A semi-infinite constraint generation algorithm for adjustable robust optimization

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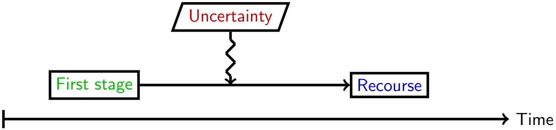
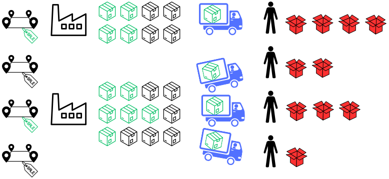
Adjustable robust optimization



Formulation

$$\begin{aligned} \min_{x\in\mathcal{X}} \quad c^\top x + \max_{\xi\in\Xi} \min_{y\in\mathbb{R}^{n_y}_+} \quad & 0 \\ \text{s.t.} \quad & Wy \geq h(\xi) - T(\xi)x \end{aligned} \tag{P}$$

With (P) feasible and bounded; $\mathcal{X}$ compact; Ξ polytope; $T(\cdot)$ and $h(\cdot)$ affine functions of ξ.



Adjustable robust optimization

Formulation

$$\min_{x\in\mathcal{X}} \quad c^\top x + \max_{\xi\in\Xi} \min_{y\in\mathbb{R}_+^{n_y}} \quad 0 \qquad (P)$$
$$\text{s.t.} \quad Wy \geq h(\xi) - T(\xi)x$$

With (P) feasible and bounded; $\mathcal{X}$ compact; Ξ polytope; $T(\cdot)$ and $h(\cdot)$ affine functions of ξ.

$$\begin{aligned}\min_{x\in\mathcal{X}} \quad & c^\top x + \max_{\xi\in\Xi} \min_{y\in\mathbb{R}_+^{n_y}} \quad f^\top y \\ & \qquad \text{s.t.} \quad Wy \geq h(\xi) - T(\xi)x\end{aligned} \quad \Leftrightarrow \quad \begin{aligned}\min_{x\in\mathcal{X},\theta\in\mathbb{R}} \quad & c^\top x + \theta + \max_{\xi\in\Xi} \min_{y\in\mathbb{R}_+^{n_y}} \quad 0 \\ & \qquad \text{s.t.} \quad f^\top y \leq \theta \\ & \qquad\qquad Wy \geq h(\xi) - T(\xi)x\end{aligned}$$

Adjustable robust optimization solution methods

Formulation

$$\min_{x\in\mathcal{X}} \quad c^\top x + \max_{\xi\in\Xi} \min_{y\in\mathbb{R}^{n_y}_+} \quad 0 \qquad\qquad (P)$$
$$\text{s.t.} \quad Wy \geq h(\xi) - T(\xi)x$$

With (P) feasible and bounded; $\mathcal{X}$ compact; Ξ polytope; $T(\cdot)$ and $h(\cdot)$ affine functions of ξ.

Approximate	Exact
$\min\limits_{\substack{x\in\mathcal{X}\\ y(\cdot):\Xi\to\mathbb{R}^{n_y}_+}} \quad c^\top x$ $\text{s.t.} \quad Wy(\xi) \geq h(\xi) - T(\xi)x \quad \forall \xi \in \Xi$	$\min\limits_{x\in\mathcal{X}} \quad c^\top x + \max\limits_{\xi\in\Xi} \min\limits_{y\in\mathbb{R}^{n_y}_+} \quad 0$ $\text{s.t.} \quad Wy \geq h(\xi) - T(\xi)x$
Ben-Tal et al. (2004), Chen and Zhang (2009) Bertsimas and Georghiou (2015) Bertsimas and Dunning (2016) Postek and den Hertog (2016) Subramanyam et al. (2018)	CG algorithm, Bertsimas et al. (2012) C&CG algorithm, Zeng and Zhao (2013) FM elimination, Zhen et al. (2018) Primal-Dual, Georghiou et al. (2020)

Monolithic reformulation

$$\min_{x \in \mathcal{X}} \quad c^\top x + \max_{\xi \in \Xi} \min_{y \in \mathbb{R}_+^{n_y}} \quad 0 \qquad (P)$$
$$\text{s.t.} \quad Wy \geq h(\xi) - T(\xi)x$$

