ADVANCES IN REPRESENTATION THEORY OF ALGEBRAS: GEOMETRY AND HOMOLOGY

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Edson Ribeiro Alvares (Universidade Federal do Paraná)

(m,n)-Quasitilted and (m,n)-Almost hereditary algebras

Abstract: We propose a generalisation to quasitilted and almost hereditary algebras, introducing two parameters that arise naturally, which we call (m,n)-quasitilted and (m,n)-almost hereditary. On the one hand, when dealing with the generalisation of quasitilted algebras, these parameters control the quantity of tilting and cotilting processes. On the other hand, for the generalisation of almost hereditary algebras, these parameters control projective and injective dimensions of the indecomposable modules. In both cases, the global dimension of the algebra is the sum of these parameters. Differently from case $m = n = 1$, not all (m,n)-almost hereditary are (m,n)-quasitilted algebras, although the converse holds true. In this talk, we present some results that can be obtained for these classes of algebras, including a discussion about one-point extension of $(m,1)$-almost hereditary algebras.

Raymundo Bautista (Universidad Nacional Autonoma de Mexico)

Tame-wild dichotomy for the category of filtered by standard modules for a quasi-hereditary algebra

Abstract: Let k be an algebraically closed field. By a theorem by Koenig, Külshammer and Ovsienko, the category of filtered by standard modules for a quasi hereditary k-lgebra is equivalent to the category of representations of a bocs. The bocs appearing in this theorem can be realized by certain differential tensor algebra

$$T = (T_A(M), \delta),$$

where A is kQ/I, with Q a finite quiver without oriented cycles and I is an admissible ideal of kQ. By means of reduction functors we can prove the tame-wild dichotomy for the category of representations of T and then for the category of filtered by standard modules.

Grzegorz Bobinski (Nicolaus Copernicus University)

Geometrically irreducible algebras

Abstract: A (finite-dimensional) algebra is called geometrically irreducible if, for each dimension vector, the corresponding module variety is irreducible. Our aim is to classify geometrically irreducible algebras. This a report on a work in progress with Jan Schröer (Bonn)

Thomas Brüstle (Bishop's University and Université de Sherbrooke)

On kissing numbers and a basis for Ext^1 ***over gentle algebras***

Abstract: This is a report on joint work with Guillaume Douville, Kaveh Mousavand, Hugh Thomas and Emine Yildirim. We like to point out that Yann Palu, Vincent Pilaud, and Pierre-Guy Plamondon informed us that they are working on similar ideas. Furthermore, we ought to mention that Ilke Canakci, David Pauksztello and Sibylle Schroll have obtained similar results in arXiv:1609.09688, however based on different techniques.

The basic question is to find a nice combinatorial description of a basis for $\mathrm{Ext}^1_A(X,Y)$ when A is a gentle algebra. In fact, one might wonder: Why do we do not yet have such a nice description, given that the work of Crawley-Boevey provides a nice combinatorial basis for $\mathrm{Hom}^1_A(X,Y)$, and Butler and Ringel gave thirty years ago a nice combinatorial description of the Auslander-Reiten translate, both results valid for string algebras? Well, it turns out that all experts found insurmountable difficulties when trying to put things together for the case of string algebras. And only recently, things start to look nice when one

restricts to gentle algebras: Canakci, Pauksztello and Schroll manage to describe Hom-spaces between indecomposable in the derived category, which yields the desired description of Ext^1.

Another approach follows the idea of adding some arrows to the quiver to make what we call a "fringed" quiver. This new, enlarged algebra is still gentle, but any any vertex of the original algebra has two incoming and two outgoing arrows - which makes the combinatorics of Butler and Ringel a lot more pleasant. Another pleasant surprise is that Crawley-Boevey's basis elements re-appear in the work of Thomas McConville as "kisses" in a certain situation. The idea is to count kisses to get the dimension of $\mathrm{Hom}(X, \tau Y)$, and show how to use kisses to calculate the dimension of $\mathrm{Ext}^1(X, Y)$.

Aslak Bakke Buan (NTNU)

From classical tilting to 2-term silting

Abstract: We give a generalization of the Brenner-Butler tilting theorem to 2-term silting complexes. As an application we give a characterization of algebras of small homological dimension (SHOD-algebras).

Ilke Canakci (Durham University)

Perfect matchings and Grassmannian cluster categories

Abstract: Joint work with Alastair KING and Matthew PRESSLAND.

A categorification of Grassmannian cluster algebras was introduced by Jensen-King-Su, and the so-called *dimer algebras* were described as endomorphism algebras of cluster-tilting objects in this category by Baur-King-Marsh. Certain cluster variables, namely the *twisted Plücker coordinates*, in Grassmannian cluster algebras have been expressed by Marsh-Scott as dimer partition functions, i.e. as sum over perfect matchings. The twisted Plücker coordinates can also be computed by the Caldero-Chapoton cluster character (CC) formula, given in this generality by Fu-Keller. In joint work with Alastair King and Matt Pressland, we relate the Marsh-Scott formula to the CC formula by defining perfect matching modules over dimer algebras associated to perfect matchings appearing in the Marsh-Scott formula.

