Handling of long-term storage in multi-horizon stochastic programs

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ICSP, August 1, 2025

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Motivation: infrastructure-planning models

Goal: build or upgrade some infrastructure, for ex.:

- energy-supply system for a remote location
- power supply system for a ship

Constraints: the infrastructure has to be able fulfil its *purpose*:

- satisfying specified energy demands
- being able to propel the ship on a given route

Measure: normally minimizing overall costs

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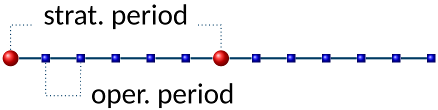
Measure: normally minimizing overall costs

Problem:

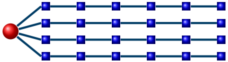
- Typically, long time horizons (many years)
- But the operational part needs time resolution of hours or finer.
- How to handle this in one model, esp. if we want uncertainty as well?

Multi-horizon models

Structure of the optimization problem:



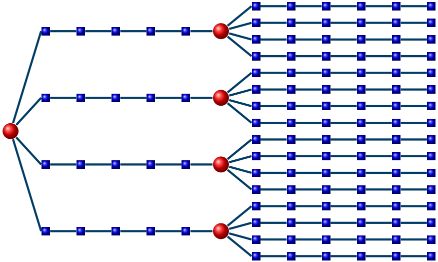
Robust solutions require use of several *operational profiles*:



- This works fine with 1 strategic period
- With multiple strat. periods, the size grows very fast...
  - used for for upgrades, ageing/degradation, etc.

Multi-horizon models

Normal approach:



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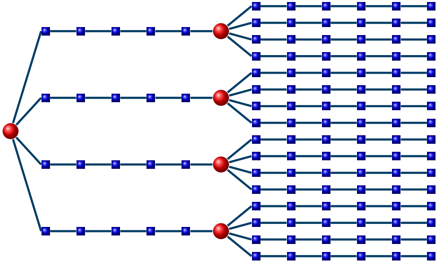
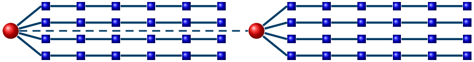


Multi-horizon approach:



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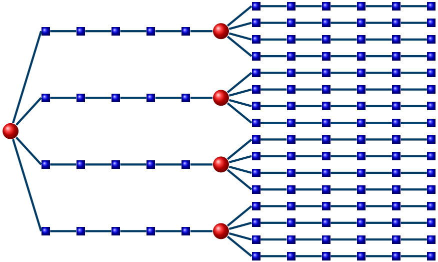
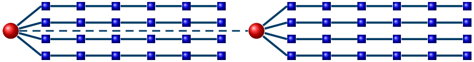


Multi-horizon approach:



- No connection between the last oper. period and the next strat. period.
- This stops the exponential growth in size.

Multi-horizon models

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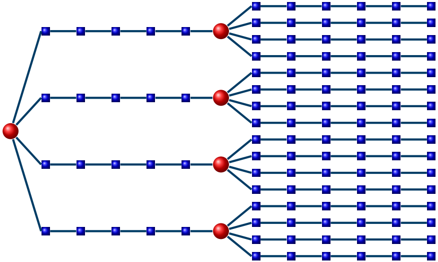

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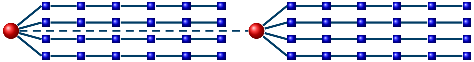
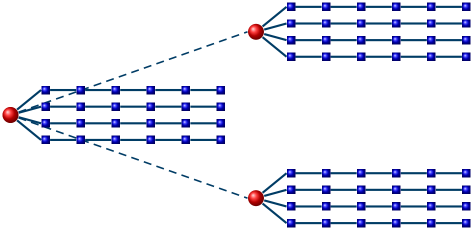


- No connection between the last oper. period and the next strat. period.
- This stops the exponential growth in size.
- It also allows the usage of short *representative periods/scenarios*
  - this again decreases the model size
  - very useful feature of the approach
  - can be useful even with 1 scenario

Multi-horizon models

The multi-horizon approach works also with *strategic uncertainty*:



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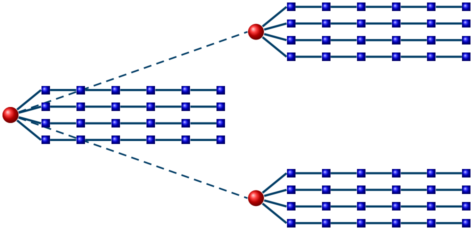
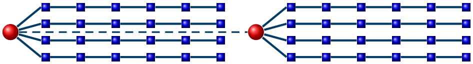


This is used for modelling of:

- step-by step building of the infrastructure
- ageing/degradation of the infrastructure
- technology changes (performance and/or price)
- regulatory changes

Multi-horizon vs. long-term storages

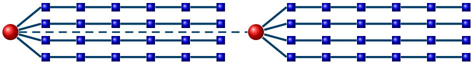


The disconnect of operational periods in different strategic periods makes it difficult to model *long-term storages*.

