# Colloque Physique mathématique et opérateurs pseudodifférentiels

# Reims, 5-7 novembre 2025







## Conférenciers

Zied Ammari Cristina Caraci Michele Correggi Marco Falconi Véronique Fischer Magnus Goffeng

Rennes Geneve Milan Milan Bath Lund Gerd Grubb Bernard Helffer Laurent Laflèche Nicolas Lerner Peter Pickl Didier Robert Robert Yuncken

Copenhague Nantes Lyon Sorbonne Tübingen Nantes Metz

#### Comité d'organisation

Sébastien Breteaux Guillaume Dollé Jérémy Faupin Victor Gayral Lisette Jager Christelle Marion

#### Comité scientifique

Laurent Amour Clotilde Fermanian Kammerer Frédéric Hérau

https://indico.math.cnrs.fr/event/13798/















	Wed Nov 5	Thu Nov 6	Fri Nov 7
09:00 – 09:45		M. Correggi	L. Lafleche
		Short break	Short break
09:55 – 10:40		G. Grubb	C. Caraci
		Coffee break	Coffee break
11:10 – 11:55		M. Falconi	D. Robert
		Lunch (univ. refectory )	Lunch (cold meal)
14:00 – 14:10	Opening (Lilian Joly)		
14:10 – 14:55	B. Helffer	Z. Ammari	
	Short break	Short break	
15:05 – 15:50	R. Yuncken	N. Lerner	
	Coffee break	Coffee break	
16:20 – 17:05	V. Fischer	P. Pickl	
	Short break		
17:15 – 18:00	M. Goffeng		
20:00 –		Conference dinner	

**Restaurant :** Le Continental 95 Place Drouet d'Erlon

# Workshop on mathematical physics and pseudodifferential operators

# **Celebrating Jean Nourrigat's 80th birthday**

### Titles and abstracts:

#### Zied AMMARI

The Kubo-Martin-Schwinger condition for Hamiltonian systems: Bose-Hubbard model

This talk explores the connection between quantum and classical equilibrium states through the Kubo–Martin–Schwinger (KMS) condition in the Bose–Hubbard model. On finite graphs, we study the semiclassical high-temperature limit, showing that quantum Gibbs (KMS) states converge to the Gibbs measures of the discrete nonlinear Schrödinger equation (DNLS). The result establishes that Wigner measures of quantum KMS states satisfy the classical KMS condition, linking quantum statistical mechanics to classical Hamiltonian dynamics.

#### **Cristina CARACI**

Euclidean field theories as limit of interacting Bose gases

Euclidean field theories have been extensively studied in the mathematical literature since the sixties, motivated by high-energy physics and statistical mechanics. Formally, they can be described by Gibbs measures associated with Euclidean action functionals over spaces of distributions.

In the latest years it has been shown how such theories emerge as high-density limit of interacting Bose gases at positive temperature, giving a rigorous derivation from a realistic microscopic model of statistical mechanics.

In this talk, I will present a recent result providing the derivation of such a field theory, hence of the invariant Gibbs measure, with a quartic local interaction in two dimensions as a limit of an inhomogeneous interacting Bose gas.

Based on joint work with Antti Knowles, Alessio Ranallo and Pedro Torres Giesteira.

#### Michele CORREGGI

Semiclassical Limit of Entropies and Free Energies

Entropy and free energy are central concepts in both statistical physics and information theory, with quantum and classical facets. In mathematics these concepts appear quite often in different contexts (dynamical systems, probability theory, von Neumann algebras, etc.). In this work, we study the von Neumann and Wehrl entropies from the point of view of semiclassical analysis. We first prove the semiclassical convergence of the von Neumann to the Wehrl entropy for quantum Gibbs states (thermal equilibrium), after a suitable renormalization has been taken into account. Then, we show that, in the same limit, the free energy functional defined with the Wehrl entropy  $\Gamma$ -converges to its classical counterpart, so implying convergence of the minima and the associated minimizers.

Joint work with Z. Ammari (Besancon), M. Falconi (Polimi), R. Gautier (Polimi & Rennes).

#### **Marco FALCONI**

Renormalization of the spin-boson model

In this talk I will present new results concerning the well-posedness of the spin-boson dynamics for arbitrarily singular form factors. The Hamiltonian operator is obtained through a self-energy and wave function renormalization procedure, and it lies on a non-Fock representation of the canonical commutation relations. This renormalization is non-trivial, thus overcoming the problem of triviality in Fock representation renormalization schemes.

Based on a joint work with B. Hinrichs and J. Valentín Martín.

## Véronique FISCHER

Quantization on filtered manifolds

In this talk, we will discuss a natural construction of a pseudo-differential calculus on filtered manifolds with symbols given in terms of the representations of the nilpotentization. In particular, we will see that the sub-calculus corresponding to poly-homogeneous symbols coincides with the calculus obtained from the groupoid approach.