Claudia Chaio (Universidad Nacional de Mar del Plata)

On the representation type of a category of complexes of fixed size and the strong global dimension

Abstract: Joint work with Alfredo González Chaio and Isabel Pratti.

Let A be a finite dimensional algebra over an algebraically closed field. We denote by $\mathrm{mod}\, A$ the finitely generated module category and by $\mathrm{proj}\, A$ the full subcategory of $\mathrm{mod}\, A$ of the finitely generated projective A-modules.

In [BSZ], the authors defined and studied the categories $\mathbf{C_n}(\mathrm{proj}\, A)$ of complexes of fixed size. We say that a category $\mathbf{C_n}(\mathrm{proj}\, A)$ is representation-finite if there are a finite number of classes of isomorphic indecomposable complexes in $\mathbf{C_n}(\mathrm{proj}\, A)$.

The concept of strong global dimension has been introduced by Skowroński in [S]. Given a complex $X \in \mathbf{K}^b(\mathrm{proy}\, A)$,

$$\cdots \to 0 \to 0 \to X^r \to X^{r+1} \to \cdots \to X^{s-1} \to X^s \to 0 \to 0 \cdots$$

with $X^r \neq 0$ and $X^s \neq 0$, we define the length of X as follows: $\ell(X) = s - r$. The strong global dimension of A is defined as:

$$\mathrm{s.gl.dim}\, A = \ \mathrm{Sup}\, \{\ell(X) \mid X \in K^b(\mathrm{proj}\ A) \ \text{ is indecomposable}\}.$$

In this work, we prove that if A is an algebra with $s.gl.dim\, A = \eta < \infty$ then, for each $n \geqslant 2$, $\mathbf{C_n}(\mathrm{proj}\, A)$ is representation-finite if and only if $\mathbf{C_{J+1}}(\mathrm{proj}\, A)$ is representation-finite.

We also study implications which allow us to decide if for some positive integer m the category $\mathbf{C_m}(\mathrm{proj}\, A)$ is representation-infinite taking into account different notions.

On the other hand, we consider some piecewise hereditary algebras A and we prove how to read their strong global dimension taking into account their ordinary quiver with relations. Moreover, we also show the entries of the complexes of maximal length in such cases.

References

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José Antonio de la Peña (Universidad Autonoma de Mexico)
Lehmer conjecture and algebras

Abstract: Let $p(z) = (z - a_1)...(z - a_n)$ be a monic integer polynomial of degree n. The Mahler measure $M(p)$ of p is defined as the product of the absolute value of the zeros of p that lie outside the unit circle. Kronecker had proved that $M(p) = 1$ implies that p is a product of cyclotomic polynomials and some x^m (we shall say that p is of *cyclotomic type*). For non-cyclotomic polynomials, it is clear that $M(p) > 1$. Lehmer exhibited the polynomial,

$$L(z) = z^{10} + z^9 - z^7 - z^6 - z^5 - z^4 - z^3 + z + 1,$$

whose largest zero is $\mu_0 = 1.17628...$, and moreover $M(L) = \mu_0$. The conjecture states that μ_0 is the smallest Mahler measure not 1.

Given an algebra A, the Mahler measure of A is the Mahler measure of its Coxeter matrix Φ_A. The following theorem is to be understood as a representation theory variant of Lehmer's problem.

Theorem: *Let A be an accessible algebra not of cyclotomic type. Then there is a convex subcategory B of A satisfying the following properties:*
(a) B is minimal not of cyclotomic type, that is, if C is any proper convex subcategory of B, then C is of cyclotomic type;
(b) the Mahler measure of B is $M(\chi_B) \geqslant \mu_0$.

Proposition. *Let C be a strongly simply connected algebra. The following are equivalent:*
(a) C *is critical of non-cyclotomic type;*
(b) C *is derived equivalent to a minimal wild hereditary algebra.*

Karin Erdmann (University of Oxford)
Algebras of generalized dihedral type

Abstract: This is part of a joint project with Andrzej Skowroński, on the representation theory of tame symmetric algebras, generalizing tame blocks of group algebras of finite groups. An algebra is of generalized dihedral type if it is tame, symmetric and indecomposable (and not simple), and if its stable Auslander-Reitenn quiver has the following components: (i) stable tubes of ranks 1 and 3, (ii) non-periodic components of the form ZA_∞^∞ or $Z\tilde{A}_n$ and if in addition, Ω fixes each stable tube of rank 3. The aim of this talk is first to present a complete description: an algebra (over an algebraically closed field) is of generalized dihedral type if and only if it is Morita equivalent to an idempotent algebra of a biserial weighted surface algebra, for a distinguished idempotent. Furthermore, we explain that any Brauer graph algebra is an idempotent algebra of some biserial weighted surface algebra.