Full-length operational periods:

- *Initial inventory* level in a strat. period is equal to the weighted average of final inventory levels from scenarios in previous period.
- *Storage capacity* is such that it can handle inventory levels in all oper. scenarios.

Multi-horizon vs. long-term storages



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Representative scenarios as operational periods:

- None of the above holds.
- Handling of inventory levels has been shown in Strømholm and Rolfsen (2021).
  - total inventory-level *change* is a weighted sum of *scaled* per-scenario changes
- Determining the storage capacity is the main contribution of the presented paper.

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Storages with representative periods – example

oper. sc.	$\Delta T^{\mathrm{d}}_{sc}$	$M^{\mathrm{SC}}_{sn,sc}$	days	$W^{\mathrm{SC}}_{sn,sc}$	$\mathbf{inv}^{\Delta}_{i,sn,sc}$	$\mathbf{inv}^{\Delta\mathrm{SP}}_{i,sn,sc}$
winter	7	13	91	91/365	−10	−130
spring	7	13	91	91/365	15	195
summer	7	13	91	91/365	−5	−65
autumn	7	13	91	91/365	0	0
bad day	1	1	1	1/365	−5	−5
sum			365	1.0		−5

Storages with representative periods – example

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The overall inventory change is given by the weighted sum of $\mathbf{inv}^{\Delta}_{i,sn,sc}$.

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bad day	1	1	1	1/365	−5	−5
sum			365	1.0		−5

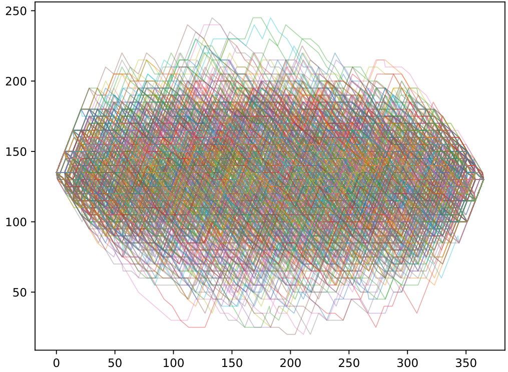
The overall inventory change is given by the weighted sum of $\mathbf{inv}^{\Delta}_{i,sn,sc}$.

But what about the required storage capacity?

Representative periods as random events

Selection of 1000 possible paths of inventory levels throughout a year

Inventory-level paths if the scenarios represented random events:



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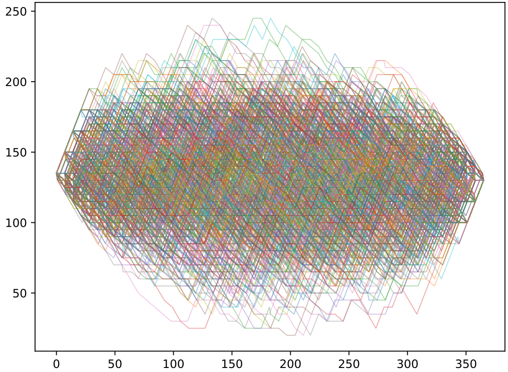

- But our scenarios are *ordered* (except for the ‘bad day’).
- $\rightarrow$ Add ordered *scenario groups* to the model.
- Inside each group, the scenarios occur in random order.

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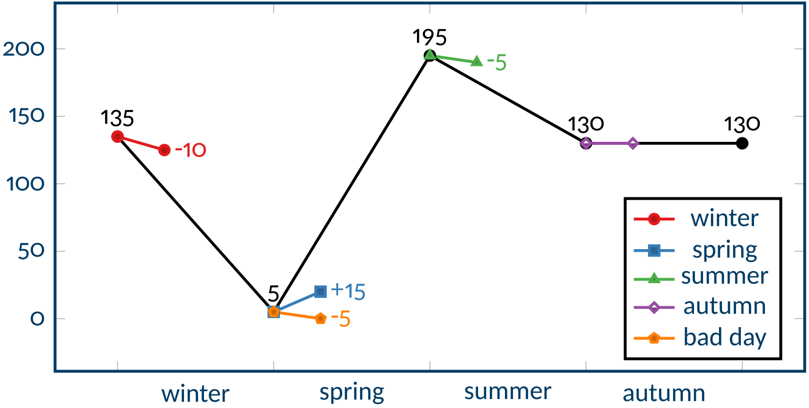
Representative periods as a sequence

