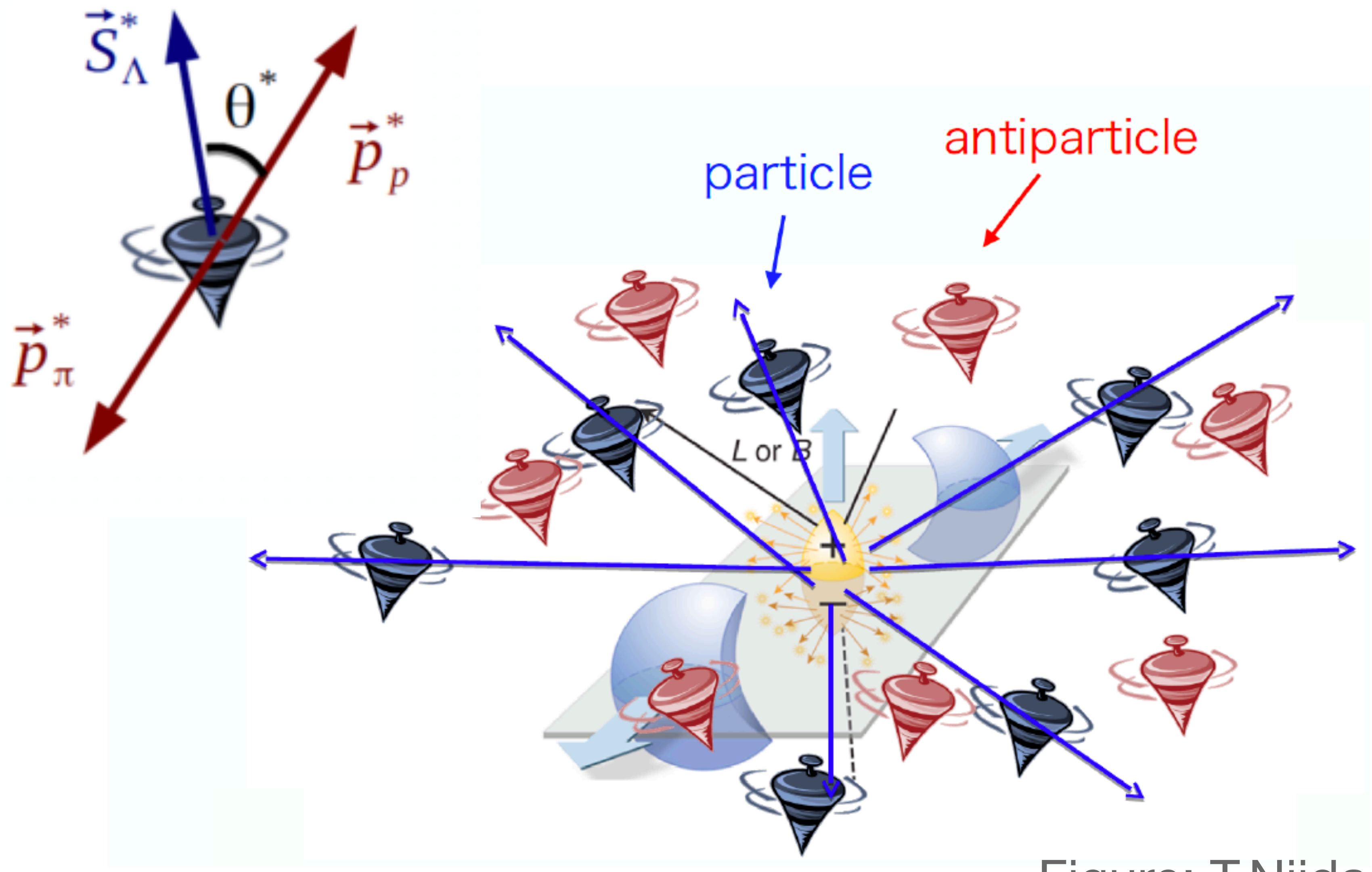


Figure: T.Niida

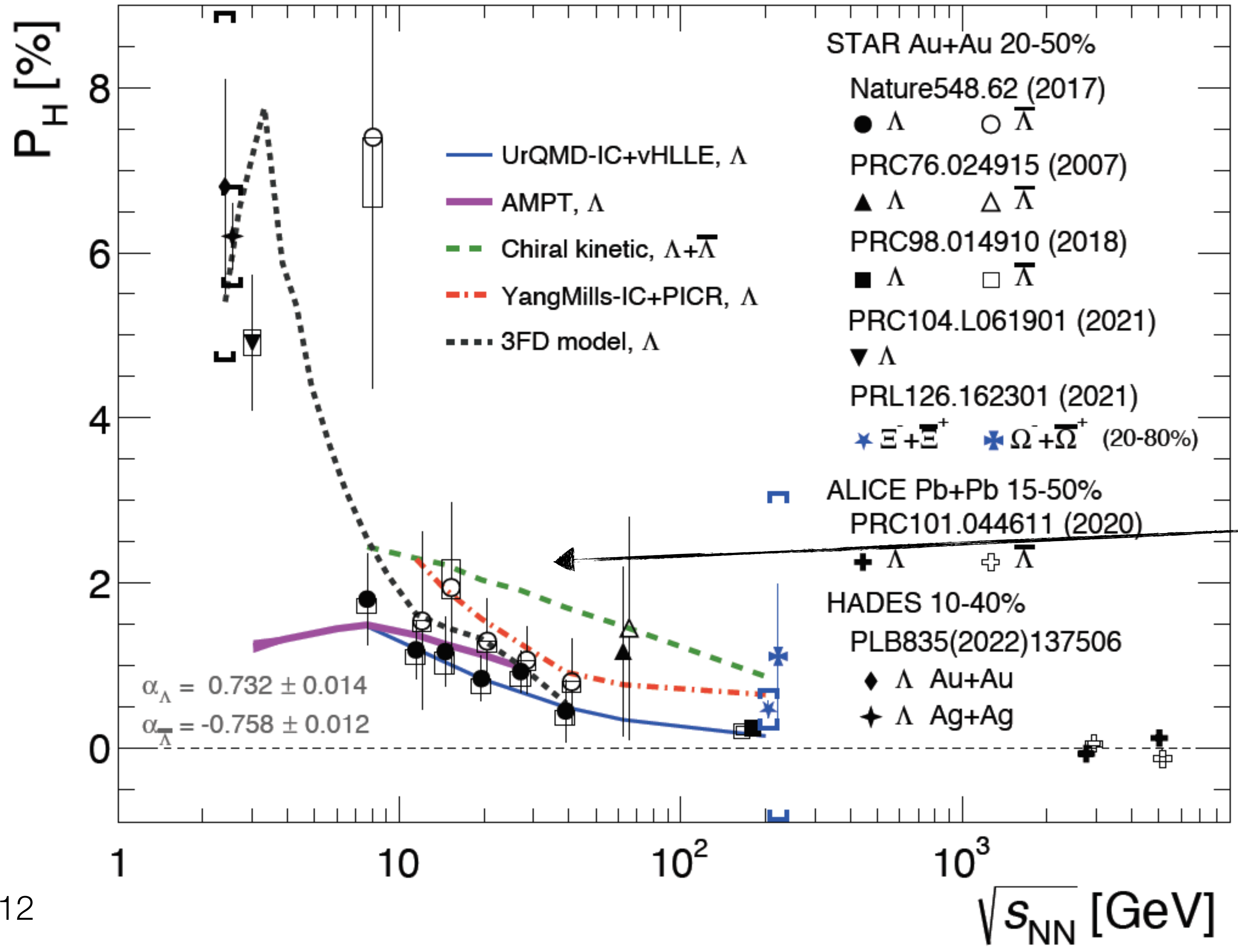
QGP **is the** ***hottest***, ***least viscous,*** **and** ***most vortical*** **fluid ever produced**

$$\omega = (P_\Lambda + P_{\bar{\Lambda}})\, k_B T/\hbar \ \sim 0.6 - 2.7 \text{ x } 10^{22}\, s^{-1}$$

$P_\Lambda \approx P_{\bar{\Lambda}}$ ➡ **first direct observation of spin**

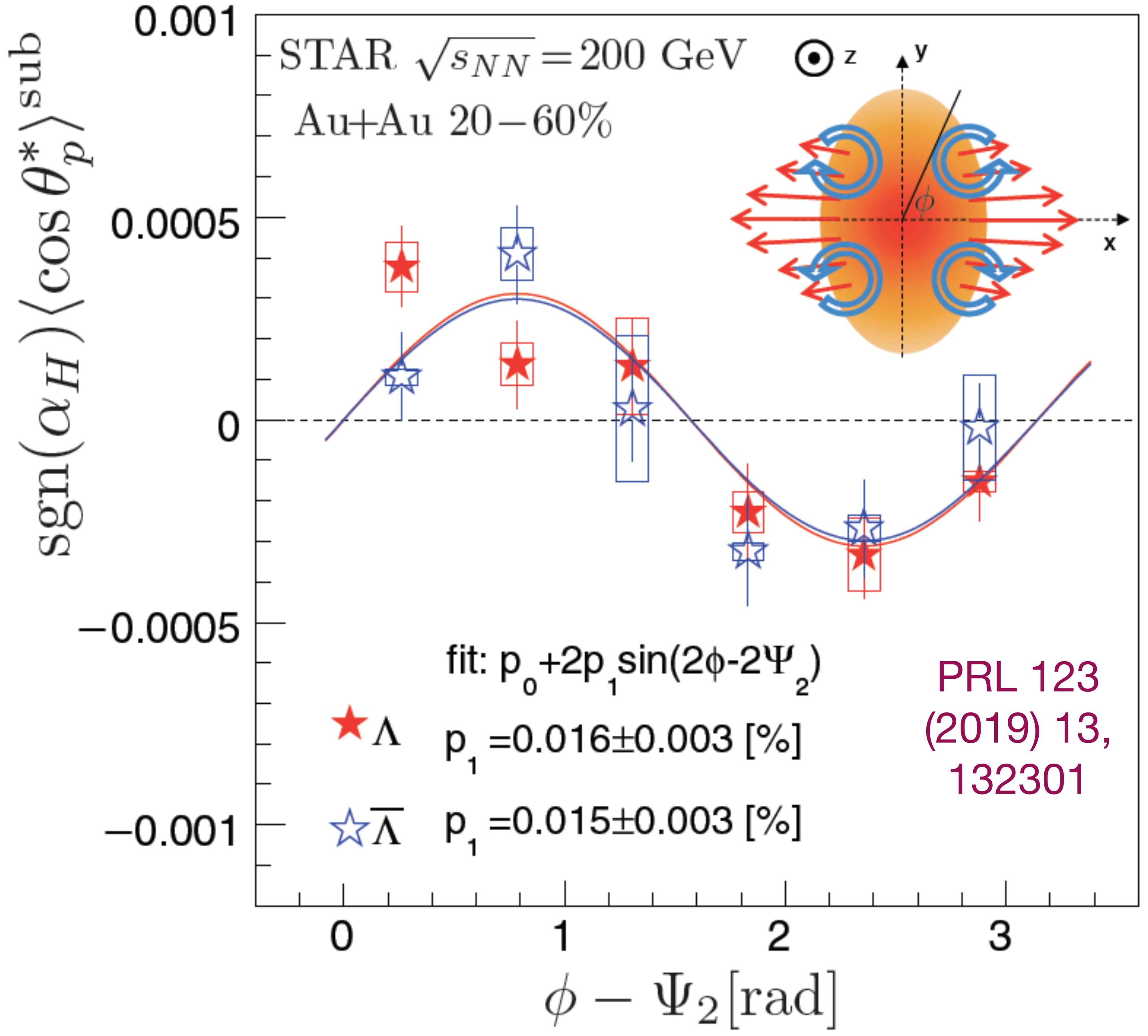
Spin polarization

- Observation of $\Lambda(\bar{\Lambda})$ global spin polarization provided evidence of QGP vortical structure.
- Shows decreasing behavior with increase in $\sqrt{s_{NN}}$.
- Differences between $\Lambda - \bar{\Lambda}$ polarization may be due to initial EM fields caused during the collisions.