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$$\text{s.t.} \quad Wy \geq h(\xi) - T(\xi)x$$

$$\Updownarrow \qquad \text{with an infeasible min} := +\infty$$

$$\min_{x\in\mathcal{X}} \quad c^\top x$$
$$\text{s.t.} \quad \max_{\xi\in\Xi} \min_{\substack{y\in\mathbb{R}_+^{n_y} \\ Wy\geq h(\xi)-T(\xi)x}} \quad 0 = 0$$

Monolithic reformulation

$$\min_{x \in \mathcal{X}} \quad c^\top x + \max_{\xi \in \Xi} \min_{y \in \mathbb{R}_+^{n_y}} \quad 0 \qquad (P)$$
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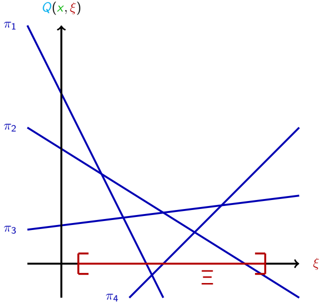
$$\min_{x \in \mathcal{X}} \quad c^\top x$$
$$\text{s.t.} \quad \max_{\xi \in \Xi} \min_{\substack{y \in \mathbb{R}_+^{n_y} \\ Wy \geq h(\xi) - T(\xi)x}} \quad 0 = 0$$

$$\Updownarrow \qquad \text{with } \Pi := \left\{ \pi \in \mathbb{R}_+^{m_y} \mid W^\top \pi \leq \mathbf{0} \right\}$$

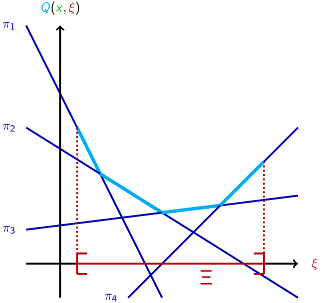
$$\min_{x \in \mathcal{X}} \quad c^\top x$$
$$\text{s.t.} \quad \max_{\xi \in \Xi} \max_{\pi \in \Pi} \pi^\top (h(\xi) - T(\xi)x) = 0$$

Monolithic reformulation



$$Q(x,\xi) := \max_{\pi\in\Pi} \pi^\top(h(\xi) - T(\xi)x)$$

Monolithic reformulation



$$Q(x, \xi) := \max_{\pi \in \Pi} \pi^\top (h(\xi) - T(\xi)x)$$

Monolithic reformulation

$$\begin{aligned} \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & \max_{\xi \in \Xi} \max_{\pi \in \Pi} \pi^\top (h(\xi) - T(\xi)x) = 0 \end{aligned}$$

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$$\Updownarrow$$

$$\begin{aligned} \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & \max_{\xi \in \mathrm{ext}(\Xi)} \max_{\pi \in \Pi} \pi^\top (h(\xi) - T(\xi)x) = 0 \end{aligned}$$

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CG and C&CG reformulations

$$\begin{aligned} \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & \max_{\pi \in \Pi} \pi^\top (h(\xi) - T(\xi)x) \leq 0 \quad \forall \xi \in \mathrm{ext}(\Xi) \end{aligned}$$

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Constraint generation (CG) reformulation:

$$\begin{aligned} \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & \pi^\top (h(\xi) - T(\xi)x) \leq 0 \quad \forall (\xi, \pi) \in \mathrm{ext}(\Xi) \times \mathrm{dir}(\Pi) \end{aligned}$$

Constraint-and-column generation (C&CG) reformulation:

$$\begin{aligned} \min_{\substack{x \in \mathcal{X} \\ y \in \mathbb{R}_+^{|\mathrm{ext}(\Xi)| \times n_y}}} \quad & c^\top x \\ \text{s.t.} \quad & W y^\xi \geq h(\xi) - T(\xi)x \quad \forall \xi \in \mathrm{ext}(\Xi) \end{aligned}$$

CG and C&CG reformulations

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$\swarrow\!\!\nearrow$ $\qquad$ $\nwarrow\!\!\searrow$

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$\Downarrow$

CG master problem:

$$\begin{aligned} \min_{x\in\mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & \pi^\top (h(\xi) - T(\xi)x) \leq 0 \quad \forall (\xi, \pi) \in \tilde{\Omega} \subseteq \operatorname{ext}(\Xi) \times \operatorname{dir}(\Pi) \end{aligned}$$