With the ‘bad day’ scenario assigned to spring



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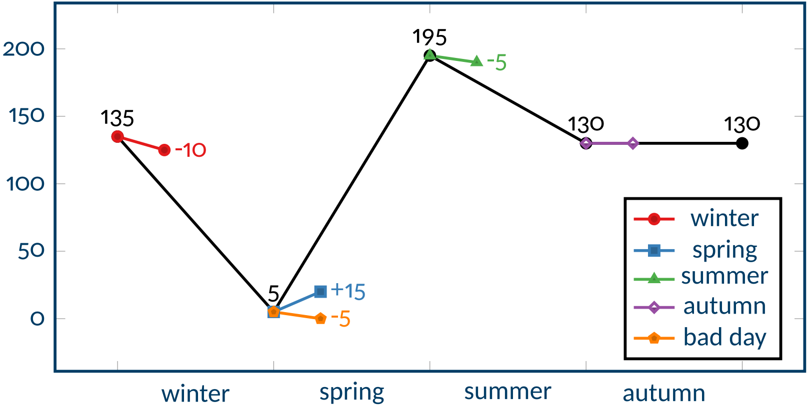

The ‘main path’ requires storage capacity of $195 - 5 = \mathbf{190}$.

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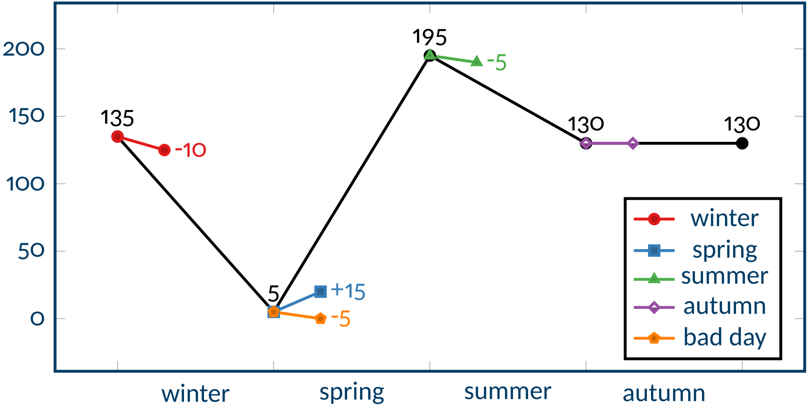
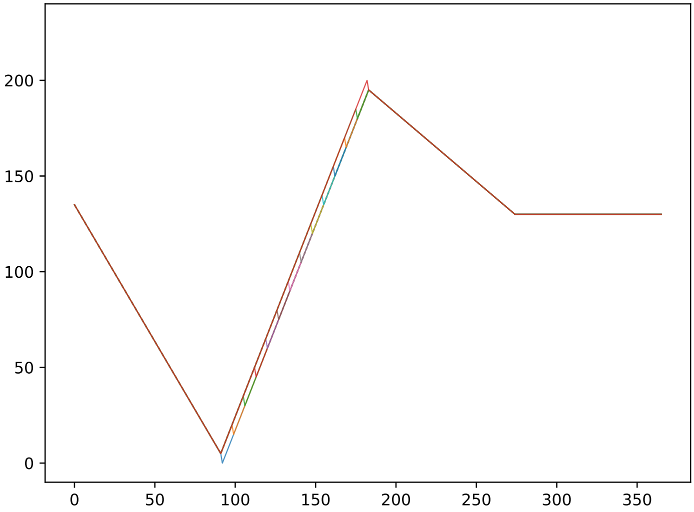


The ‘main path’ requires storage capacity of $195 - 5 =$ **190**.
To handle the ‘bad day’ at the start of spring, this increases to **195**.

Representative periods as a sequence

With the ‘bad day’ scenario assigned to spring – all paths



Representative periods as a sequence

With the 'bad day' scenario assigned to spring and extra summer scenarios

For summer, we replace the one week scen. by the following three:

oper. scen.	$M^{SC,G}_{sn,g,sc}$	$W^{SC,G}_{sn,g,sc}$	$W^{SC}_{sn,sc}$	$\mathbf{inv}^{\Delta}_{i,sn,sc}$	$\mathbf{inv}^{\Delta G}_{i,sn,g,sc}$
summer-1	6	6/13	42/365		
summer-2	6	6/13	42/365		
summer-3	1	1/13	7/365		
sum	13	1.0	91/365		

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summer-1	6	6/13	42/365	5	30
summer-2	6	6/13	42/365	− 12	−72
summer-3	1	1/13	7/365	− 23	−23
sum	13	1.0	91/365		−65

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summer-1	6	6/13	42/365	5	30
summer-2	6	6/13	42/365	− 12	−72
summer-3	1	1/13	7/365	− 23	−23
sum	13	1.0	91/365		−65

The total inventory change is the same as before – was $13 \times -5 = -65$.