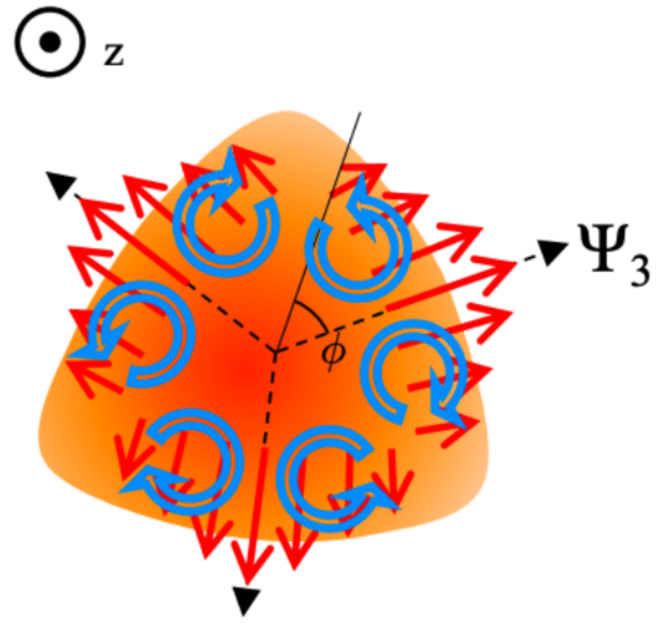


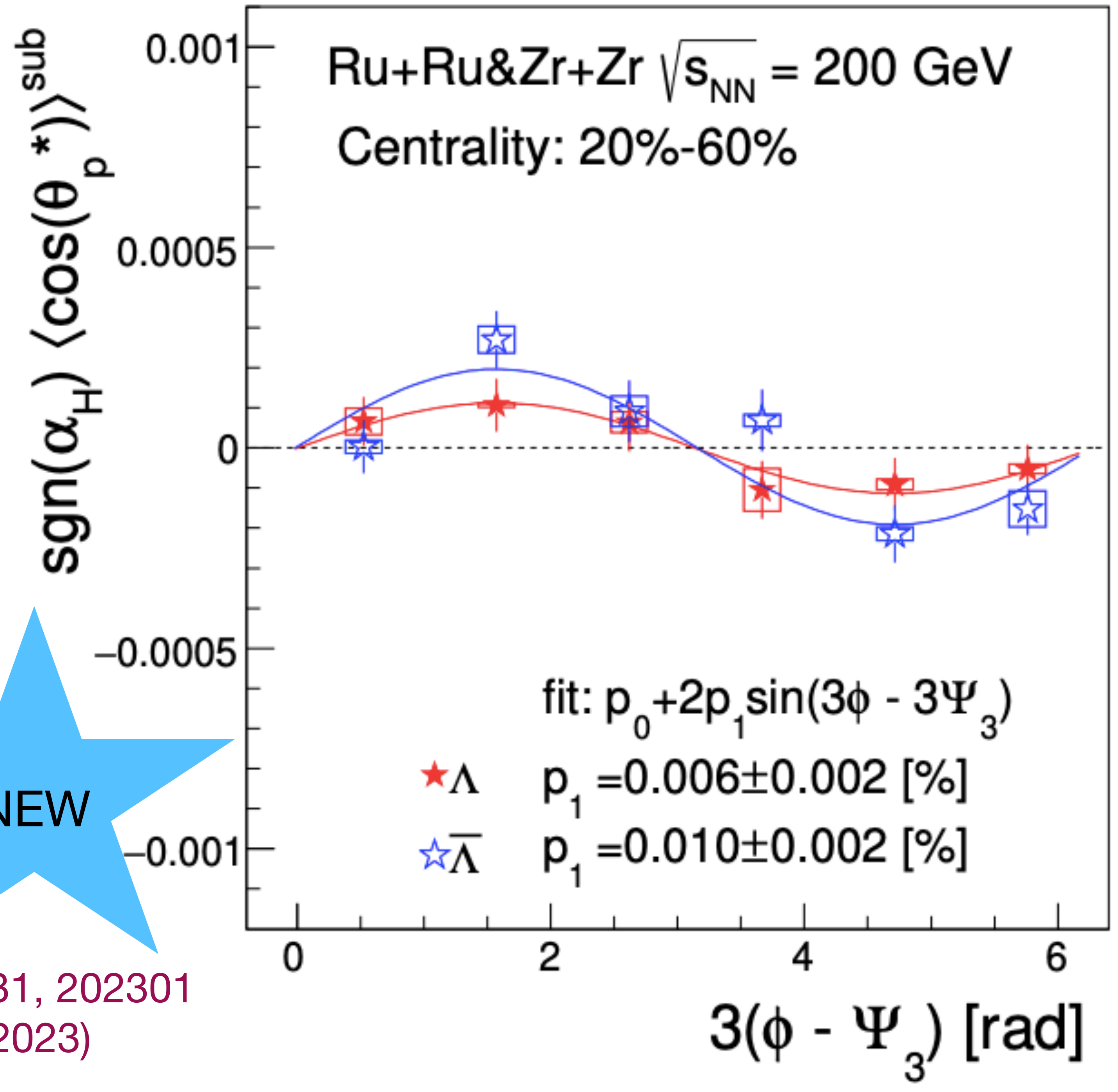
Finite differences
(Open question!)

- Spin polarization along the beam may result from the transverse plane flow structure.



PRL 123 (2019) 13, 132301





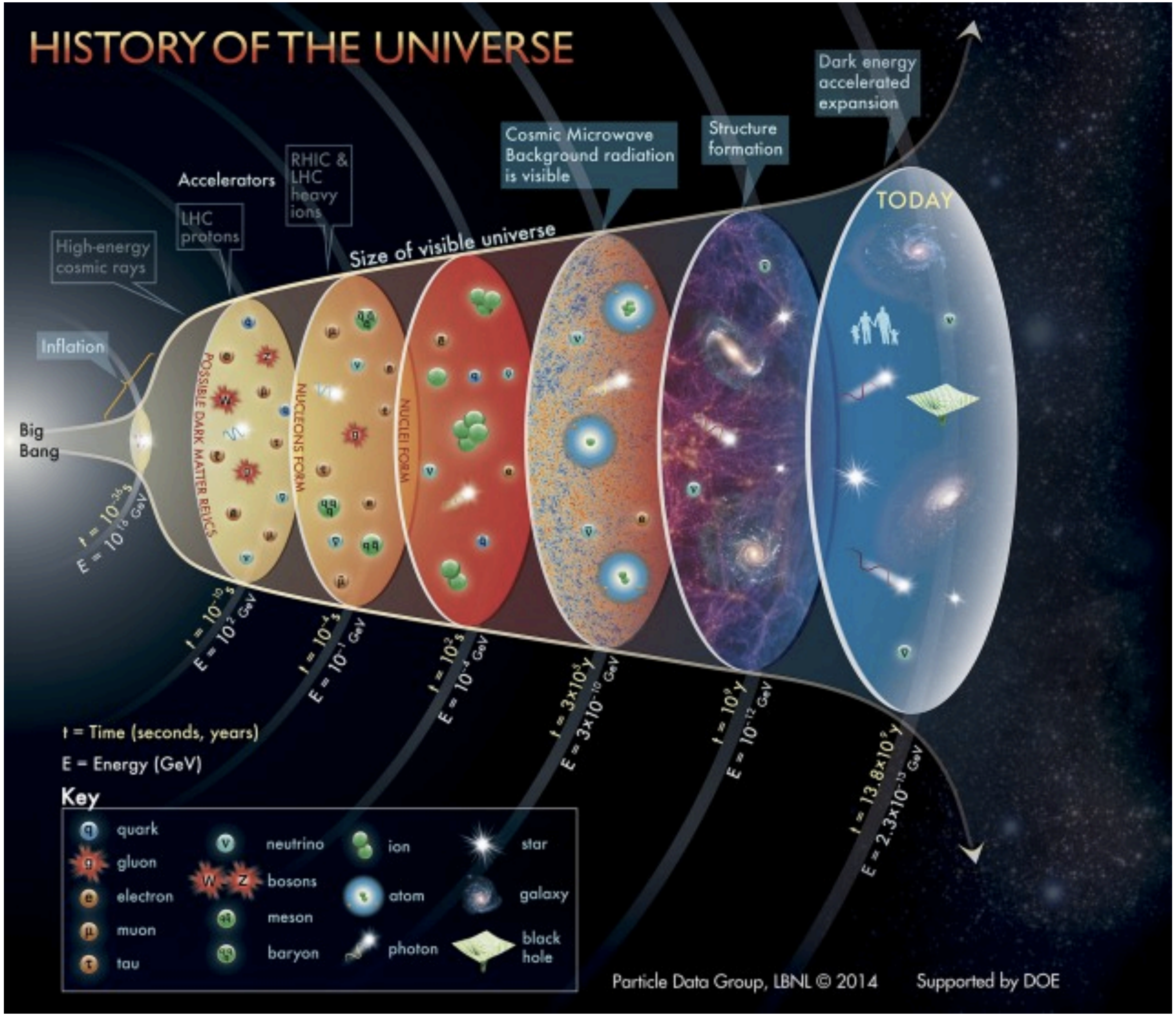
NEW

PRL 131, 202301 (2023)

Bigger picture

Spin polarization is a new and interesting probe to help us to know the formation and characteristics of the QGP

Detecting and understanding the QGP indirectly allows us to understand better the universe in the moments after the Big Bang



Recent developments

Lagrangian effective field theory approach

D. Montenegro, G. Torrieri, Phys.Rev. D94 (2016) no.6, 065042
D. Montenegro, L. Tinti, G. Torrieri, Phys. Rev. D 96(5) (2017) 056012; Phys. Rev. D 96(7) (2017) 076016
D. Montenegro, G. Torrieri, Phys. Rev. D 100, 056011 (2019)

Hydrodynamics with spin based on entropy-current analysis

K. Hattori, M. Hongo, X-G Huang, M. Matsuo, H. Taya, PLB 795 (2019) 100-106
K. Fukushima, S. Pu, PLB 817 (2021) 136346

Hydrodynamics of spin currents using presence of torsion

D. Gallegos, U. Gursoy, A. Yarom arXiv:2101.04759

Relativistic viscous hydrodynamics with spin using Navier-Stokes type gradient expansion analysis

D. She, A. Huang, D. Hou, J. Liao, arXiv:2105.04060

Relativistic viscous spin hydrodynamics from chiral kinetic theory

S. Shi, C. Gale, and S. Jeon, Phys. Rev. C 103, 044906 (2021)

Spin polarization generation from vorticity through nonlocal collisions

N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke, arXiv:2005.01506, arXiv:2103.04896

Spin polarisation due to thermal shear

F. Becattini, M. Buzzegoli, and A. Palermo, arXiv:2103.10917
S. Y. F. Liu and Y. Yin, arXiv:2103.09200

Not enough space to include all papers. I apologize!

