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Orders and shuffles on nestohedra

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In the 1990s, two Hopf algebras were defined in terms of shuffles: shuffles of permutations for Malvenuto-Reutenauer Hopf algebras and shuffles of rooted binary trees for Loday-Ronco Hopf algebras. Permutations and binary trees label the vertices of the permutohedron and the associahedron respectively. The 1-skeletons of these polytopes correspond moreover to two well-known posets: the weak Bruhat order (for the permutohedron) and the Tamari lattice (for the associahedron). In 2002, Loday and Ronco established a "magic formula" linking the Tamari order to the shuffle product on binary trees and the weak Bruhat order to the shuffle product on permutations.

These shuffle products were then extended to the faces of the permutohedron (surjections) and of the associahedron (planar trees) by Burgunder-Ronco and Loday-Ronco respectively. Palacios and Ronco then extended the Tamari order and the Bruhat order to all faces of the corresponding polytopes. They extended the correspondance of Loday-Ronco between the shuffle product and the order on the vertices to all the faces of these polytopes. Carr and Devadoss introduced an order on the vertices of a subclass of polytopes called graph associahedra. Forcey and Ronco extended this order to the faces of a subclass of these polytopes.

In a recent work with Pierre-Louis Curien and Jovana Obradovic, we introduce a condition on families of nestohedra to get a shuffle product (more precisely, tridendriform products) on its faces. This condition includes the previous works and new examples. In this framework, we define an order on faces of the nestohedra and link it with the shuffle product.

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