Ana Garcia Elsener (Universidad Nacional de Mar del Plata)
Short exact sequences and skein relations

Abstract: We study short exact sequences over jacobian algebras arising from surfaces with punctures, establishing a connection with cluster algebras arising from surfaces and punctured skein relations. This is a work in progress.

Victoria Guazzelli (Universidad Nacional de Mar del Plata)
On tilting theory and the radical

Abstract: Joint work with Claudia Chaio.

Let A be a finite dimensional algebra over an algebraically closed field, and $\operatorname{mod} A$ the category of finitely generated left A-modules. We denote by $\operatorname{Re}(\operatorname{mod} A)$ the Jacobson radical of $\operatorname{mod} A$.

M. Auslander characterized the algebras of finite representation type through the radical of its module category. More precisely, the author proved that an artin algebra A is representation-finite if and only if there is a positive integer n such that $\mathrm{Re}^n(\mathrm{mod}\, A) = 0$.

We consider A a representation-finite algebra, T a separating tilting A-module and $B = \mathrm{End}_A T$. The aim of this work is to establish a relation between the nilpotency of $\mathrm{Re}(\mathrm{mod}\, A)$ and the nilpotency of $\mathrm{Re}(\mathrm{mod}\, B)$. In particular, if r_A and r_B denote the nilpotency indices of $\mathrm{Re}(\mathrm{mod}\, A)$ and $\mathrm{Re}(\mathrm{mod}\, B)$, respectively, then $r_A \leqslant r_B$.

We also obtain some information related to the nilpotency of $\mathrm{Re}(\mathrm{mod}\, A)$ when A is an iterated tilted algebra of Dynkin type.

Osamu Iyama (Nagoya University)

Auslander-Gorenstein algebras from Serre-formal algebras via replication

Abstract: This is a joint work with Aaron Chan, and Rene Marczinzik.

We introduce a new family of algebras, called Serre-formal algebras. They are Iwanaga- Gorenstein algebras for which applying any power of the Serre functor on any indecomposable projective module remains a stalk complex. Typical examples are given by (higher) hereditary algebras and selfinjective algebras; it turns out that other interesting algebras such as (higher) canonical algebras are also Serre-formal. Starting from a Serre-formal algebra, we consider a series of algebras - called the replicated algebras - given by certain subquotients of its repetitive algebra. We calculate the selfinjective dimension and dominant dimension of all such replicated algebras and determine which of them are minimal Auslander-Gorenstein, i.e. when the two dimensions are finite and equal to each other. In particular, we show that there exist infinitely many minimal Auslander-Gorenstien algebras in such a series if, and only if, the Serre-formal algebra is twisted fractionally Calabi-Yau. We apply these results to a construction of algebras from Yamagata, called SGC extensions, given by iteratively taking the endomorphism ring of the smallest generator-cogenerator. We give a sufficient condition so that the SGC extensions and replicated algebras coincide. Consequently, in such a case, we obtain explicit formulae for the self-injective dimension and dominant dimension of the SGC extension algebras.

Gustavo Jasso (Universität Bonn)

Triangulations of cyclic apeirotopes and higher dimensional tilting combinatorics

Abstract: This a report of joint work with Julian Külshammer. It is well known that there is a bijective correspondence between triangulations of the regular polygon with $n+2$ vertices and isomorphism classes of basic tilting modules over the path algebra of the quiver of type A_n with linear orientation. Similarly, by work of Holm and Jørgensen for example, triangulations of the cyclic apeirogon are in bijective correspondence with tilting subcategories of the category of finite dimensional representations of the quiver of type A_∞^∞ with linear orientation. The purpose of this talk is to describe a bijection between triangulations of the d-dimensional cyclic apeirotope and certain tilting subcategories of the category of finite dimensional representations over a certain d-dimensional analogue of the quiver of type A_∞^∞ with linear orientation, which we also introduce. This bijection makes use of an earlier bijection, established by Oppermann and Thomas, between triangulations of a d-dimensional cyclic polytope on $n+2d$ vertices and isomorphism classes of certain basic tilting modules over the d-Auslander algebra of type A_n.

Alicja Jaworska-Pastuszak (Nicolaus Copernicus University)

Selfinjective algebras without short cycles of indecomposable modules

Abstract: This is report on a joint work with A. Skowroński.

Let A be a basic indecomposable finite dimensional K-algebra over a field K. Following Reiten, Skowroński and Smalø, a short cycle in the category $\mathrm{mod}\, A$ of finite dimensional right A-modules is a sequence

$$X \xrightarrow{f} Y \xrightarrow{g} X$$

of two nonzero nonisomorphisms between indecomposable modules X and Y. In this lecture we are concerned with the problem of describing the structure of selfinjective algebras without short cycles in their module category.

