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A continuous-time optimal control approach based on Pontryagin's maximum principle with chance constraints

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In this talk, we present a general method for solving the optimal control problem of trajectory planning for autonomous vehicles in continuous time with robustness to uncertainty. In a precedent work [1], we proposed a formulation as a non-linear optimisation problem with an integral cost function including chance constraints. Our present work uses Pontryagin's maximum principle to solve autonomous vehicles' reference trajectory planning problem in continuous-time. We obtain a system of ordinary differential equations (ODE) equivalent to minimising an objective function under constraints preserving the continuous-time formulation. We include the chance constraints in the system thanks to their deterministic equivalent formulation as a second-order conic programming problem [2] [3].

Let $\mathbf{z} \in \mathcal{Z}, \mathbf{u} \in \mathcal{U}$ be the state and control under constraints, respectively. For $t \in \mathbb{R}$ and $n, m \in \mathbb{N}$, let $z(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$. We have $\mathbf{z} = (z(t))_{t\in[0,t_f]}$ and $\mathbf{u} = (u(t))_{t\in[0,t_f]}$. The integral cost function to minimise is $J = \int_0^{t_f} l(z(t), u(t))dt$ with $l(z(t), u(t)) : \mathbb{R}^n \times \mathbb{R}^m \longmapsto \mathbb{R}$ the integrand and $t_f \in \mathbb{R}_*^+$ the final time. We denote $\frac{dz(t)}{dt} = f(z(t), u(t))$ with $f(z(t), u(t)) : \mathbb{R}^n \times \mathbb{R}^m \longmapsto \mathbb{R}^n$ the system dynamics and $\lambda(t) \in \mathbb{R}^n$ the costates associated with $z(t)$.

For $k \in \mathbb{N}$, we denote $S(z(t)) \in \mathbb{R}^k$ the deterministic equivalent formulation of the chance constraints over the state vector $z(t)$ [4]. Let $q \in \mathbb{N}$ the order of temporal derivative such that $\frac{d^q S(z(t))}{dt^q}$ depends explicitly on the state $z(t)$ and the command $u(t)$ i.e there exists a function $g(z(t), u(t)) : \mathbb{R}^n \times \mathbb{R}^m \longmapsto \mathbb{R}^k$ such as $\frac{d^q S(z(t))}{dt^q} = g(z(t), u(t))$. We include this term in the Hamiltonian with time-dependent Lagrange multipliers $\eta(t) \geq 0$. The Hamiltonian is then given by:

\begin{align}
H(z(t),u(t),\lambda(t), \eta(t)) = l(z(t),u(t)) + \lambda^T(t) f(z(t), u(t)) + \eta^T(t) \frac{d^qS(z(t))}{dt^q}
\end{align}

By applying Maximum Principle, we derive a two-point boundary value problem (TPBVP), which can be solved as an optimisation problem of finding the values for the costates at initial time, denoted $\lambda(0) \in \mathbb{R}^n$ [5]. To initialise the algorithm, we will use the solution from direct methods optimisation approach, for the discrete model, from our previous work [1].
We will evaluate the proposed approach based on numerical simulations of real scenarios from industrial applications of autonomous vehicle trajectories, highlighting its efficiency.

References
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