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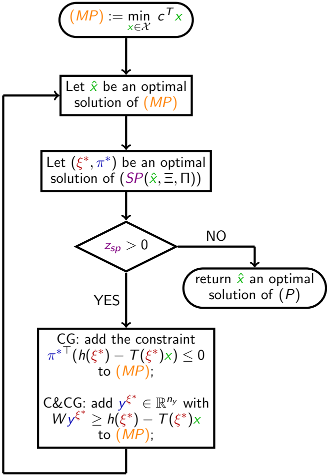
$\searrow$ $\swarrow$

$$z_{sp} := \max_{\xi\in\Xi} \max_{\pi\in\Pi} \pi^\top(h(\xi) - T(\xi)x) \qquad (SP(x,\Xi,\Pi))$$

CG and C&CG algorithms



Unified flow chart for CG and C&CG algorithms.

Semi-infinite constraint generation (SICG) reformulation

$$\min_{x\in\mathcal{X}} \quad c^\top x + \max_{\xi\in\Xi} \min_{y\in\mathbb{R}_+^{n_y}} \quad 0 \qquad (P)$$
$$\text{s.t.} \quad Wy \geq h(\xi) - T(\xi)x$$

$$\Updownarrow$$

$$\begin{aligned} \min_{x\in\mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & \max_{\pi\in\Pi} \pi^\top (h(\xi) - T(\xi)x) = 0 \quad \forall \xi \in \mathrm{ext}(\Xi) \end{aligned}$$

$\Swarrow$

CG reformulation

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Advantage: no new variables

$\Searrow$

C&CG reformulation

$$\begin{aligned} \min_{\substack{x\in\mathcal{X} \\ y\in\mathbb{R}_+^{|\mathrm{ext}(\Xi)|\times n_y}}} \quad & c^\top x \\ \text{s.t.} \quad & Wy^\xi \geq h(\xi) - T(\xi)x \quad \forall \xi \in \mathrm{ext}(\Xi) \end{aligned}$$

Advantage: better worst-case number of iterations

Semi-infinite constraint generation (SICG) reformulation

$$\begin{array}{llll} \min\limits_{x\in\mathcal{X}} & c^\top x + \max\limits_{\xi\in\Xi} \min\limits_{y\in\mathbb{R}^{n_y}_+} & 0 & (P) \\ & \text{s.t.} & Wy \geq h(\xi) - T(\xi)x & \end{array}$$

$$\Updownarrow$$

$$\begin{array}{ll} \min\limits_{x\in\mathcal{X}} & c^\top x \\ \text{s.t.} & \max\limits_{\pi\in\Pi} \pi^\top (h(\xi) - T(\xi)x) = 0 \quad \forall \xi \in \text{ext}(\Xi) \end{array}$$

$\swarrow$ $\searrow$

CG reformulation

$$\begin{array}{ll} \min\limits_{x\in\mathcal{X}} & c^\top x \\ \text{s.t.} & \pi^\top (h(\xi) - T(\xi)x) \leq 0 \quad \forall (\xi, \pi) \in \text{ext}(\Xi) \times \text{dir}(\Pi) \end{array}$$

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Advantage: better worst-case number of iterations

$\searrow$ $\swarrow$

Semi-infinite constraint generation (SICG) reformulation

$$\begin{array}{ll} \min\limits_{x\in\mathcal{X}} & c^\top x \\ \text{s.t.} & \pi^\top (h(\xi) - T(\xi)x) \leq 0 \quad \forall \xi \in \text{ext}(\Xi), \forall \pi \in \Pi \end{array}$$

Semi-infinite constraint generation (SICG) algorithm overview

SICG reformulation

$$\begin{aligned} \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & \pi^\top (h(\xi) - T(\xi)x) \le 0 \quad \forall \xi \in \mathrm{ext}(\Xi), \forall \pi \in \Pi \end{aligned}$$