By a result of Happel and Liu, every algebra A having no short cycle in $\mathrm{mod}\, A$ is of finite representation type. The class of selfinjective algebras of finite representation type has been intensively studied since the early 80's when Riedtmann and Todorov described the shape of the stable Auslander-Reiten quiver of such algebras. Moreover, Riedtmann in a series of articles presented the classification of selfinjective algebras of finite representation type over an algebraically closed field. For an arbitrary field, the classification is not known yet but it is hoped that selfinjective algebras of finite representation type are strongly related to selfinjective algebras of Dynkin type. Recall that, for a Dynkin quiver Δ of type $\mathbb{A}_n$ ($n \geqslant 1$), $\mathbb{B}_n$ ($n \geqslant 2$), $\mathbb{C}_n$ ($n \geqslant 3$), $\mathbb{D}_n$ ($n \geqslant 4$), $\mathbb{E}_6$, $\mathbb{E}_7$, $\mathbb{E}_8$, $\mathbb{F}_4$ or $\mathbb{G}_2$, a selfinjective algebra of type Δ is an orbit algebra $\widehat{B}/G$, where $\widehat{B}$ is the repetitive category of a tilted algebra B of type Δ and G is an admissible group of automorphisms of $\widehat{B}$. Using the theory of selfinjective algebras with deforming ideals, developed by Skowroński and Yamagata, we prove that a selfinjective algebra A has no short cycle in $\operatorname{mod} A$ if and only if it is isomorphic to a selfinjective algebra $\widehat{B}/G$ of Dynkin type with $G = (\varphi\nu_{\widehat{B}}^2)$, where $\nu_{\widehat{B}}$ is the Nakayama automorphism of $\widehat{B}$ and φ is a strictly positive automorphism of $\widehat{B}$.

Otto Kerner (H. Heine Universität)

Thick subcategories of the stable category of modules over the exterior algebra

Abstract: Let $R = R(V)$ be the exterior algebra of a finite dimensional vector space V over an algebraically closed field. R is a graded algebra in the obvious way. We consider the category $\underline{\operatorname{mod}}\, R$ of finite dimensional $\mathbb{Z}$-graded R-modules (with homomorphisms of degree zero). The corresponding stable category $\underline{\operatorname{mod}}\, R$ is a triangulated category and is equivalent as a triangulated category to the derived category of coherent sheaves over the corresponding projective space $P(V)$. We describe the thick subcategories of $\underline{\operatorname{mod}}\, R$, generated by R-modules of complexity one.

This is joint work with Dan Zacharia.

Justyna Kosakowska (Nicolaus Copernicus University)

Generic extensions and Hall polynomials for invariant subspaces of nilpotent linear operators

Abstract: Let k be an arbitrary field. For a partition $\alpha = (\alpha_1, \ldots, \alpha_n)$ we consider the nilpotent $k[x]$-module $N_\alpha = N_\alpha(k) = \bigoplus_{i=1}^n k[x]/x^i$ and call it the nilpotent linear operator of type α. We are interested in the category $\mathcal{S}_1 = \mathcal{S}_1(k)$ of all short exact sequences

$$0 \to N_\alpha \to N_\beta \to N_\gamma \to 0 \tag{1}$$

where α, β, γ are partitions and $\alpha_1 \leqslant 1$.

We discuss the existence of Hall polynomials and generic extensions in the category $\mathcal{S}_1$.

This is a report about a joint project with M. Kaniecki and S. Kasjan from Nicolaus Copernicus University.

Helmut Lenzing (Universität Paderborn)

Weighted projective lines and Riemann surfaces

Abstract: We work over the base field of complex numbers and address the relationship between weighted projective lines and compact Riemann surfaces. To avoid case distinctions, we discard weighted projective lines with only one or two weights.

Theorem. For each weighted projective line X (as above) there exists a compact Riemann surface Y together with a finite group G of automorphisms of Y such that the category $\mathrm{coh}(X)$ of coherent sheaves on X is equivalent to the skew group category $(\mathrm{coh}(Y)[G]$ of $\mathrm{coh}(Y)$, that is the category of G-equivariant coherent sheaves on Y.

The theorem thus links the representation theory of canonical algebras (whose module categories are tilting equivalent to categories of the form $\mathrm{coh}(X)$) to the theory of (compact) Riemann surfaces.

This has been known for some time - by work with Geigle - in the domestic case (Euler characteristic of X greater 0); in the tubular case (Euler characteristic of X equal to 0) it was recently established by [Chen-Chen-Zhou, 2015].

For the wild case (Euler characteristic less than 0) this will be derived from classical results on discrete group actions on the hyperbolic plane. A key role in the proof is taken by an analysis of the orbifold fundamental group of X.

Shiping Liu (Université de Sherbrooke)

The derived Auslander-Reiten components of an algebra with radical squared zero

Abstract: Joint work with R. Bautista.