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D. Montenegro, G. Torrieri, Phys. Rev. D 100, 056011 (2019)

as of June 02

Hydrodynan

Hydrodynan

Relativistic v … nalysis

	Citeable	Published
Papers	844	562
Citations	22,449	20,099
h-index	79	76
Citations/paper (avg)	26.6	35.8

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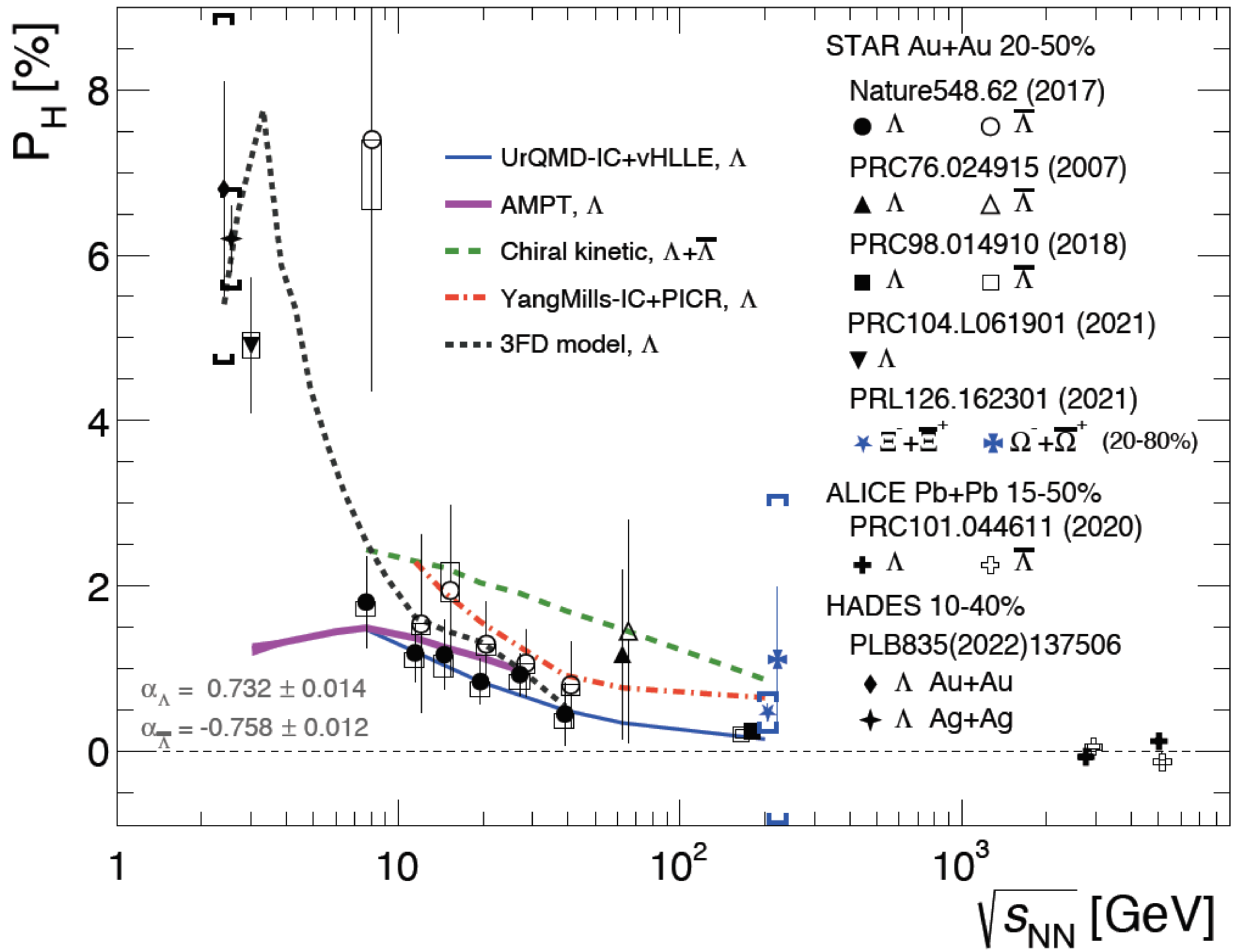
Theoretical efforts

→ Models that assume LTE of spin degrees of freedom are able to explain global spin polarization measurement.

What does it mean?



Average global spin polarization for $\Lambda(\bar{\Lambda})$ hyperons in 20-50% centrality $\mathrm{Au}+\mathrm{Au}$ collisions as a function of collision energy. (Nature 548 (2017) 62-65)

Theoretical efforts

In local thermodynamic equilibrium, one can establish a link between spin and thermal vorticity

Ann. Phys. 323:2452 (2008), Ann. Phys. 338:32 (2013)
Phys. Rev. C 94:024904 (2016)

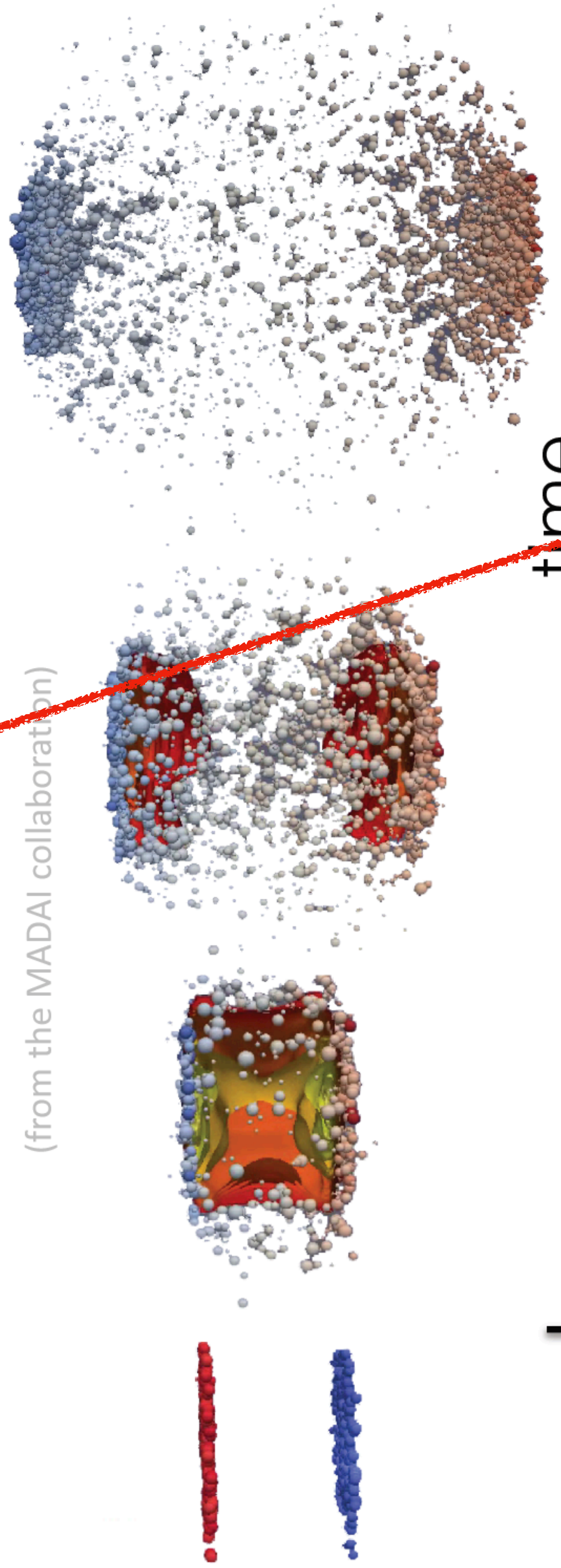
$$\partial_\alpha N^\alpha = 0 \qquad \partial_\alpha T^{\alpha\beta} = 0$$

$$\boxed{S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int d\Sigma_\lambda p^\lambda n_F(1-n_F)\boxed{\varpi_{\rho\sigma}}}{\int d\Sigma_\lambda p^\lambda n_F}}$$

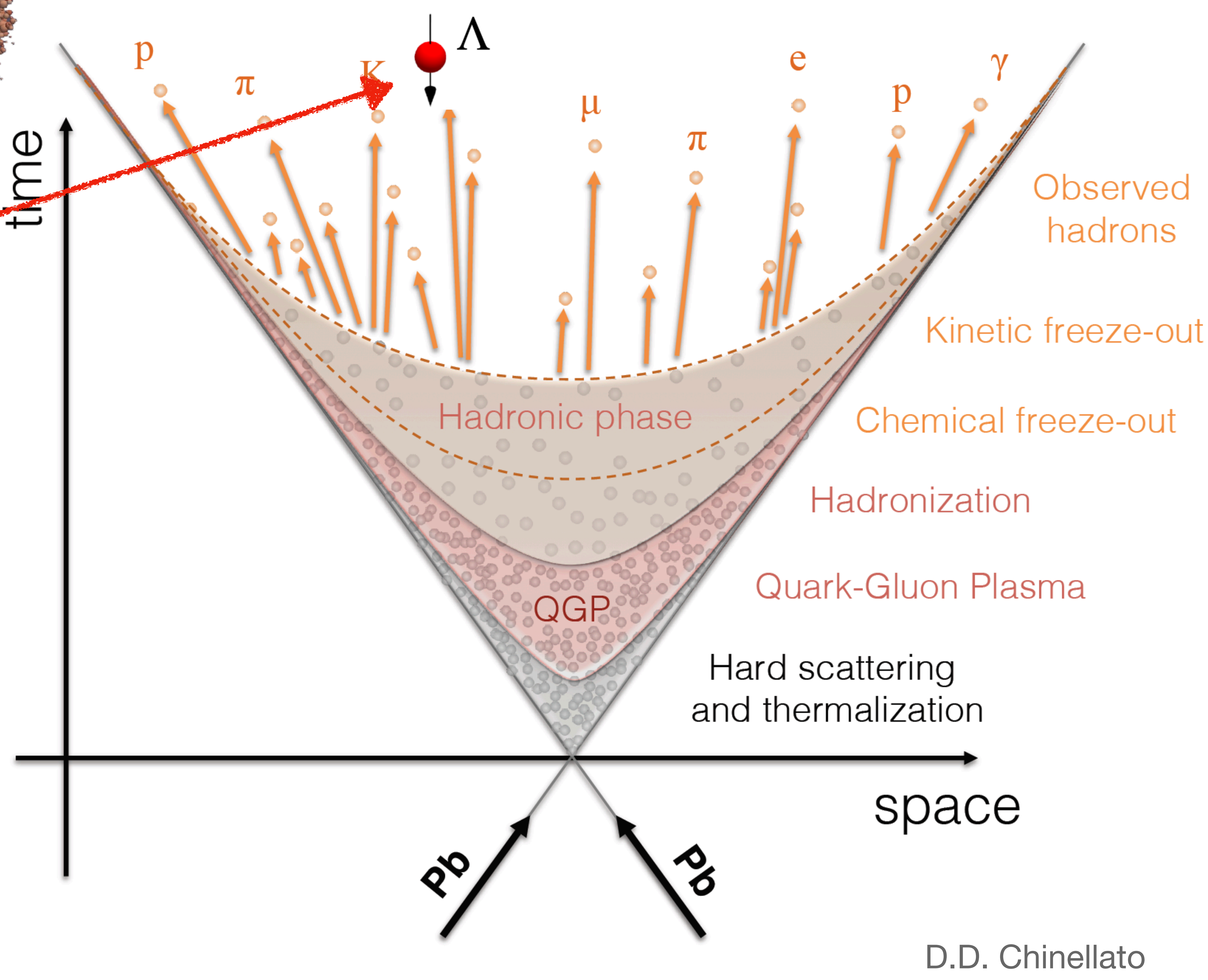
$$\boxed{\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) \qquad \beta^\mu = \frac{u^\mu}{T}}$$

$$n_F = (1+\exp[\beta\cdot p - \mu Q/T])^{-1}$$

Allows to extract polarization at the freeze-out hypersurface in any model which provides u^μ, T and μ



relativistic heavy-ion collision



Theoretical efforts

→ Models that assume LTE of spin degrees of freedom are able to explain global spin polarization measurement.

→ But, unsuccessful to provide clear explanation for the azimuthal angle dependence of longitudinal polarization. Recent progress with thermal shear have some agreement.