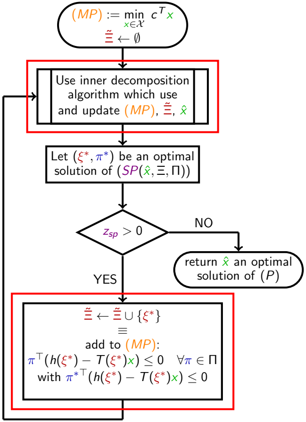
- Exponential (relax and generate semi-infinite constraints):

$$\begin{aligned} \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & \pi^\top (h(\xi) - T(\xi)x) \le 0 \qquad \forall \xi \in \tilde{\Xi} \subseteq \mathrm{ext}(\Xi), \forall \pi \in \Pi \end{aligned}$$

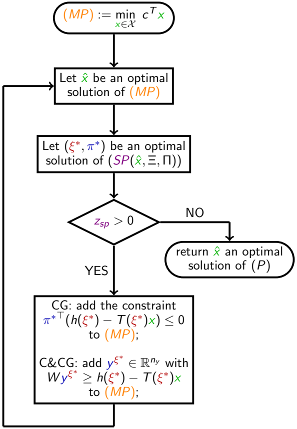
- Semi-infinite (relax further and generate constraints):

$$\begin{aligned} \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & \pi^\top (h(\xi) - T(\xi)x) \le 0 \qquad \forall \xi \in \tilde{\Xi} \subseteq \mathrm{ext}(\Xi), \forall \pi \in \tilde{\Pi}^\xi \subseteq \mathrm{dir}(\Pi) \end{aligned}$$

Semi-infinite constraint generation (SICG) algorithm overview



Flow chart for SICG outer decomposition algorithm.



Unified flow chart for CG and C&CG algorithms.

SICG inner decomposition algorithm

- For $\tilde{\Xi}, (\tilde{\Pi}^{\xi})_{\xi \in \tilde{\Xi}}$, let $\hat{x}$ be an optimal solution of:

$$\begin{aligned} \min_{x \in \mathcal{X}} \quad & c^\top x && \text{(RMP)} \\ \text{s.t.} \quad & \pi^\top (h(\xi) - T(\xi)x) \leq 0 && \forall \xi \in \tilde{\Xi} \subseteq \mathrm{ext}(\Xi), \forall \pi \in \tilde{\Pi}^{\xi} \subseteq \mathrm{dir}(\Pi) \end{aligned}$$

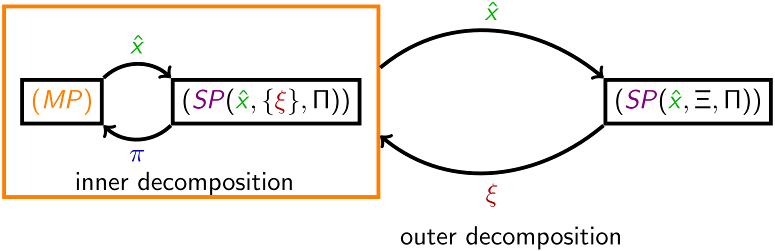
- Then $\hat{x}$ is an optimal solution of:

$$\begin{aligned} \min_{x \in \mathcal{X}} \quad & c^\top x && \text{(MP)} \\ \text{s.t.} \quad & \pi^\top (h(\xi) - T(\xi)x) \leq 0 && \forall \xi \in \tilde{\Xi} \subseteq \mathrm{ext}(\Xi), \forall \pi \in \Pi \end{aligned}$$

- iff for $\forall \xi \in \tilde{\Xi}$:

$$z^i_{sp} := \max_{\pi \in \Pi} \pi^\top (h(\xi) - T(\xi)\hat{x}) \leq 0 \qquad (SP(\hat{x}, \{\xi\}, \Pi))$$

SICG inner decomposition algorithm



Improving the inner decomposition algorithm

- Assume a violated cut is found for $\xi \in \tilde{\Xi}$ and $\hat{x}$:

$$\pi^{*\top}(h(\xi) - T(\xi)\hat{x}) > 0$$

- Does there exist a deeper cut? Solve:

$$\max_{\xi \in \Xi} \pi^{*\top}(h(\xi) - T(\xi)\hat{x}) \qquad (SP(\hat{x}, \Xi, \{\pi^*\}))$$

- Let ξ' be an optimal solution:

$$\pi^{*\top}(h(\xi') - T(\xi')x) \leq 0$$

  cuts off $\hat{x}$ and is (potentially) stronger than the initial cut. (Cut selection)
- Do we have $\xi' \in \tilde{\Xi}$? If not:

$$\tilde{\Xi} \leftarrow \tilde{\Xi} \cup \{\xi'\}$$

  adds a new semi-infinite constraint. (Pool expansion)