#### **Magnus GOFFENG**

A quantitative version of the Helgason conjecture

The classical Helgason conjecture claimed that the Poisson transform isomorphically maps the space of distributions on the Furstenberg boundary G/P on a semisimple Lie group G to the space of joint eigenfunctions on its symmetric space G/K, as proven in the '70s by Kashiwara et al. In a related scenario, Knapp-Wallach studied the Poisson transform infinitesimally intertwining certain non-unitary principal series representations with discrete series representations. In this talk, we will show how the Heisenberg calculus give quantitative control on Sobolev mapping properties of Knapp-Wallach's Poisson transform for groups of real rank one. Along the way we prove that this Poisson transform is compatible with smooth functions on the Furstenberg compactification up to compact operators, a result that constituted the last missing piece in Julg's program for the Baum-Connes conjecture for subgroups of real rank one groups.

Joint work with Heiko Gimperlein.

#### **Gerd GRUBB**

Boundary problems for a class of operators containing the fractional Laplacian

A survey of recent results for 2a-order pseudodifferential operators generalizing  $(1-\Delta)^a$ , on a bounded domain in IR<sup>n</sup>. The emphasis will be on:

- 1- The precise solution space for the Dirichlet problem (involving the use of  $\mu$ -transmission spaces),
- 2- Evolution problems, where maximal L<sup>p</sup>-regularity has recently been obtained.

## **Bernard HELFFER**

Lower and Upper bounds for the magnetic lowest Dirichlet-to-Neumann eigenvalue in the strong magnetic limit

Inspired by some questions presented in a recent ArXiv preprint (version v1) by T. Chakradhar, K. Gittins, G. Habib and N. Peyerimhoff, we analyze their conjecture that the ground state energy of the magnetic Dirichlet-to-Neumann operator tends to  $+\infty$  as the magnetic field tends to  $+\infty$ . More precisely, we explore refined conjectures for general domains in IR<sup>2</sup> or IR<sup>3</sup> based on the previous analysis in the case of the half-plane and the disk.

This part is a work in collaboration with Ayman Kachmar and François Nicoleau. In connexion with old works on the magnetic Schrödinger operator with J. Nourrigat, we will also discuss recent results by Zhongwei Shen.

#### **Laurent LAFLECHE**

Commutator Estimates and Semiclassical Mean-Field Limits with Singular Potentials

Due to the singularity of the interactions, the derivation of the Vlasov-Poisson equation with Coulomb or gravitational interaction remains an open problem. There have been recent advances in the study of singular potentials, which now allow the treatment of square-integrable interaction potentials. On the quantum side, a key ingredient in the strategy is the use of propagation of uniform-in-hbar commutator estimates, which are the quantum analogue of the Sobolev regularity for classical phase space densities

In this talk, I will discuss the properties of these quantum Sobolev spaces, and show their applications to semiclassical mean-field limits, as well as to the study of ground states, such as the spectral projections on the negative eigenvalues of Schrödinger operators with non-smooth potentials. In particular, they allow us to obtain quantitative Weyl laws in phase space in strong topologies.

#### **Nicolas LERNER**

Singular Integrals Methods for Liouville Theorems

We shall begin our talk with the classical Liouville theorem for harmonic functions and, in the sequel, we shall review the various results obtained for the Navier-Stokes Stationary System for Incompressible Fluids, following in particular the works by D. Chae, G. Galdi, G. Seregin & W. Wang. We'll see that it is possible to obtain some precised regularity results involving several versions of the Wiener Algebra, stable at the same time under products and for the action of Fourier multipliers linked to singular integrals. This will allow us to describe some improvements of the classical results in terms of frequency localisation.

#### Peter PICKL

Derivation of the time-dependent Hartree equations for strongly interacting dense Fermionic systems

The time-dependent Hartree and Hartree-Fock equations provide effective mean-field descriptions for the dynamics of large Fermionic systems and play a fundamental role in many areas of physics. In the talk I will present a rigorous derivation of the time-dependent Hartree equations as the large-N limit of the microscopic Schrödinger dynamics of N Fermions confined to a volume of order one and interacting via strong pair potentials. A central step in our analysis is the implementation of time-dependent gauge transformations, which eliminate the dominant contribution from the interaction potential in both the Schrödinger and Hartree evolutions. In contrast to other results we will not have to assume semiclassicality of the gas.

#### **Didier ROBERT**

Spin-Orbit Interactions, Coherent States and Large Spin

The aim of the talk is to explain some results about perturbations of scalar Schrödinger Hamiltonians by spin matrices in irreducible representations of SU(2) in large dimension.

In the first part we consider propagation of coherent states (for orbit and spin) in the semi-classical regime for the time dependent Schrödinger equation.

In the second part we revisit an old result obtained by Hepp and Lieb (Annals of Physics, 1973) concerning the Dicke model and the super-radiance phenomenon for the interaction light-matter.

#### Robert YUNCKEN

The Helffer-Nourrigat Conjecture: Smoothness of solutions to linear PDEs

In 1979, Helffer and Nourrigat proved the Rockland conjecture, which proposes a sufficient and necessary condition for the hypo-ellipticity (smoothness of solutions) of a left-invariant differential operator on a graded nilpotent Lie group. The condition is stated in terms of the representation theory of the nilpotent group. Helffer and Nourrigat quickly realized that this conjecture can be vastly generalized to arbitrary polynomials in bracket-generating vector fields, extending Hörmander's famous sums-of-squares theorem. In this talk, I will present the Helffer-Nourrigat conjecture, as well as its solution using groupoid techniques.

Joint work with I. Androulidakis and O. Mohsen.