Phys.Rev.Lett. 127 (2021) 27, 272302, Phys.Rev.Lett. 127 (2021) 14, 142301

There are two differences in the thermal shear-spin formula

$$\mathcal{A}^\mu_{\rm BBP} = -\varepsilon^{\mu\rho\sigma\tau}\left(\frac{1}{2}\omega_{\rho\sigma}p_\tau + \frac{1}{E}\hat{t}_\rho\xi_{\sigma\lambda}p^\lambda p_\tau\right)$$

F. B., M. Buzzegoli, A. Palermo, Phys. Lett. B 820 (2021) 136519

$$S^\mu(p^\alpha) = \frac{1}{4m}\frac{\int d\Sigma\cdot p\, n_0(1-n_0)\mathcal{A}^\mu}{\int d\Sigma\cdot p\, n_0},$$

S. Liu, Y. Yin, JHEP 07 (2021) 188

$$\mathcal{A}^\mu_{\rm LY} = -\varepsilon^{\mu\rho\sigma\tau}\left[\frac{1}{2}\omega_{\rho\sigma}p_\tau + \frac{1}{E}u_\rho\xi_{\sigma\lambda}p^\lambda_\perp p_\tau + \frac{b_i}{\beta E}u_\rho p^\perp_\sigma \partial^\perp_\tau(\beta\mu_B)\right].$$

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right)$$

$$p^\lambda_\perp = p^\lambda - (u\cdot p)u^\lambda.$$

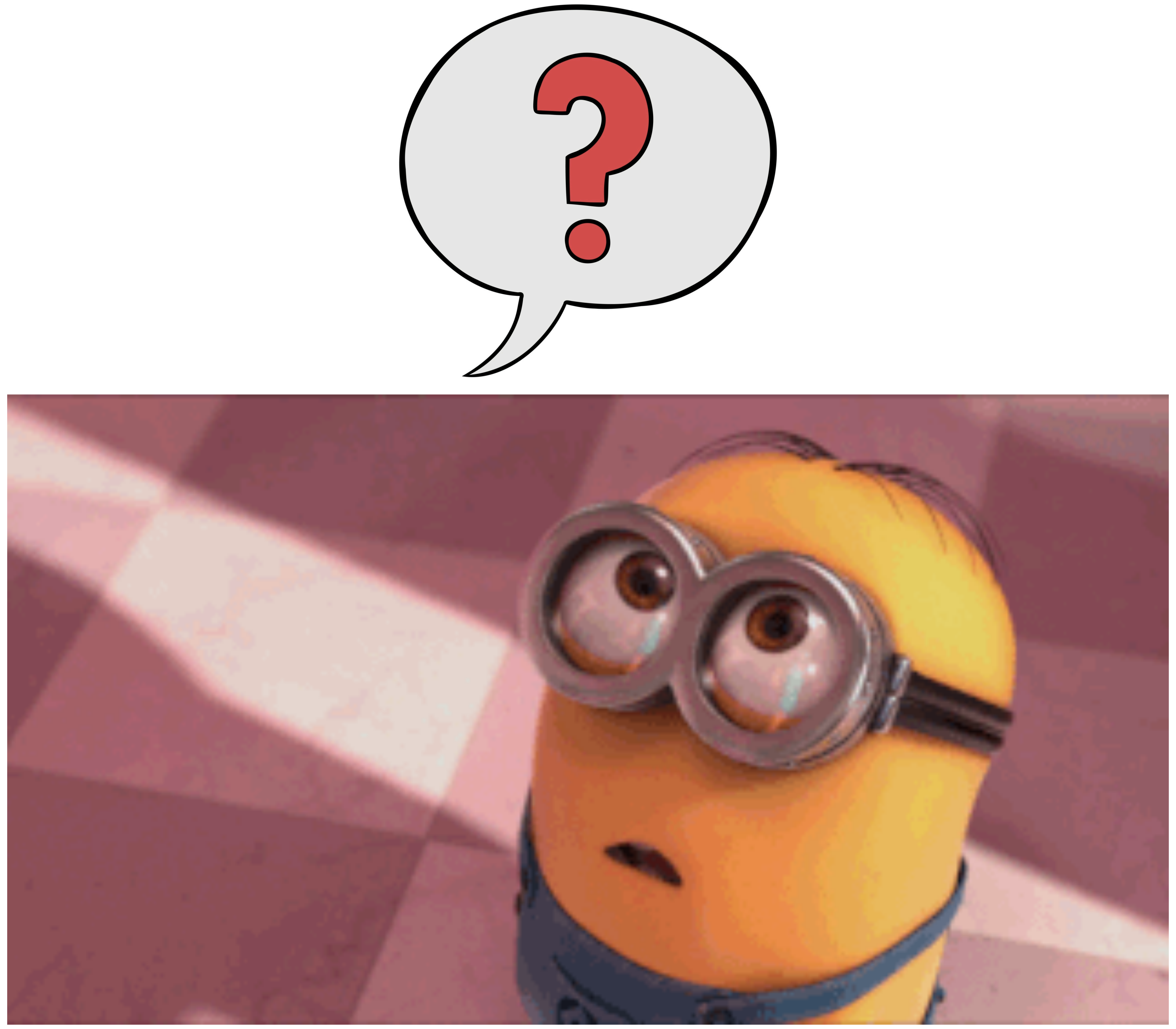
$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right)$$

From F. Becattini talk

$$\xi_{\sigma\rho} = \frac{1}{2}\partial_\sigma\left(\frac{1}{T}\right)u_\rho + \frac{1}{2}\partial_\rho\left(\frac{1}{T}\right)u_\sigma + \frac{1}{2T}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \frac{1}{T}\sigma_{\rho\sigma} + \frac{1}{3T}\theta\Delta_{\rho\sigma}$$

In the LY formula, both the thermal gradient and acceleration terms are killed by the four-velocity u replacing t and the p⊥ replacing p

These discrepancies raise some questions



Why we need spin hydro?

- Why spin-thermal approach does not fully capture differential observables?
- Is spin polarization always enslaved to thermal vorticity?
- Is there non-trivial space-time dynamics of spin?

Why we need spin hydro?

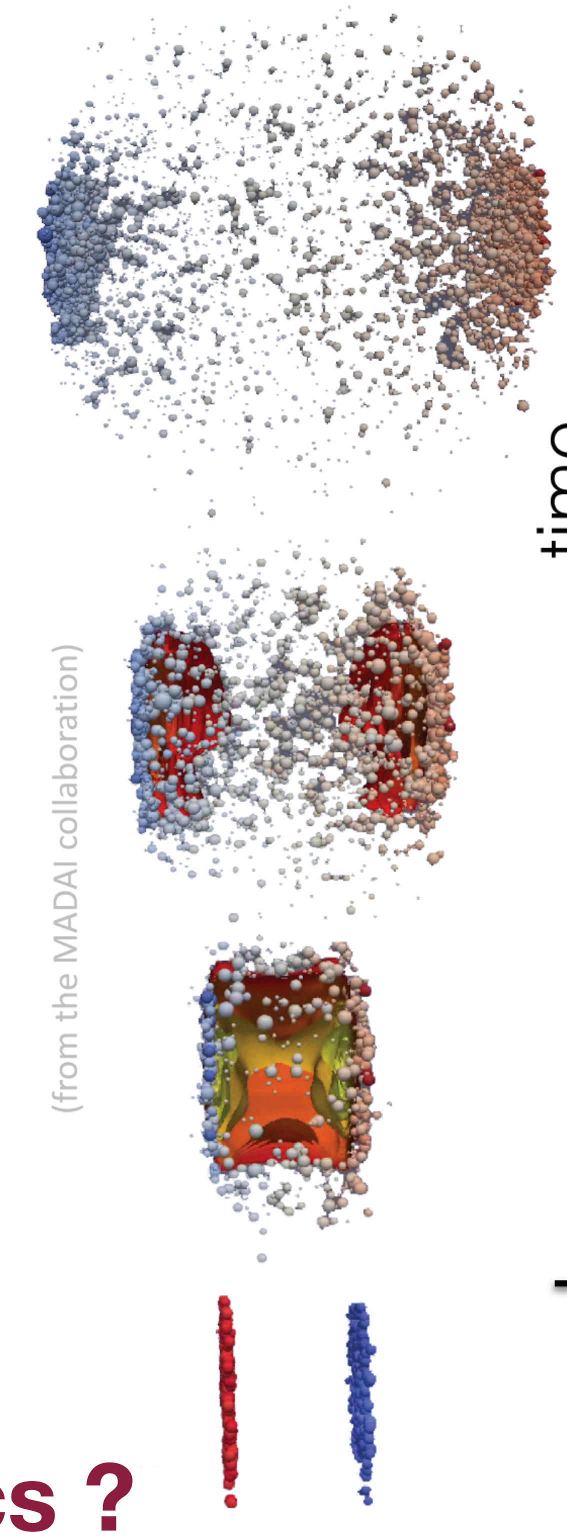
Relativistic fluid dynamics forms the basis of HIC models

→ Why spin-thermal approach does not fully capture differential observables?

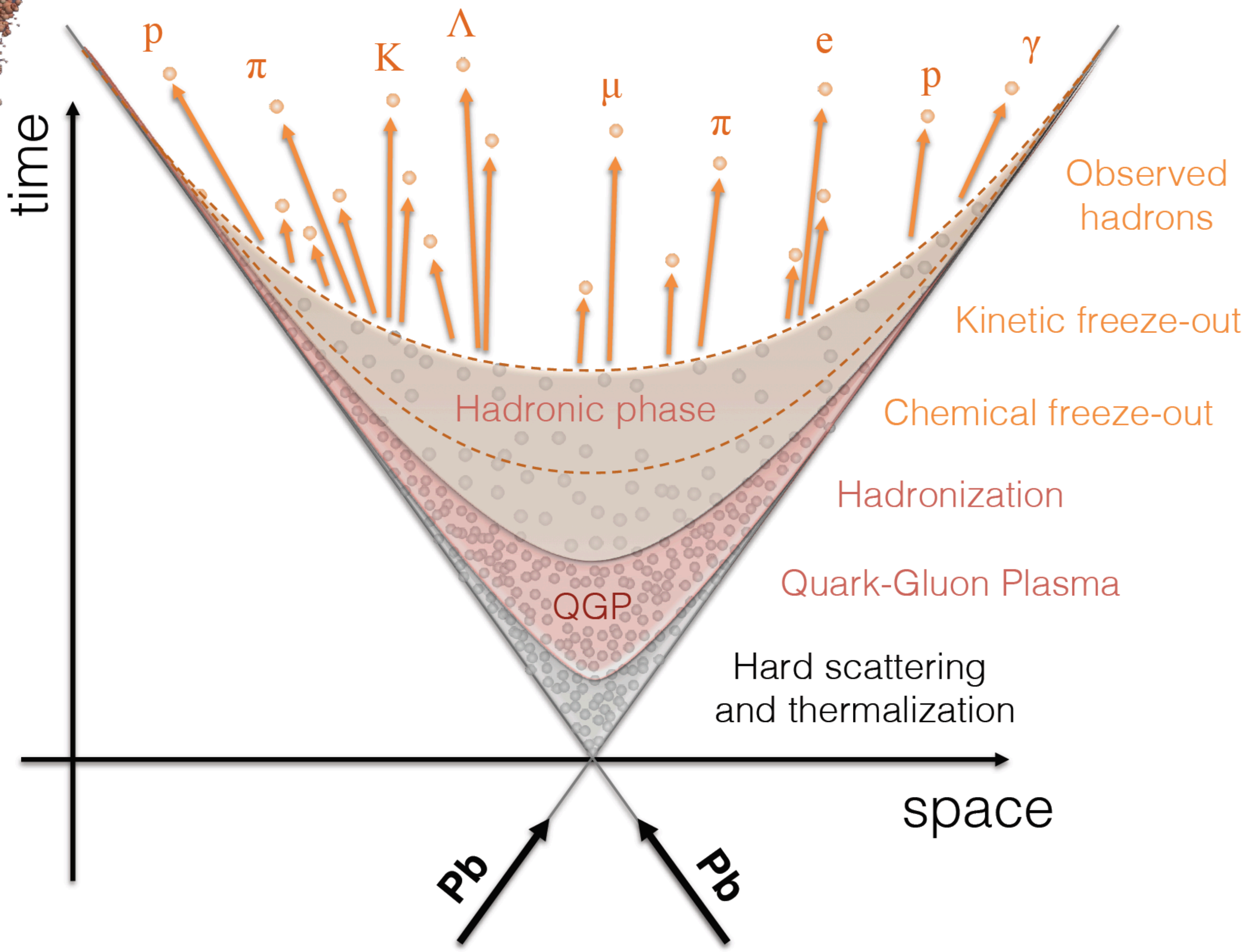
→ Is spin polarization always enslaved to thermal vorticity?

→ Is there non-trivial space-time dynamics of spin?