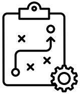
Facility Location Transportation (FLT) problem

$$\min_{\substack{x\in\{0,1\}^{|I|},\ z\in\mathbb{R}_+^{|I|} \\ z_i\leq c_i x_i\ \forall i\in I}} \sum_{i\in I} s_i x_i + a_i z_i + \max_{\substack{\gamma\in B_I^{\Gamma} \\ \delta\in B_J^{\Delta}}} \min_{\substack{y\in\mathbb{R}_+^{|I|\times|J|} \\ h\in\mathbb{R}_+^{|J|}}} \sum_{(i,j)\in I\times J} f_{ij} y_{ij} + \sum_{j\in J} \hat{f} h_j$$

$$\text{s.t.} \quad \sum_{j\in J} y_{ij} \leq z_i(1-\gamma_i) \qquad \forall i\in I$$

$$\sum_{i\in I} y_{ij} + h_j \geq \bar{d}_j + \hat{d}_j \delta_j \qquad \forall j\in J$$

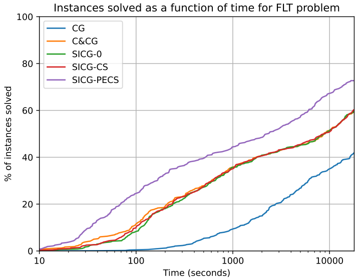
- Facilities $I \subset \mathbb{N}$, Customers $J \subset \mathbb{N}$
- $B_I^{\Gamma} := \{\gamma \in [0,1]^{|I|} \,|\, \sum_{i\in I} \gamma_i \leq \Gamma\}$
- $B_J^{\Delta} := \{\delta \in [0,1]^{|J|} \,|\, \sum_{j\in J} \delta_j \leq \Delta\}$

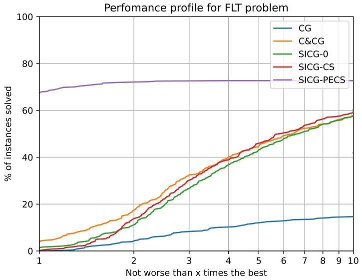


- $|I| = |J| = 30$
- 80 randomly generated instances[a]
- $\Gamma \in \{0, 6, 6.5\}$, $\Delta \in \{10, 10.5\}$
- 480 tests / algorithm

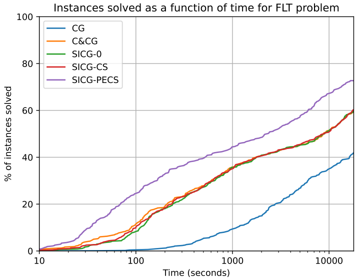
[a] Zeng and Zhao (2013)

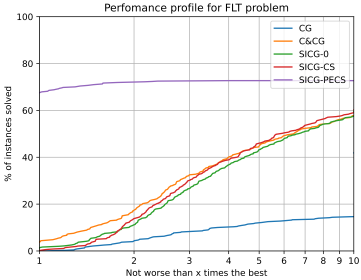
Results for FLT problem





Results for FLT problem





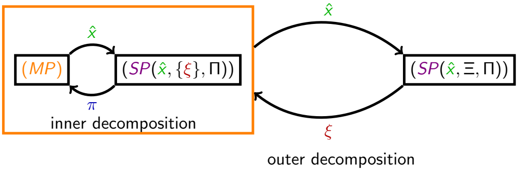
Method	Total Time [s]	% Time $SP(x, \Xi, \Pi)$	# $SP(x, \Xi, \Pi)$	# Ctr	$\lvert\tilde{\Xi}\rvert$
CG	4873.5	96.6%	255.9	254.9	-
C&CG	1192.5	98.6%	12.2	682.5	11.2
SICG-0	1203.5	87.1%	12.2	216.0	11.2
SICG-CS	1248.3	90.2%	10.7	145.5	9.7
SICG-PECS	583.9	74.7%	3.8	626.3	70.8

Detailed results over instances solved by all methods (201/480).