Let Λ be a connected elementary locally bounded linear category over a field with radical squared zero, whose ordinary quiver is Q_Λ. Combining the Koszul equivalence with the derived push-down functor of a minimal gradable Galois covering $\tilde{Q}$ of Q_Λ, we obtain a Galois covering functor from $D^b(\text{rep}^-(\tilde{Q}))$ to $D^b(\text{mod}\Lambda)$, where $\text{rep}^-(\tilde{Q})$ denotes the category of finitely co-presented representations of $\tilde{Q}$. In this way, the indecomposable complexes in $D^b(\text{mod}\Lambda)$ are described in terms of the indecomposable representations in $\text{rep}^-(\tilde{Q})$. The shape of an Auslander-Reiten component of $D^b(\text{mod}\Lambda)$ is determined according to the following situations: it contains simple complexes; it contains non-simple non-perfect complexes; it contains only non-simple perfect complexes. Finally, $D^b(\text{mod}\Lambda)$ has only finitely many Auslander-Reiten components precisely when Q is of Dynkin type or of type $\tilde{\mathbb{A}}_n$ with the number of clockwise oriented arrows different from the number of counter-clockwise oriented arrows; and in these cases, all Auslander-Reiten components are explicitly described.

Andrzej Mróz (Nicolaus Copernicus University)

Coxeter energy of graphs and algebras

Abstract: The Coxeter spectrum of a finite-dimensional algebra Λ together with its "measures" like spectral radius or Mahler measure were studied by many authors. It has been shown that these derived equivalence invariants reflect several ring-theoretic, homological and categorical properties of Λ; moreover, their study establishes nice links from representation theory of algebras to graph theory, number theory and Lie theory. We consider an alternative spectral invariant for a finite-dimensional algebra of finite global dimension $\Lambda =: \mathcal{G}$ or a "Cox-regular" (bi)graph $\Delta =: \mathcal{G}$, defined to be the sum of absolute values of all eigenvalues of the Coxeter matrix $\text{Cox}_\mathcal{G} \in \text{Gl}_n(\mathbb{Z})$ associated with $\mathcal{G}$. This quantity, called the Coxeter energy of $\mathcal{G}$, is inspired by concepts from (classical) spectral graph theory having their origins in computational chemistry.

We will discuss basic properties of the Coxeter energy and its modified variants, provide some classification results, and point out certain relationship with representation type of algebras. Moreover (if it is enough time), we plan to outline few related observations of a number-theoretic flavour which are close to some problems considered by Lakatos, Lenzing and de la Peña, connected to Salem trees and general properties of the Coxeter spectrum.

Yann Palu (Université de Picardie - Jules Verne)

The non-kissing complex and tau-tilting over gentle algebras

Abstract: Gentle algebras form a class of algebras, described in terms of quivers and relations, whose representations are well understood and can be described combinatorially. In this talk, I will introduce, following T. McConville, a combinatorial notion called "non-kissing". I will then explain how the non-kissing condition can be interpreted in terms of the representation theory of gentle algebras. This is joint work with V. Pilaud and P-G. Plamondon.

Maria Julia Redondo (Universidad Nacional del Sur)

Cohomology of partial smash products

Abstract: We define the partial group cohomology as the right derived functor of the functor of partial invariants, we relate this cohomology with partial derivations and with the partial augmentation ideal. Finally we show that there exists a Grothendieck spectral sequence relating cohomology of partial smash algebras with partial group cohomology and algebra cohomology.

Manuel Saorín (Universidad de Murcia)
Silting theory revisited

Abstract: (based on joint work with Alexandra Zvonareva)

Silting sets in triangulated categories, introduced by Keller and Vossieck ([5]), have had in recent times an important impact in the representation theory of finite dimensional algebras. They are a generalization of tilting objects and, due to a paper of Aihara and Iyama ([1]), unlike tilting objects, they are shown to be apt to mutation. As a final point of the contribution of several authors (see [3], [4], [7],...), a result of König and Yang ([3]) gave, for a finite dimensional algebra Λ, bijections between: i) Silting objects on $\mathcal{K}^b(\text{proj}-\Lambda)$ (up to add-equivalence); ii) Bounded co-t-structures in $\mathcal{K}^b(\text{proj}-\Lambda)$; iii) Bounded t-structures in $\mathcal{D}^b(\text{mod}-\Lambda)$, and iv) Simple minded collections in $\mathcal{D}^b(\text{mod}-\Lambda)$. Moreover, these bijections commute with the corresponding mutations.

In this talk we will show that, except of the case of simple minded collections which is genuine of the context of finite dimensional algebras, the other bijections are particular instances of more general situations. All triangulated categories considered in the talk are supposed to have split idempotents. Let us say that a set $\mathcal{T}$ in the triangulated category $\mathcal{D}$ is *nonpositive* when $\text{Hom}_{\mathcal{D}}(T, T'[i]) = 0$, for all $T, T' \in \mathcal{T}$ and all integers $i > 0$. We will then show:

Theorem 0.1. *Let $\mathcal{D}$ be skeletally small. There is a bijection between weakly preenveloping (resp. weakly precovering) nonpositive sets in $\mathcal{D}$, up to add-equivalence, and left (resp. right) bounded co-t-structure in $\mathcal{D}$. This bijection restricts to another one between silting sets and bounded co-t-structures in $\mathcal{D}$.*

The definition of 'weakly preenveloping' will be made precise in the talk, but let us mention here that when $\mathcal{T}$ is finite and $\mathcal{D}$ is Hom finite (over some commutative ring K), it just amounts to say that, for each $M \in \mathcal{D}$, the functor $\text{Hom}_{\mathcal{D}}(M, ?[k])$ vanishes on $\mathcal{T}$ for $k \gg 0$.