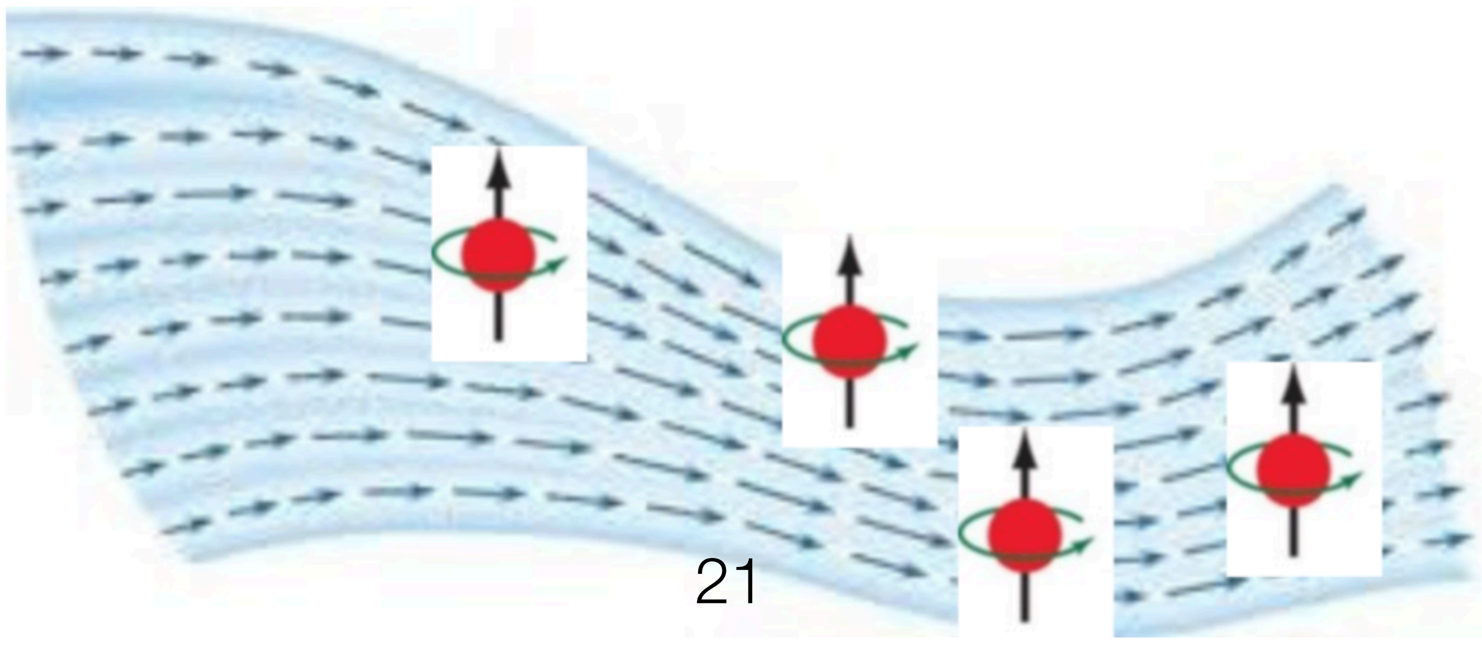
(from the MADAI collaboration)



A heavy ion collision



Spin Hydrodynamics ?



Most of the time close to equilibrium but the dissipation is also important

Relativistic (ideal) hydrodynamics with spin

Prog.Part.Nucl.Phys. 108 (2019) 103709

$\longrightarrow$ Using Wigner function (in equilibrium) $W_{\rm eq}(x,k) = W^+_{\rm eq}(x,k) + W^-_{\rm eq}(x,k)$

$$\left(i\hbar\frac{\gamma^\mu\partial_\mu}{2} + \gamma^\mu k_\mu - m\right) W_{\rm eq}(x,k) = \hbar\, C[W_{\rm eq}(x,k)]$$

Transport equation

$X^\pm = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu\right]\left[1 \pm \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]$

$\Sigma^{\mu\nu} = (i/4)\left[\gamma^\mu, \gamma^\nu\right], \quad \xi(x) = \mu_B/T, \quad \beta_\mu(x) = U^\mu/T$

$z = m/T$

"+" means particle contribution

"−" means antiparticle contribution

$$W^+_{\rm eq}(x,k) = \frac{1}{2}\sum_{r,s}\int {\rm dP}\,\delta^{(4)}(k-p)\,\mathcal{U}^r(p)\,\bar{\mathcal{U}}^s(p)\,f^+_{rs}(x,p)$$

$$W^-_{\rm eq}(x,k) = -\frac{1}{2}\sum_{r,s}\int {\rm dP}\,\delta^{(4)}(k+p)\,\mathcal{V}^s(p)\,\bar{\mathcal{V}}^r(p)\,f^-_{rs}(x,p)$$

Groot, Leeuwen, Weert, Relativistic Kinetic Theory. Principles and Applications. North Holland, 1, 1980

and ansatz for local equilibrium distribution functions

$$f^+_{rs}(x,p) = \frac{1}{2m}\bar{\mathcal{U}}_r(p)\,X^+\,\mathcal{U}_s(p) = \frac{1}{2m}\bar{\mathcal{U}}_r(p)\,\exp\left[-\beta_\mu(x)p^\mu + \xi(x)\right]\left[1 + \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]\mathcal{U}_s(p)$$

$$f^-_{rs}(x,p) = -\frac{1}{2m}\bar{\mathcal{V}}_s(p)\,X^-\,\mathcal{V}_r(p) = -\frac{1}{2m}\bar{\mathcal{V}}_s(p)\,\exp\left[-\beta_\mu(x)p^\mu - \xi(x)\right]\left[1 - \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]\mathcal{V}_r(p)$$

Dirac spinors

Spin polarization tensor
(Spin chemical potential)

We obtain

$$W^\pm_{\rm eq}(x,k) = \frac{1}{4m}\int {\rm dP}\,e^{-\beta\cdot p\pm\xi}\,\delta^{(4)}(k\mp p)\left[2m(m\pm\gamma^\mu p_\mu) \pm \frac{1}{2}\,\omega_{\mu\nu}(\gamma^\mu p_\mu \pm m)\Sigma^{\mu\nu}(\gamma^\mu p_\mu \pm m)\right]$$

Relativistic (ideal) hydrodynamics with spin

Prog.Part.Nucl.Phys. 108 (2019) 103709

$\Sigma^{\mu\nu} = (i/4)\left[\gamma^\mu, \gamma^\nu\right], \quad \xi(x) = \mu_B/T, \quad \beta_\mu(x) = U^\mu/T$

$\mathscr{C} = \cosh(\xi), \quad \Delta^{\mu\nu} = g^{\mu\nu} - (U^\mu U^\nu)/(U\cdot U)$

$\mathscr{B}_{(0)} = -\frac{2}{z^2}\frac{\mathscr{E}_{(0)}+\mathscr{P}_{(0)}}{T}, \quad \mathscr{A}_{(0)} = 2\mathscr{N}_{(0)} - 3\mathscr{B}_{(0)}$

$z = m/T$

$\longrightarrow$ Decomposing Wigner function using Clifford algebra expansion

One can derive the <u>constitutive relations</u> for

- Net baryon current

$$N^\alpha(x) = \langle : \bar\psi \gamma^\alpha \psi : \rangle = \mathrm{tr}\int d^4k\, \gamma^\alpha \left(W^+_{\rm eq}(x,k) - W^-_{\rm eq}(x,k)\right)$$

$$N^\alpha(x) = \mathscr{N}\, U^\alpha$$

with

$$\mathscr{N} = 4\sinh(\xi)\,\mathscr{N}_{(0)}(T)$$

$$\mathscr{N}_{(0)}(T) = \frac{T^3}{2\pi^2} z^2 K_2(z)$$

$$\boxed{\partial_\alpha N^\alpha = 0}$$

- Energy-momentum tensor

$$T^{\mu\nu}_{\rm GLW}(x) = \langle : \hat T^{\mu\nu}_{\rm GLW} : \rangle = \frac{1}{m}\,\mathrm{tr}\int d^4k\, k^\mu k^\nu \left(W^+_{\rm eq}(x,k) + W^-_{\rm eq}(x,k)\right)$$

$$T^{\mu\nu}_{\rm GLW}(x) = (\mathscr{E}+\mathscr{P})\,U^\mu U^\nu - \mathscr{P}\, g^{\mu\nu}$$

with

$$\mathscr{E} = 4\cosh(\xi)\,\mathscr{E}_{(0)}(T)$$

$$\mathscr{P} = 4\cosh(\xi)\,\mathscr{P}_{(0)}(T)$$

$$\mathscr{E}_{(0)}(T) = \frac{T^4}{2\pi^2} z^2 \left[zK_1(z) + 3K_2(z)\right]$$

$$\mathscr{P}_{(0)}(T) = T\mathscr{N}_{(0)}(T)$$

$$\boxed{\partial_\alpha T^{\alpha\beta}_{\rm GLW} = 0}$$

Relativistic (ideal) hydrodynamics with spin

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- Spin tensor $$S^{\alpha,\beta\gamma}_{\rm GLW} = \langle : \hat{S}^{\alpha,\beta\gamma}_{\rm GLW} : \rangle = \frac{\hbar}{4}\int d^4k \,\mathrm{tr}\left[\left(\left\{\sigma^{\beta\gamma},\gamma^\alpha\right\} + \frac{2i}{m}\left(\gamma^{[\beta}k^{\gamma]}\gamma^\alpha - \gamma^\alpha\gamma^{[\beta}k^{\gamma]}\right)\right)\left(W^+_{\rm eq}(x,k) + W^-_{\rm eq}(x,k)\right)\right]$$

$$S^{\alpha,\beta\gamma}_{\rm GLW} = U^\alpha\left(\mathscr{A}_1\,\omega^{\beta\gamma} + \mathscr{A}_2\,U^{[\beta}\,\omega^{\gamma]}{}_\delta\,U^\delta\right) + \mathscr{A}_3\left(U^{[\beta}\,\omega^{\gamma]\alpha} + g^{\alpha[\beta}\,\omega^{\gamma]}{}_\delta\,U^\delta\right)$$

Fluid-flow four-velocity

Spin polarization tensor

with

$$\left.\begin{aligned}\mathscr{A}_1 &= \mathscr{C}\left(\mathscr{N}_{(0)} - \mathscr{B}_{(0)}\right)\\ \mathscr{A}_2 &= \mathscr{C}\left(\mathscr{A}_{(0)} - 3\mathscr{B}_{(0)}\right)\\ \mathscr{A}_3 &= \mathscr{C}\,\mathscr{B}_{(0)}\end{aligned}\right\}$$

Thermodynamic coefficients coming from $\partial_\alpha N^\alpha = 0\,,\ \partial_\alpha T^{\alpha\beta}_{\rm GLW} = 0$

$$\rightarrow \boxed{\partial_\alpha N^\alpha = 0\,,\ \partial_\alpha T^{\alpha\beta}_{\rm GLW} = 0} \quad \boxed{\partial_\alpha S^{\alpha,\beta\gamma}_{\rm GLW} = 0}$$

$\Sigma^{\mu\nu} = (i/4)\left[\gamma^\mu,\gamma^\nu\right]$, $\xi(x) = \mu_B/T$, $\beta_\mu(x) = U^\mu/T$

$\mathscr{C} = \cosh(\xi)$, $\Delta^{\mu\nu} = g^{\mu\nu} - (U^\mu U^\nu)/(U\cdot U)$

$\mathscr{B}_{(0)} = -\frac{2}{z^2}\frac{\mathscr{E}_{(0)} + \mathscr{P}_{(0)}}{T}$, $\mathscr{A}_{(0)} = 2\mathscr{N}_{(0)} - 3\mathscr{B}_{(0)}$

$z = m/T$

Modeling of the spin polarization dynamics

→ Mean spin polarization per particle (momentum-dependent)