Conclusions and future directions

SICG reformulation

$$\begin{aligned} \min_{x \in \mathcal{X}} \quad & c^\top x \\ \text{s.t.} \quad & \pi^\top (h(\xi) - T(\xi)x) \le 0 \quad \forall \xi \in \mathrm{ext}(\Xi), \forall \pi \in \Pi \end{aligned}$$



Conclusions and future directions

SICG reformulation

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- Solution of separation problems/certifying feasibility optimality.
- Branch-and-cut for when master relaxations are difficult to solve.
- Adjustable robust optimization problems with decision-dependent uncertainty.

Thank you for your attention!

Patxi Flambard, Ayșe N Arslan, Boris Detienne. A semi-infinite constraint generation algorithm for two-stage robust optimization problems. 2025. ⟨hal-05184998⟩

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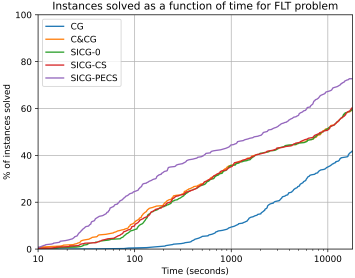
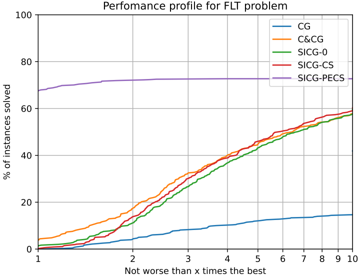
Results for FLT problem without CG





Method	Total Time [s]	% Time $SP(x, \Xi, \Pi)$	# $SP(x, \Xi, \Pi)$	# Ctr	$\lvert\tilde{\Xi}\rvert$
C&CG	2878.4	98.9%	15.2	868.0	14.2
SICG-0	2907.4	90.2%	15.2	302.3	14.2
SICG-CS	2935.8	92.7%	12.9	178.1	11.9
SICG-PECS	1118.9	79.0%	4.1	950.1	165.6

Detailed results over instances solved by all methods except CG (279/480).

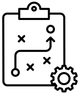
Network Design (ND) problem

$$\min_{x \in \mathbb{R}_+^{|A|}} \sum_{a \in A} c_a x_a + \max_{\substack{\gamma \in B_A^\Gamma \\ \delta \in B_K^\Delta}} \min_{\substack{y \in \mathbb{R}_{|A|}^{|K|} \\ y \geq \mathbf{0}}} 0$$

$$\text{s.t.} \quad \sum_{a \in \sigma_v^-} y_a^k - \sum_{a \in \sigma_v^+} y_a^k = \begin{cases} -\bar{d}_k - \hat{d}_k \delta_k & \textit{if } v = s_k \\ +\bar{d}_k + \hat{d}_k \delta_k & \textit{if } v = t_k \\ 0 & \textit{otherwise} \end{cases} \qquad \forall (v,k) \in V \times K$$

$$\sum_{k \in K} y_a^k \leq x_a (1 - \gamma_a) \qquad \forall a \in A$$

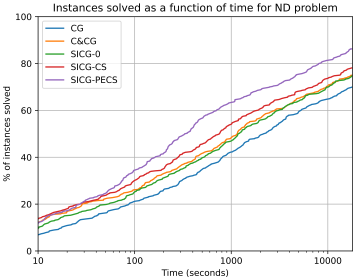
- Node $V \subseteq \mathbb{N}$, Arcs $A \subseteq V \times V$
- $B_A^\Gamma := \{\gamma \in [0,1]^{|A|} | \sum_{a \in A} \gamma_a \leq \Gamma\}$
- $B_K^\Delta := \{\delta \in [0,1]^{|K|} | \sum_{k \in K} \delta_k \leq \Delta\}$
- σ_v^+ and σ_v^- the set of outgoing and incoming arcs of a vertex $v \in V$
- s_k and t_k the start and end node of the commodity $k \in K$

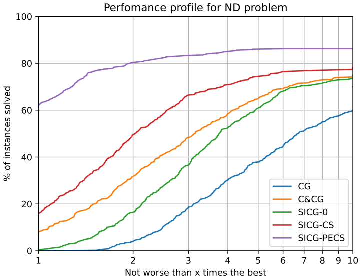


- 4 network instances from SNDlib[a]
- $|K| \in \{10, 20, 30, 40, 50\}$
- $\Gamma \in \{0, 1, 1.5\}$
- $\Delta \in \{1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5\}$
- 480 tests / algorithm