On what concerns t-structures, for any triangulated category $\mathcal{D}$, we say that a set $\mathcal{T}$ of objects is *partial silting* when the orthogonal pair generated by $\bigcup_{k\geqslant 0} \mathcal{T}[k]$, namely $(\mathcal{U}_{\mathcal{T}}, \mathcal{U}_{\mathcal{T}}^{\perp}) := ({}^{\perp}(\mathcal{T}^{\perp_{\leqslant 0}}), \mathcal{T}^{\perp_{\leqslant 0}})$, is a torsion pair in $\mathcal{D}$ and $\text{Hom}_{\mathcal{D}}(T, ?[1])$ vanishes on $\mathcal{U}_{\mathcal{T}}$, for all $T \in \mathcal{T}$. Note that, for a finite dimensional algebra Λ, any silting set in $\mathcal{K}^b(\text{proj}-\Lambda)$ is partial silting in $\mathcal{D}^b(\text{mod}-\Lambda)$. We will then show:

Theorem 0.2. *Let $\mathcal{D}$ be any skeletally small triangulated category. If $\mathcal{T}$ is a partial silting (resp. finite partial silting) set in $\mathcal{D}$ such that add$(\mathcal{T})$ is a precovering class in $\mathcal{D}$, then the t-structure $(\mathcal{U}_{\mathcal{T}}, \mathcal{U}_{\mathcal{T}}^{\perp}[1])$ has a heart which is equivalent to the category mod $- A$ of finitely presented modules over a (right) coherent algebra with enough idempotents A (resp. right coherent unital algebra A). Moreover, when $\mathcal{D}$ satisfies some Brown-like representativity conditions (as in [2] or [8]), any bounded t-structure in $\mathcal{D}$ whose heart is equivalent to mod $- A$, for some A as above, is the one associated to an (add-)precovering partial silting set.*

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Ralf Schiffler (University of Connecticut)
Cluster-tilted algebras and quasi-tilted algebras

Abstract: This is joint work with Ibrahim Assem and Khrystyna Serhiyenko [1, 2]. We consider the map ϕ from tilted algebras to cluster-tilted algebras given by sending a tilted algebra C to its relation extension, the trivial extension of C by $\mathrm{Ext}^2(DC, C)$. It is known that this map is surjective. For a given cluster-tilted algebra B, we consider its fiber $\phi^{-1}(B)$. Using local slices in the module category of B, we define the operation of reflections and show that for any two tilted algebras C_1, C_2 in the fiber there is a sequence of reflections that transforms C_1 into C_2. We also show that almost every transjective indecomposable B-module lies on a local slice, and we characterize those that don't.

More generally, we show that relation extensions of quasi-tilted algebras are 2-Calabi-Yau tilted and extend the map to quasi-tilted algebras. For a tame cluster-tilted algebra B, we show that the tilted algebras in the fiber are given as a quotient of B by the annihilator of a local slice, and the quasi-tilted algebras as a quotient of B by a 'partition ideal'. Roughly speaking, the first case corresponds to cutting the cluster category in the transjective component, namely in the local slice. The second case corresponds to cutting the cluster category between regular components and partitioning the non-homogeneous tubes into predecessors and successors of the cut.

[1] I. Assem, R. Schiffler and K. Serhiyenko, Cluster-tilted and quasi-tilted algebras, to appear in J. Pure Appl. Alg., arXiv:1606.00938.

[2] I. Assem, R. Schiffler and K. Serhiyenko, Modules that do not lie on local slices,arXiv:1606.05161.

Markus Schmidmeier (Florida Atlantic University)
The support of a finitely presented functor

Abstract: For a finitely presented functor $F : \mathrm{mod}\, A \to \mathrm{mod}\, k$, the map $\dim_k F$ defines a "hammock function" on the Auslander-Reiten quiver Γ_A. Suppose F has projective resolution

$$0 \to (Z, -) \to (Y, -) \to (X, -) \to F \to 0.$$

We discuss how the indecomposable direct summands of X, Z, Y define sources, sinks and tangent points for the hammock function. Examples show properties of modules which can be captured by the hammock function.