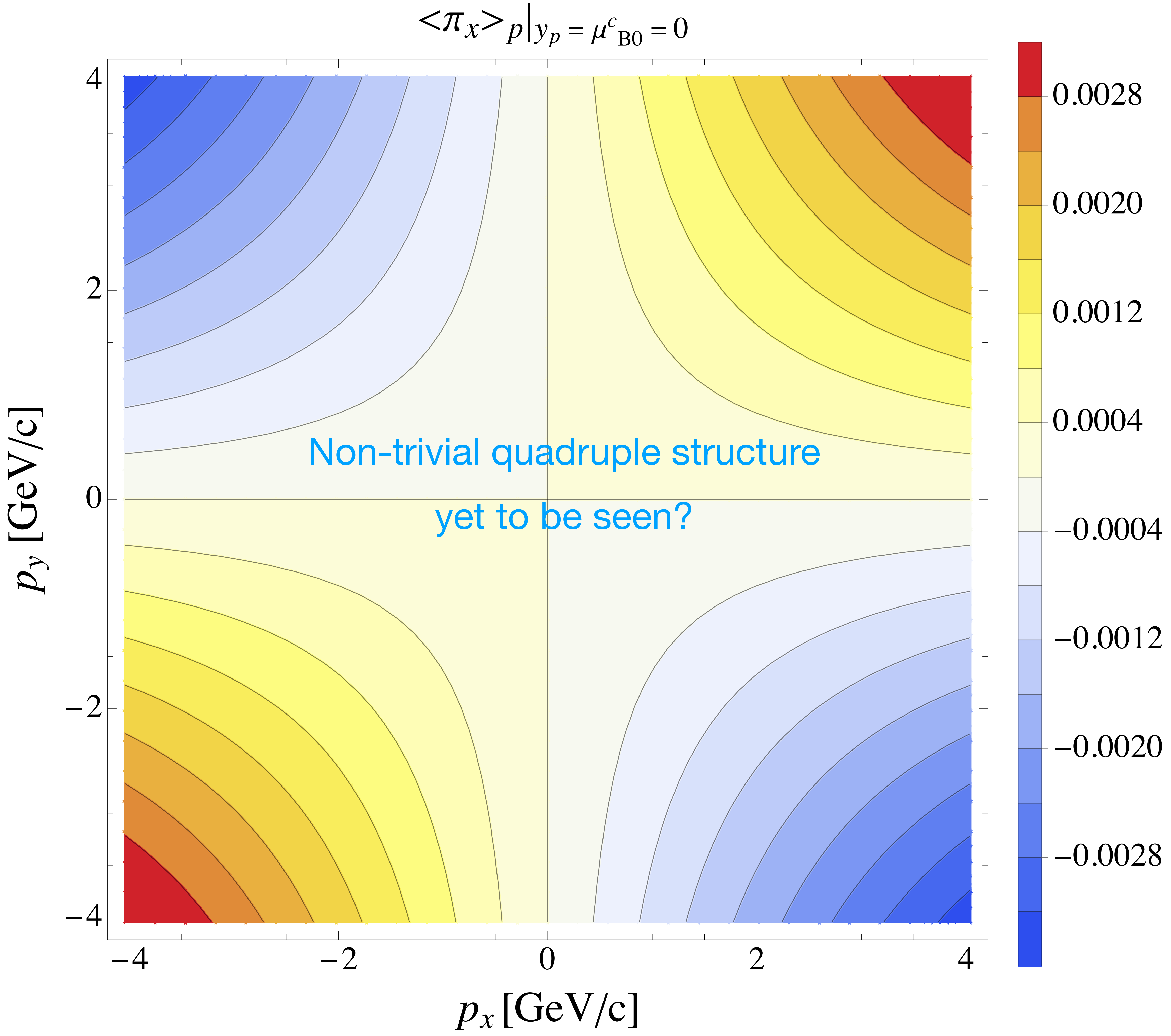
$$\langle \pi_\mu \rangle_p = \frac{E_p \frac{d\Pi_\mu(p)^*}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}} = \frac{-\frac{1}{(2\pi)^3 m} \int \cosh(\xi)\, \Delta\Sigma_\lambda\, p^\lambda\, e^{-\beta\cdot p}\, (\omega^\star_{\mu\beta} p^\beta)^*}{\frac{4}{(2\pi)^3} \int \cosh(\xi)\, \Delta\Sigma_\lambda\, p^\lambda\, e^{-\beta\cdot p}}$$