[a] (Orlowski et al., 2010)

Results for ND problem

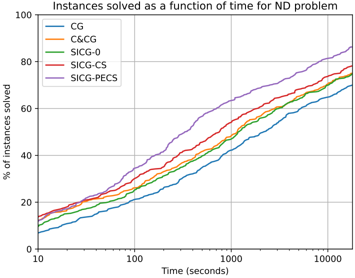


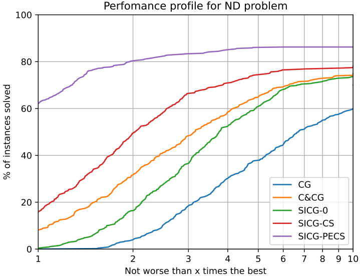


Method	Total Time [s]	% Time $SP(x, \Xi, \Pi)$	# $SP(x, \Xi, \Pi)$	# Ctr	$\lvert\tilde{\Xi}\rvert$
CG	2305.6	94.7%	132.0	131.0	-
C&CG	1345.9	83.0%	46.8	44640.2	45.8
SICG-0	1444.9	74.7%	47.2	724.9	46.2
SICG-CS	888.5	78.2%	27.8	300.0	26.8
SICG-PECS	442.0	47.6%	9.5	1655.9	298.0

Detailed results over instances solved by all methods (336 / 480).

Results[1] for ND problem without CG





Method	Total Time [s]	% Time $SP(x, \Xi, \Pi)$	# $SP(x, \Xi, \Pi)$	# Ctr	$\lvert\tilde{\Xi}\rvert$
C&CG	1868.8	83.6%	47.6	46554.4	46.6
SICG-0	1988.0	75.9%	48.0	745.5	47.0
SICG-CS	1302.4	79.3%	28.5	310.3	27.5
SICG-PECS	600.7	49.5%	9.7	1719.6	301.4

Detailed results over instances solved by all methods except CG (356 / 480).

SP reformulation as a MIP

$$\max_{\xi \in \Xi} \min_{y \in \mathbb{R}_+^{n_y}} \; 0 \quad \text{s.t.} \quad Wy \geq h(\xi) - T(\xi)x \qquad \leq 0$$

$\Updownarrow$ with α and N artificial variables and coefficients

$$\max_{\xi \in \Xi} \min_{\substack{\alpha \in \mathbb{R}_+^{n_\alpha} \\ y \in \mathbb{R}_+^{n_y}}} \; \mathbf{1}^\top \alpha \quad \text{s.t.} \quad Wy + N\alpha \geq h(\xi) - T(\xi)x \qquad \leq 0$$

KKT optimality conditions $\nearrow$

$$\max_{\xi \in \Xi} \max_{\substack{\alpha \in \mathbb{R}_+^{n_\alpha} \\ y \in \mathbb{R}_+^{n_y} \\ \pi \in \mathbb{R}_+^{m_y}}} \; \mathbf{1}^\top \alpha \qquad \leq 0$$
$$\begin{aligned} \text{s.t.} \quad & Wy + N\alpha \geq h(\xi) - T(\xi)x \\ & W^\top \pi \leq \mathbf{0} \\ & N^\top \pi \leq \mathbf{1} \\ & (Wy + N\alpha - h(\xi) + T(\xi)x) \circ \pi = \mathbf{0} \\ & (-W^\top \pi) \circ y = \mathbf{0} \\ & (\mathbf{1} - N^\top \pi) \circ \alpha = \mathbf{0} \end{aligned}$$

big-M method $\searrow$

$\searrow$ LP duality and $\Omega \subseteq \{0,1\}^{n_\omega}$ such that $\exists\, A \in \mathbb{R}^{n_\xi \times n_\omega},\ \mathrm{ext}(\Xi) \subseteq \{A\omega \mid \omega \in \Omega\} \subseteq \Xi$

$$\max_{\omega \in \Omega} \max_{\pi \in \mathbb{R}_+^{m_y}} \; \pi^\top (h(A\omega) - T(A\omega)x) \qquad \leq 0$$
$$\begin{aligned} \text{s.t.} \quad & W^\top \pi \leq \mathbf{0} \\ & N^\top \pi \leq \mathbf{1} \end{aligned}$$

$\nearrow$ McCormick envelope

MIP reformulation ≤ 0