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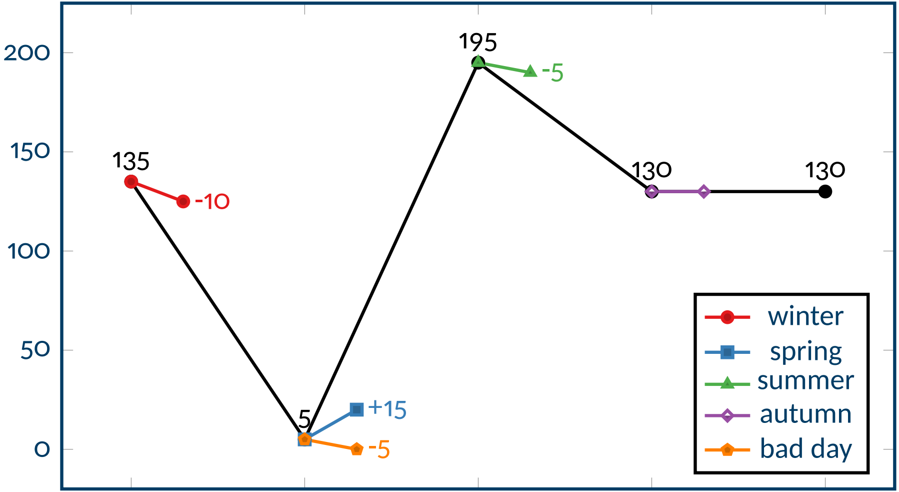

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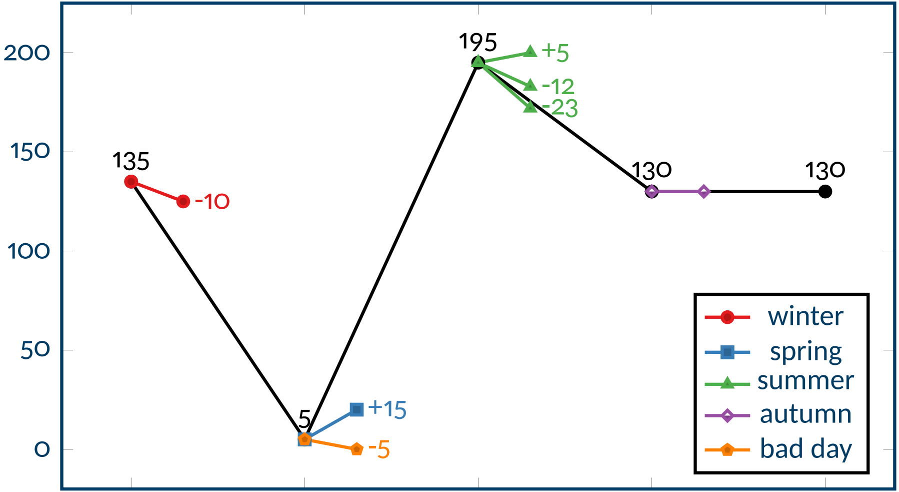

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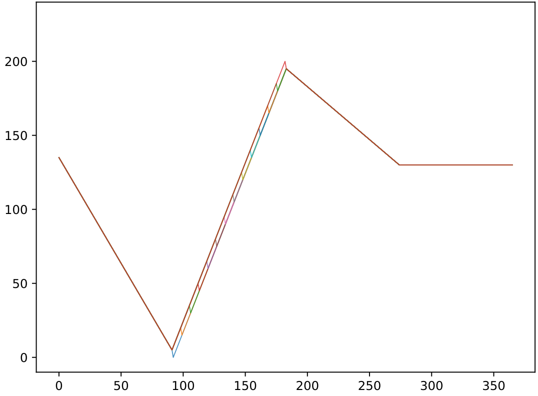

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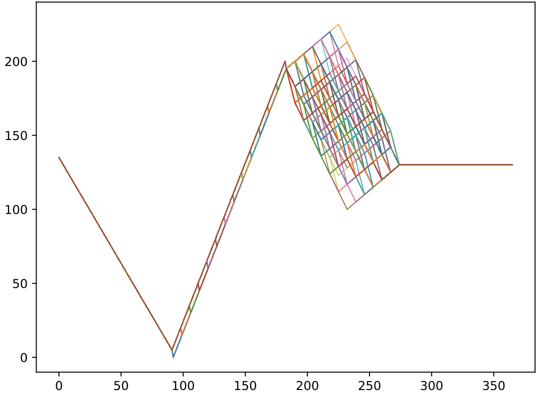