Pauli Lubanski four-vector

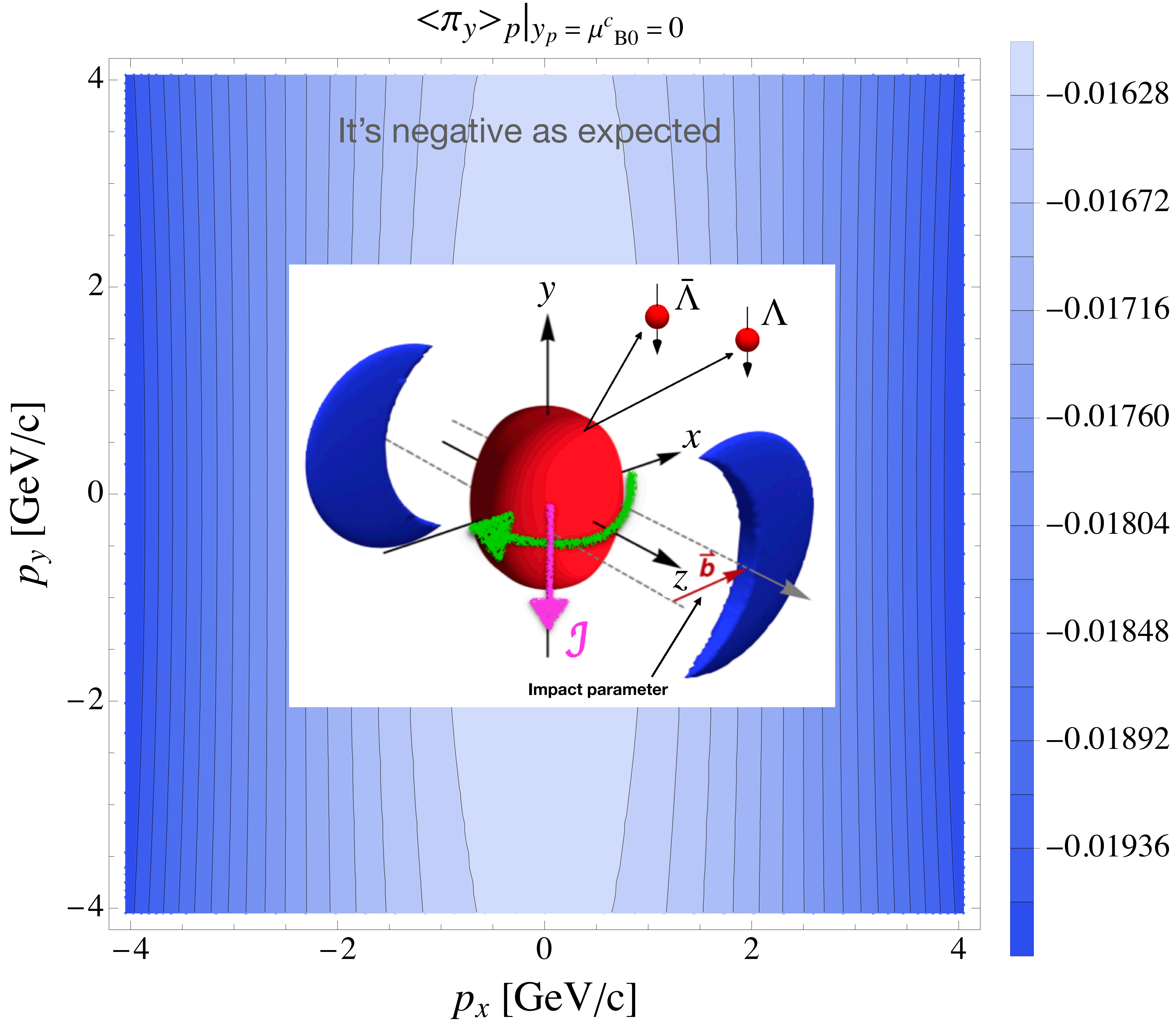
Momentum density of all particles

Freeze-out hyper-surface element

→ Non-boost-invariant and transversely homogeneous

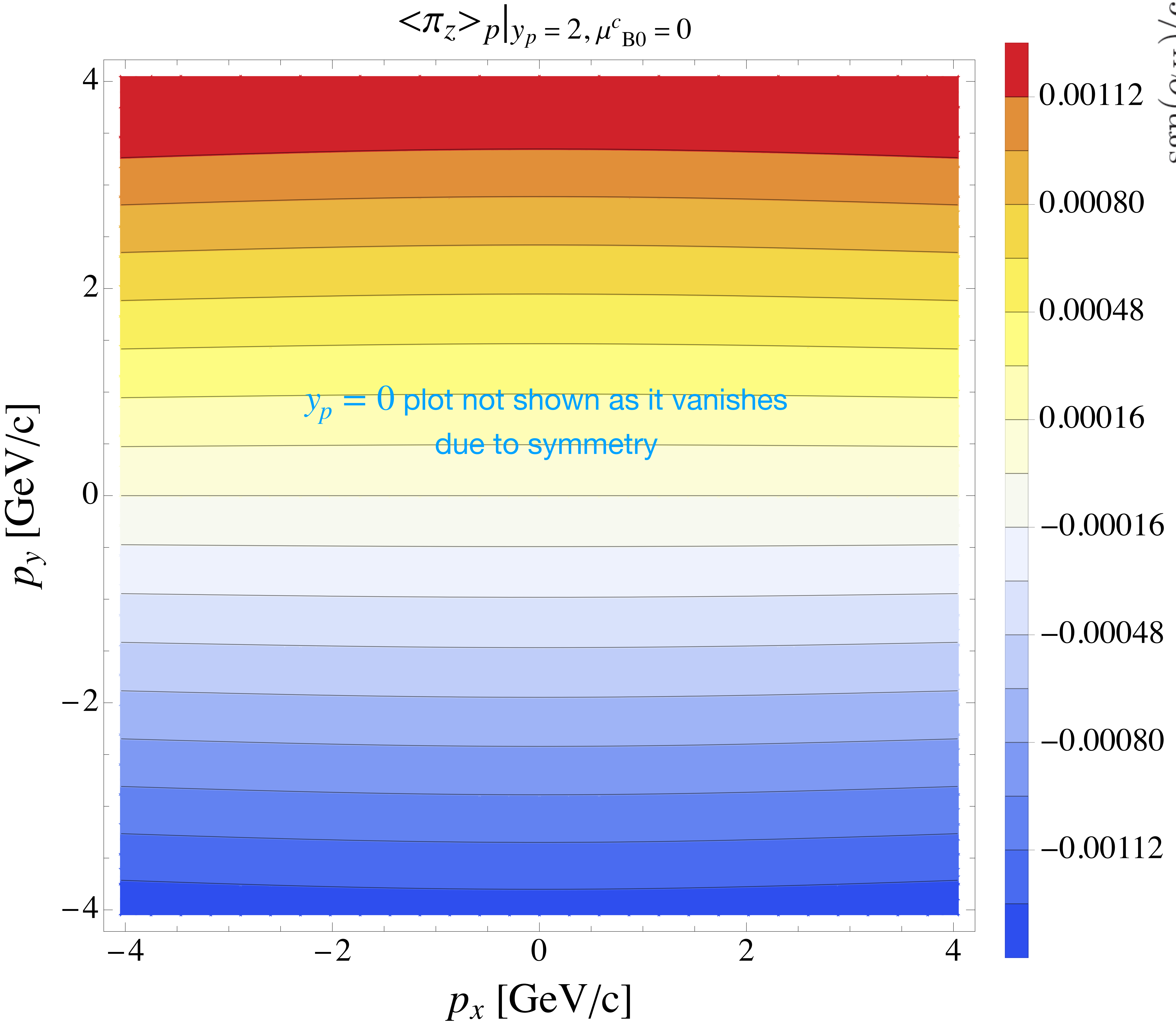


$\mu_B \neq 0$ plots not shown as they are qualitatively similar

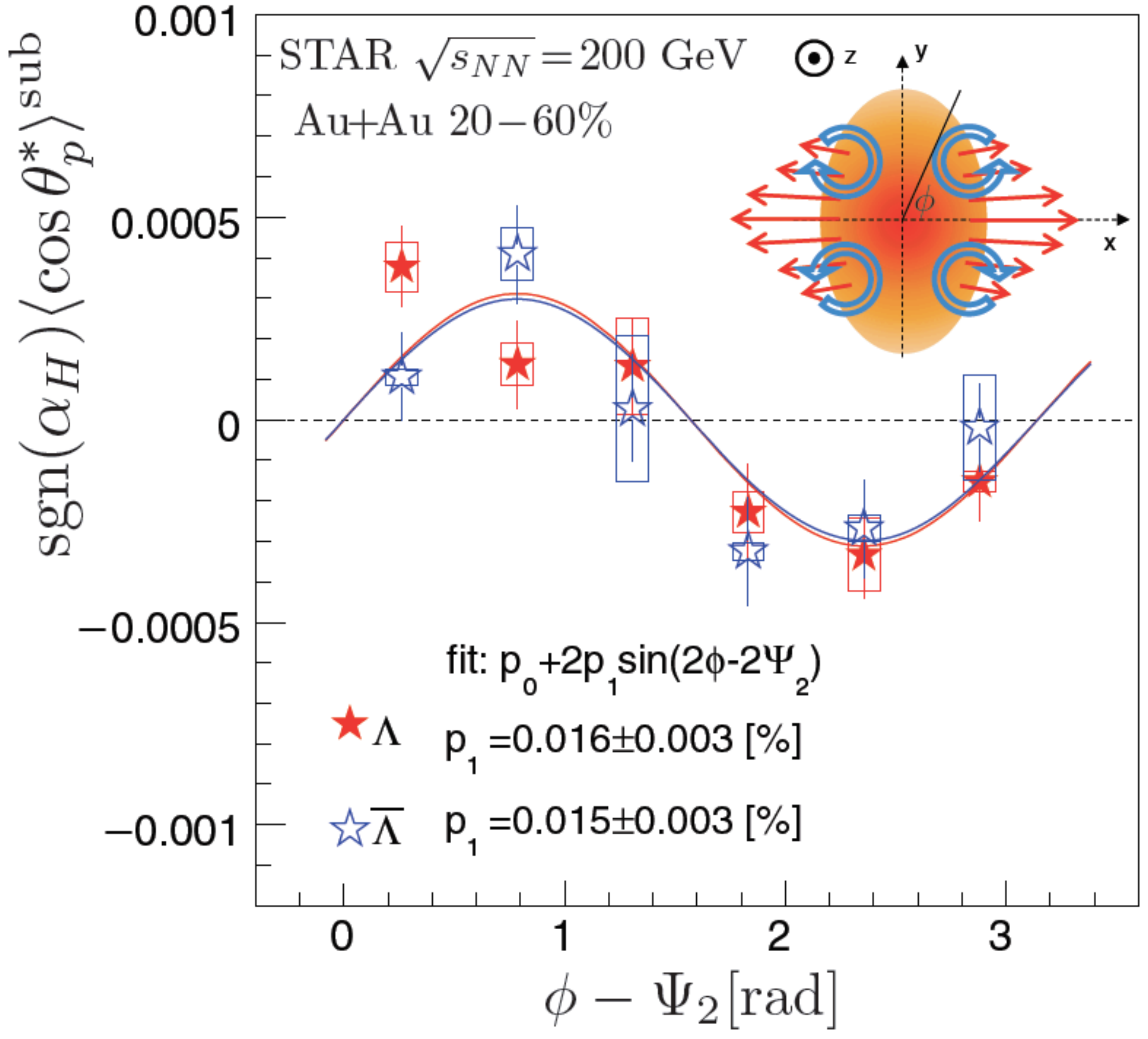


→ Non-boost-invariant and transversely homogeneous

$\mu_B \neq 0$ plots not shown as they are qualitatively similar



$y_p = 0$ plot not shown as it vanishes due to symmetry



Modeling of the spin polarization dynamics

→ Mean spin polarization per particle (momentum-dependent)

$$\langle \pi_\mu \rangle_p = \frac{E_p \frac{d\Pi_\mu(p)}{d^3p}^*}{E_p \frac{d\mathcal{N}(p)}{d^3p}} = \frac{-\frac{1}{(2\pi)^3 m} \int \cosh(\xi)\, \Delta\Sigma_\lambda\, p^\lambda\, e^{-\beta\cdot p}\, (\omega^\star_{\mu\beta} p^\beta)^*}{\frac{4}{(2\pi)^3} \int \cosh(\xi)\, \Delta\Sigma_\lambda\, p^\lambda\, e^{-\beta\cdot p}}$$

Pauli Lubanski four-vector

Momentum density of all particles

Freeze-out hyper-surface element

→ Mean spin polarization per particle (momentum-independent)

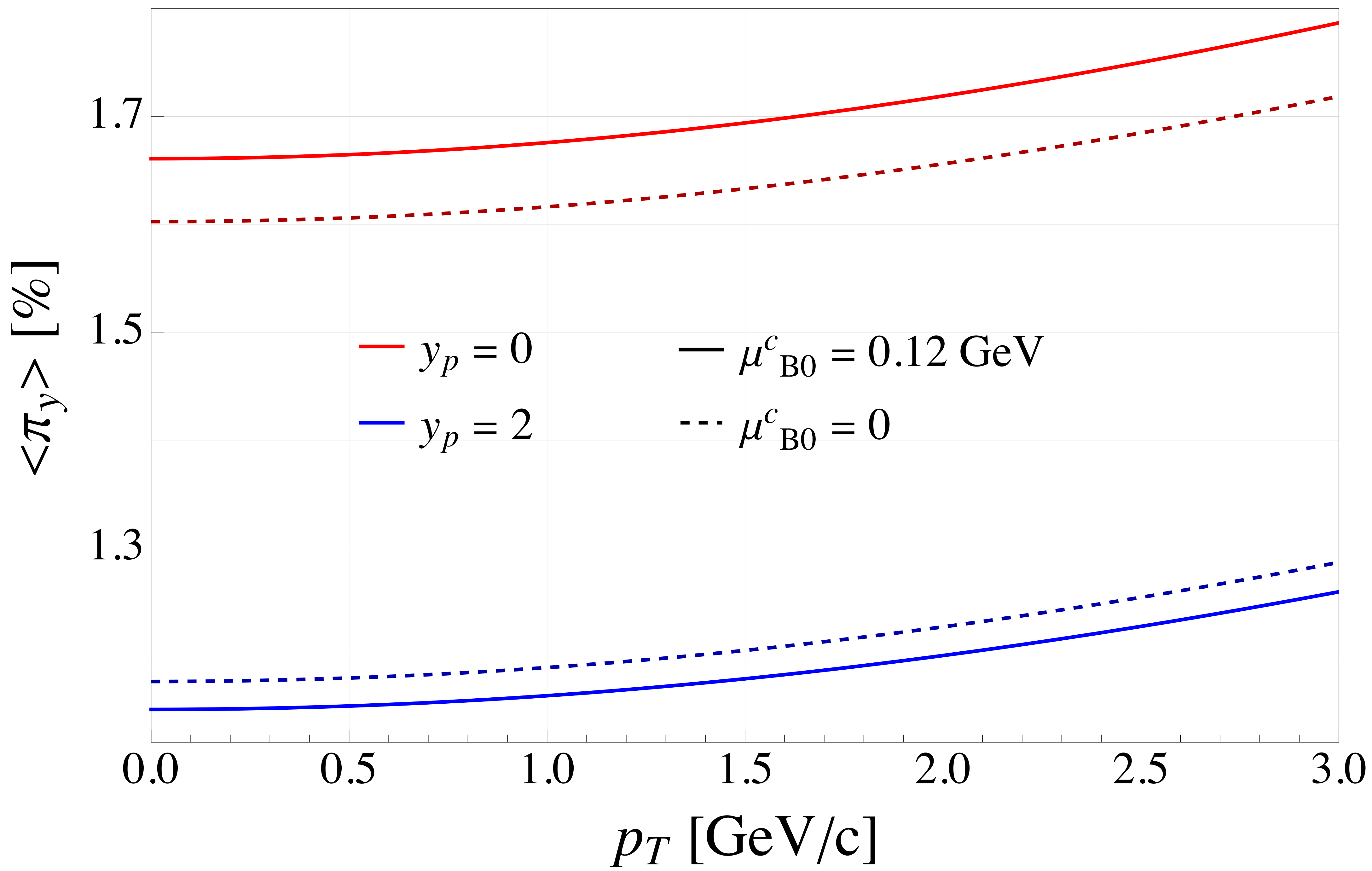
$$\langle \pi_\mu \rangle = \frac{\int dP\, \langle \pi_\mu \rangle_p\, E_p \frac{d\mathcal{N}(p)}{d^3p}}{\int dP\, E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