Sibylle Schroll (University of Leicester)
Algebras and Varieties

Abstract: In this talk I will introduce new affine algebraic varieties whose points correspond to quotients of path algebras. Each variety has a distinguished point corresponding to a monomial algebra. The properties of this monomial algebra determine many of the properties of all the algebras in the same variety. I will further show that graded algebras as well as finite dimensional algebras given by admissible ideals arise as subvarieties. This is joint work with Ed Green and Lutz Hille.

Khrystyna Serhiyenko
Mutation of Conway-Coxeter friezes

Abstract: A frieze is a lattice of shifted rows of positive integers satisfying a diamond rule: the determinant of a 2x2 matrix formed by the neighboring entries is 1. Friezes were first studied by Conway and Coxeter in 1970's, but they gained fresh interest in the last decade in relation to cluster theory. It is known that given an Aulsander-Reiten quiver of a cluster-tilted algebra B of type A we can apply the specialized Caldero-Chapoton map to every indecomposable B-module and obtain a frieze. In particular, this yields a bijection between friezes and cluster-tilted algebras of type A. An operation called mutation is the key notion in cluster theory, and we introduce mutations of friezes which are compatible with mutations of the associated cluster-tilted algebras. This is joint work with K. Baur, E. Faber, S. Gratz, and G. Todorov.

Andrzej Skowroński. (Nicolaus Copernicus University)
Algebras of generalized quaternion type

Abstract: This is report on the periodic part of a joint project with Karin Erdmann concerning the representation theory of tame symmetric algebras (over an algebraically closed field), generalizing tame blocks of group algebras of finite groups. Finding or possibly classifying periodic algebras (with respect to syzygy) is an important problem because of its interesting connections with group theory, topology, singularity theory, cluster algebras and combinatorics. Moreover, the periodicity of an algebra, and its period, are invariant under derived equivalences. Motivated by the theory of blocks of group algebras with quaternion defect groups, we introduced the algebras of generalized quaternion type as: the symmetric, indecomposable, representation-infinite tame algebras with all simple modules being periodic of period 4. The aim of the talk is to present a complete classification of all basic algebras of generalized quaternion type with 2-regular Gabriel quivers, and conclude that these algebras are periodic of period 4. In particular, we obtain that the stable Auslander-Reiten quivers of these algebras consist of stable tubes of ranks 1 and 2. The major part of the classification is given by the weighted surface algebras of triangulated surfaces with arbitrarily oriented triangles, and their socle deformations, but some exotic families of algebras also occur.

Andrea Solotar (Univerdad de Buenos Aires)
Some invariants of the super Jordan plane

Abstract: Hochschild cohomology and its Gerstenhaber algebra structure are relevant invariants: they are invariant by Morita equivalences, by tilting processes and by derived equivalences.

The computation of these invariants requires a resolution of the algebra considered as a bimodule over itself. Of course, there is always a canonical resolution available, the bar resolution, very useful from a theoretical point of view, but not very satisfactory in practice: the complexity of this resolution rarely allows explicit calculations to be carried out.

Nichols algebras are generalizations of symmetric algebras in the context of braided tensor categories. These are graded algebras, which first appeared in an article by Nichols in 1978, in which the author looked for examples of Hopf algebras. They are fundamental objects for the classification of pointed Hopf algebras, as was shown by the work of Andruskiewitsch and Schneider.

Heckenberger classified finite-dimensional Nichols algebras of diagonal type up to isomorphism. The classification separates the Nichols algebras in different classes: Nichols algebras of the Cartan type, essentially related to the finite quantum groups of Lusztig; Nichols algebras related to finite quantum supergroups and a third class related to countergradient Lie superalgebras. Later, Angiono described the definition relations of the Nichols algebras of the list of Heckenberger.

The problem of finite generation of the chomology of a Hopf algera is related to Hochschild cohomology, since for any augmented algebra, the former graded space is isomorphic to a direct summmand of the latter.

In a joint work with Sebastián Reca, we computed the Hochschild homology and cohomology of $A = \Bbbk\langle x, y | x^2, y^2x - xy^2 - xyx\rangle$, called the super Jordan plane, when $\mathrm{char}(\Bbbk) = 0$ and $\Bbbk$ is algebraically closed. This is the Nichols algebra $\mathfrak{B}(V(-1, 2))$, whose Gelfand-Kirillov dimension is 2.

The main results we obtained are the following.

- We give explicit bases for the Hochschild homology and cohomology spaces.
- We describe the cup product. From this description we see that the isomorphism between $H^{2p}(A, A)$ and $H^{2p+2}(A, A)$, where $p > 0$, is given by the multiplication with an element of $H^2(A, A)$, and similarly for the odd degrees.
- We describe Lie algebra structure of $H^1(A, A)$, which turns out to be isomorphic to a Lie subalgebra of the Virasoro algebra.
- We describe the $H^1(A, A)$-module structure of the vector spaces $H^n(A, A)$ for all $n \geqslant 2$ and classify these representations.