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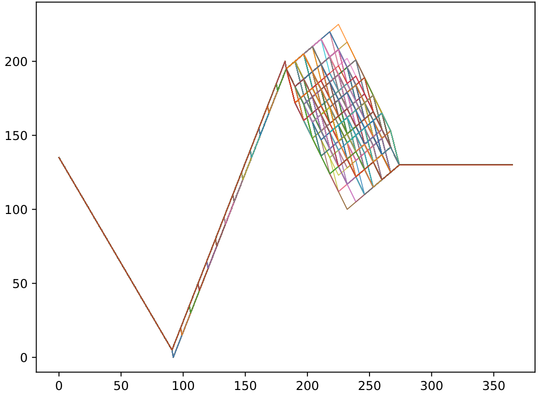
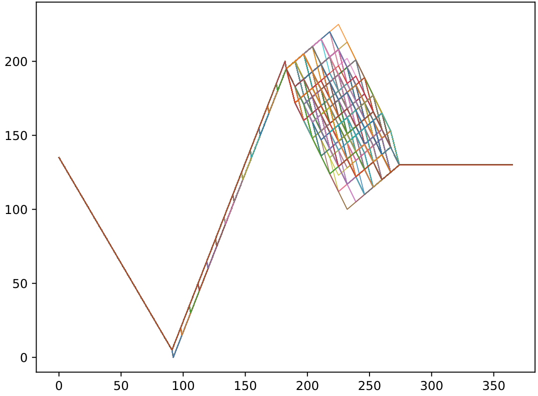


Worst case:

- Summer starts with $6 \times$ ‘summer-1’
- $\rightarrow$ Storage capacity increases by $6 \times 5 = 30$.

Representative periods as a sequence

With the ‘bad day’ scenario assigned to spring and extra summer scenarios



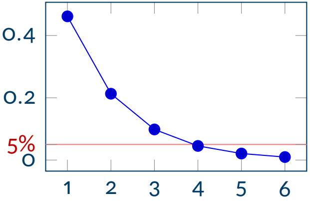
Worst case:

- Summer starts with 6 $\times$ ‘summer-1’
- $\rightarrow$ Storage capacity increases by $6 \times 5 = 30$.
- But this has probability of only $(\frac{6}{13})^6 \approx 1\%$ – do we want to include this?

Computing storage requirements

- For each scenario, we take into account repetitions up to given probability.
- By default, we set the limit to **5%**.

Computing storage requirements



- For each scenario, we take into account repetitions up to given probability.
- By default, we set the limit to **5%**.
- In our example, this means taking into account 3 repetitions of 'summer-1', since $(\frac{6}{13})^4 \approx 4.5\%$.
- This brings the highest inventory level to $195 + 3 \times 5 = 210$.
- Since min. level is zero, the required storage capacity is $210 - 0 = \mathbf{210}$.

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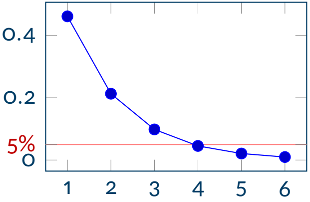
Computing storage requirements



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- This brings the highest inventory level to $195 + 3 \times 5 = 210$.
- Since min. level is zero, the required storage capacity is $210 - 0 = \mathbf{210}$.

- This assumes that 'summer-1' is never higher than +5.
- What if it first rises to +10, before falling to the (final) +5?
- This would increase the required storage cap. by +5 to **215**.
  - — have to be taken into account as well...

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Formulation and implementation

Mathematical formulation of the presented approach is in Kaut (2024).

- It is rather messy, requiring many sets and indices.
- Based on HyOpt, our infrastructure-optimization model.

The presented research has been done under projects 'LowEmission Centre' and 'Offlex', funded by the Research Council of Norway projects number 296207 and 319158.

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HyOpt implementation

- All the presented work is implemented in the open version of HyOpt, available from `https://gitlab.sintef.no/open-hyopt`.
- Test case from the paper is available there as well.
- All the presented features are available in the JSON input format for pyHyOpt, the python interface to HyOpt.
- It has been used in several projects at SINTEF.

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Bibliography

Michal Kaut. Handling of long-term storage in multi-horizon stochastic programs. *Computational Management Science*, 21(27), 2024. doi: 10.1007/s10287-024-00508-z.

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