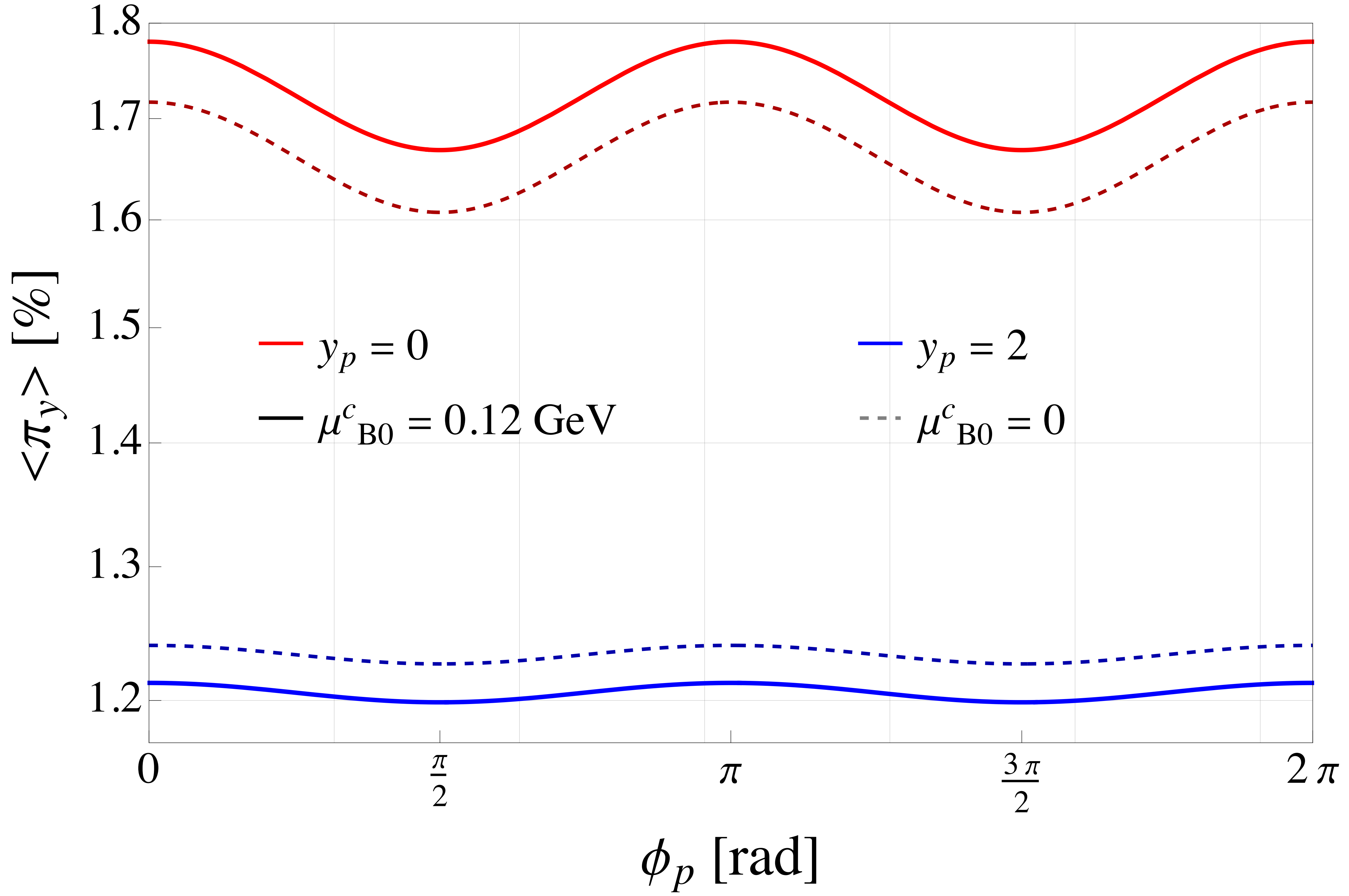
Transverse momentum

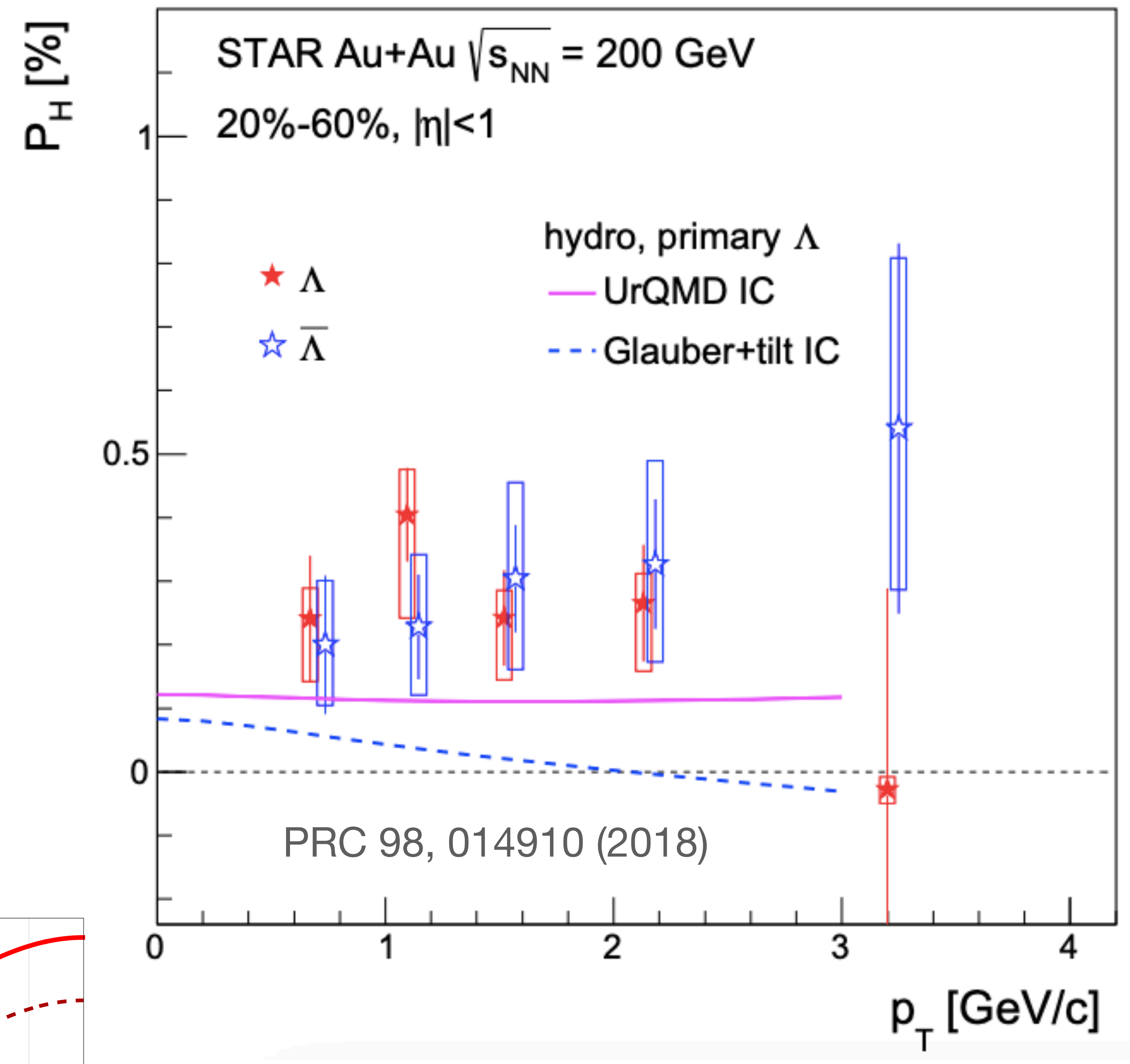
Azimuthal angle

$$\langle \pi_\mu(p_T) \rangle = \frac{\frac{1}{2\pi} \int d\phi_p\, \sin(2\phi_p)\, E_p \frac{d\Pi^*_\mu(p)}{d^3p}}{\int d\phi_p\, E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

$$\langle \pi_\mu(\phi_p) \rangle = \frac{\int p_T\, dp_T\, E_p \frac{d\Pi^*_\mu(p)}{d^3p}}{\int d\phi_p\, p_T\, dp_T\, E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

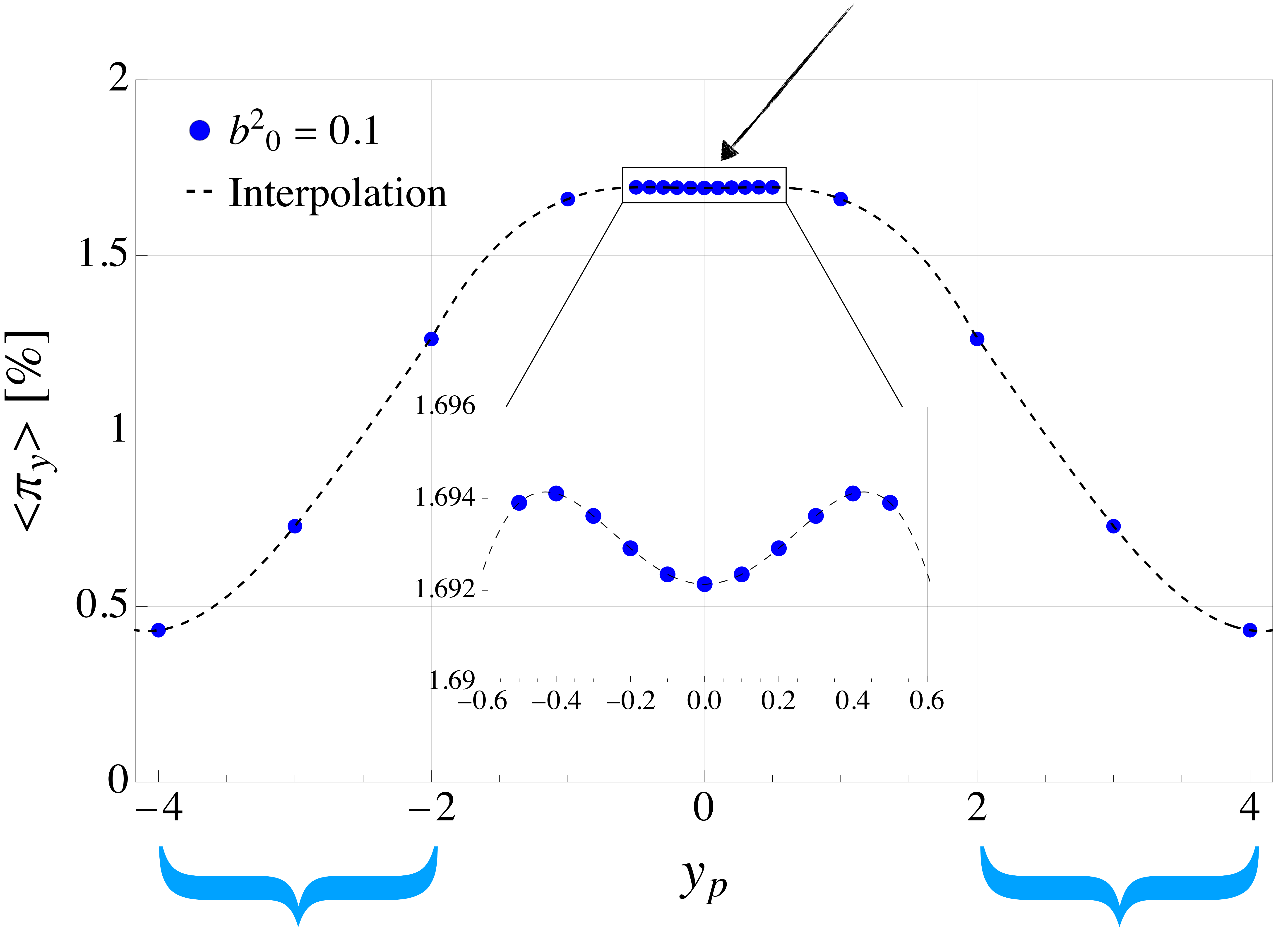






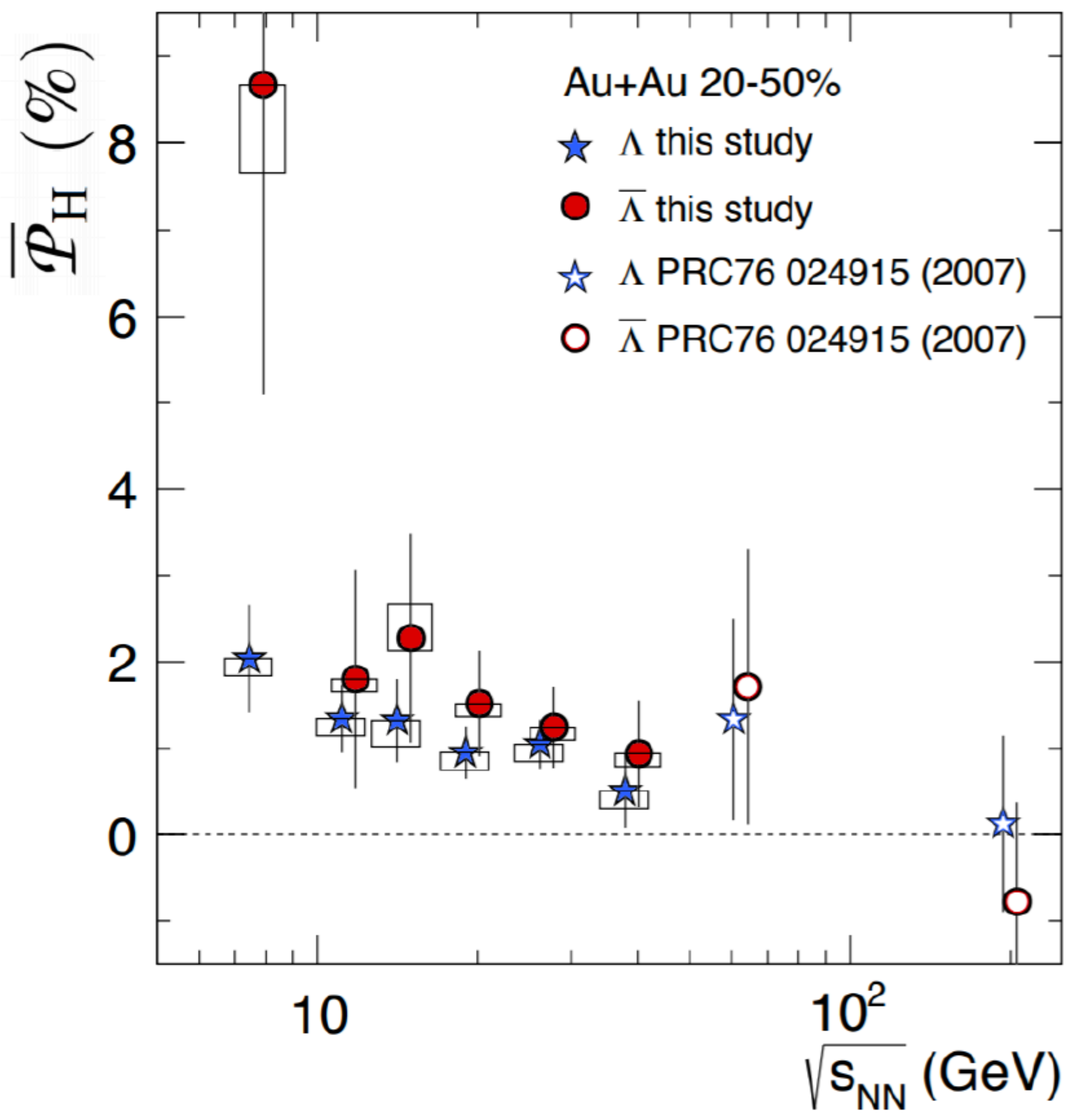
Work in progress!

mid-rapidity region



Still needs to be confirmed

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65



Work in progress!

Summary

→ Spin polarization provides a sensitive new probe of QGP properties

→ Disagreement between theory and experiment motivates development of dynamical models

Rel. viscous spin hydro is work in progress!

→ Spin can be incorporated naturally in relativistic (viscous) fluid dynamics

This can be solved, work in progress!

→ Spin hydro also depends on pseudo gauge

→ Also incorporated EM fields in spin hydro, qualitatively explaining the $\Lambda - \bar{\Lambda}$ differences

→ Inclusion of non-local collisions gave spin-orbit coupling term in our approach

→ Spin polarization dynamics can be studied in various (conformal) fluid backgrounds

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NPA 1035 (2023) 122656
PRD 105 (2022) 5, 054007
PRD 106 (2022) 1, 014018
PRD 103 (2021) 9, L091502
PRD 103 (2021) 9, 094034
PRD 103 (2021) 7, 074024
PRC 99 (2019) 4, 044910

Thank you for listening!