Pamela Suarez (Universidad Nacional de Mar del Plata)

On the global dimension of the endomorphism algebra of a τ-tilting module

Abstract: τ-tilting theory was introduced by Adachi, Iyama and Reiten as a generalization of tilting modules from the point of view of mutation. The main idea is that any indecomposable direct summand of a support τ-tilting module can be replaced in a unique way to get a new support τ-tilting module.

Given an algebra A of finite global dimension and B the endomorphism algebra of a tilting A-module, it is well-known that there exists a deep connection between the global dimension of A and the global dimension of B. Moreover, the global dimension of B is always finite.

Now, let A be an algebra of finite global dimension and B the endomorphism algebra of a τ-tilting A-module. In this talk, we present some relations between the global dimension of A and the global dimension of B.

In order to understand the problem, we study the annihilator of a τ-tilting module. We show that finiteness of the global dimension is not preserved under τ-tilting process. We present a family of algebras A_n of finite global dimension such that there exists a τ-tilting A_n-module T_n such that $\mathrm{gldimEnd}T_n = \infty$.

Finally, we show two cases where the finiteness of the global dimension is preserved under τ-tilting process. In case we deal with a monomial or special biserial algebra A of global dimension 2, the global dimension of B is always finite.

Hugh Thomas (Université du Québec à Montréal)

Nilpotent endomorphisms of quiver representations and the Robinson-Schensted-Knuth correspondence

Abstract: Let Q be a quiver, and let V be a representation of Q. We could ask: what is the Jordan form of a generic nilpotent endomorphism of V? Conversely, we could ask: given a collection of vectors spaces V_i associated to the vertices of Q, and nilpotent endomorphisms N_i of V_i, what is the generic representation V on the vectors spaces V_i such that the N_i fit together to define a nilpotent endomorphism of V? This question is already interesting in type A, where it connects up to the classical combinatorics of the Robinson-Schensted-Knuth correspondence. I will discuss this case, and also other finite type cases.

Kunio Yamagata (Tokyo University of Agriculture and Technology)

Morita algebras and canonical bimodules

Abstract: In 1958, K. Morita [1] initiated the study of the endomorphism rings of modules and characterized the endomorphism ring of a generator over a quasi-Frobenius ring, which is called a *Morita algebra* after his work, and its generalization of his theory was one of the main problems in ring theory in the 1960-1970s. In 2011, M. Fang and S. Koenig [2] revived the Morita theory in finite dimensional symmetric algebras, by introducing the bimodule $V = \mathrm{Hom}_A(D(A), A)$ which is called the *canonical bimodule* of a finite dimensional algebra A over a field K. In [3] then it is shown that the bimodule has characteristic properties of Morita algebras. Those properties are related to the double centralizer property of modules and dominant dimension of algebras. In my talk I will recall basic facts on Morita algebras and introduce a new characterization of Morita algebras by using the canonical bimodules from joint wok with Fang and Kerner.

References

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Emine Yildirim (Université du Québec à Montréal)
Auslander-Reiten translation on cominuscule posets

Abstract: Let P be a cominuscule poset which can be thought of as a parabolic analogue of the poset of positive roots of a finite root system. Let $J(P)$ be the poset of order ideals of P, and $D^b(J(P))$ be the bounded derived category of the incidence algebra of $J(P)$. The Auslander-Reiten translation τ on the bounded derived category $D^b(J(P))$ naturally defines an endomorphism on the Grothendieck group of $J(P)$ which is called *Coxeter transformation*. We show that Coxeter transformation has finite order for two of the three infinite families of cominuscule posets, and for the exceptional cases. Our motivation comes from a conjecture by Chapoton which states that $D^b(J(R))$ is fractionally Calabi-Yau, which means some non-zero power of the Auslander-Reiten translation equals some power of the shift functor, when R is the poset of positive roots of a finite root system.

Shijie Zhu (Northeastern University)
Dominant dimension and tilting modules

Abstract: We study which algebras have a tilting module which is both generated and cogenerated by projective-injective modules. Auslander algebras have such a tilting module and for algebras of global dimension 2, Auslander algebras are classified by the existence of such a tilting module. We show that, independently of global dimension, the existence of such a tilting module is equivalent to the algebra having dominant dimension at least 2. Furthermore, such a tilting module is also a cotilting module if and only if the algebra is 1-Auslander-Gorenstein. As a special family, we show that algebras extended from Auslander algebras by certain injective modules admit a tilting module which is generated and cogenerated by projective-injective modules.

Grzegorz Zwara (Nicolaus Copernicus University)
Tangent spaces to orbit closures for representations of Dynkin quivers

Abstract: Let Q be a Dynkin quiver. Any finite dimensional representation of Q corresponds to an orbit O(M) in an affine space equipped with an algebraic group action. An interesting question is to find the dimension of the tangent spaces to the (Zariski) closure C of the orbit O(M). The answer is relatively easy to obtain in case the Dynkin quiver Q is of type A, as we know the defining equations of the affine variety C. During the talk we focus on the other types of Q (D and E). As a corollary we characterize the singular locus of the closure C of the orbit O(M).