

S&P 500 optimal investing based on SDDP and implied calibrated ARIMA-GARCH model

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An implied distribution of the underlying asset price for the options expiration moment can be obtained from the market option prices [1]. However, exchange-traded options rarely expire more often than once a month. It is not enough for planning dynamic decisions in many cases. In [2] implied calibration of the dynamic ARMA(1,1)-GARCH(1,1) model using market prices of options of different expirations is considered. We use historical data from the S&P 500 index options for the numerical experiments. The parameters of the risk-neutral model are chosen in such a way that the sum of squares of the relative mistakes in option prices was minimal. During this process, model prices of options are calculated using the Monte-Carlo method. To solve the corresponding problem of optimal calibration, the Randomized Stochastic Projected Gradient Free Method [3] is used. The theorem regarding the conditions of convergence of the algorithm is given in this paper. We proved that the conditions of the theorem are satisfied for our case. Thus, the algorithm converges for our problem of ARMA(1,1)-GARCH(1,1) calibration. Then, the corresponding physical ARMA-GARCH version is obtained according to [4].

This physical ARMA(1,1)-GARCH(1,1) model is used to construct a scenario lattice as described in [5]. The problem of the position in S&P 500 management for the SDDP algorithm is formulated. The results of the historical simulation are also presented in the report.

References

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