Representations with interconnections: Lie theory and nonassociative algebras

in honor of Olivier Mathieu's 65th birthday

July 2 - 4, 2025

Thanks!

- Institut Camille Jordan, Lyon
- Université de Cergy-Pontoise
- Réseau Thématique "Algèbre" CNRS
- Laboratoire Mathématiques de Reims UMR 9008
- Institut Elie Cartan de Lorraine UMR 7502
- IRN in Representation Theory

- Maria and Vincent and Philippe

G Lie group

Ph. D. students

1. Khalid Bennani
2. Rupert Yu
3. Yasmine Sanderson
4. Boujemaâ Agrebaoui
5. Rosane Ushirobira
6. Raika Dehy
7. Jérôme Germoni
8. Georges Papadopoulo
9. Sébastien Foulle
10. Séverine Verneyre
11. Lucas Fresse
12. Benoît Dejoncheere
13. Eduardo Mendonça

Olivier is sixty-five!

Olivier's first area of expertise is the representation theory of several classes of algebras and groups. If you name a classical problem—such as the classification of Lie algebras, the classification of certain types of representations, character formulas, or the decomposition of tensor products—chances are there are several of Olivier's articles on the topic.

What is striking is his ability to apply sophisticated ideas and techniques—like reduction to characteristic p, algebraic geometry, and differential operators—in contexts where they are completely unexpected. Let us see a few examples.

Olivier's initial results are the classification of infinite-dimensional Lie algebras graded over $\mathbb{Z}$. In his Ph.D. thesis, under the supervision of Jacques Tits and Michel Duflo, under he dealt with the case where the dimensions of the graded components are less than or equal to 1. Later, he studied cases where the growth is less than 1, a highly technical result. The Zentralblatt reviewer wonders how he found his way through 59 non-trivial lemmas!

Around the same time, in the late 1980s, he applied a sophisticated method in algebraic geometry known as "Frobenius splitting." Mehta, Ramanan, and Ramanathan introduced this technique for flag varieties of algebraic groups. Olivier adapted this approach to the infinite-dimensional context of Kac-Moody groups, leading to a character formula for Weyl-Demazure modules.

He studied good and excellent filtrations and bases of representations of G and B, focusing on their compatibility with tensor products for decomposition. Using ideas like Frobenius splitting and Kac-Moody groups, he achieved results related to a finite-dimensional group!

Around 1990, Olivier introduced a groundbreaking approach by studying representation theory over characteristic zero and reducing problems modulo a prime number, enabling effective use of algebraic geometry methods. This approach allowed him to prove the bicontinuity of the Dixmier map for solvable algebras and classify bounded representations, or Harish-Chandra modules, of the Virasoro algebra. Additionally, he advanced the concept of tilting modules for algebraic groups in finite characteristic. In collaboration with Galin Georgiev, he defined a "fusion algebra" from the tensor product of tilting modules, leading to a formula akin to the Verlinde formula in quantum field theory, with parallels in quantum groups and affine Kac-Moody algebras.

Olivier used the representation theory of certain algebras and Lie algebras generated by differential operators to disprove Brylinsky's conjecture on harmonic forms in symplectic manifolds. He also explored "hidden symmetries" on Poisson manifolds, proving the existence of differential operators that cannot be expressed through products and the Poisson bracket.

In the late 1990s, alone or with George Papadopoulo, he developed character formulas in finite characteristic for the symmetric group and GL_n, addressing a challenging problem.

When classifying infinite-dimensional representations with bounded weight spaces, Olivier introduced the concept of a coherent family, enabling a classification based on highest weight modules. This approach was later applied to affine Kac-Moody algebras and superalgebras. More recently, in collaboration with Kenji Iohara, he classified additional infinite-dimensional Lie algebras and investigated the complex structure of free Jordan algebras with Michael Lau and Iryna Kashuba.

Olivier also explored different fields. For instance, he collaborated with the geometer Ingrid Irmer on systoles of hyperbolic surfaces, employing an utterly original approach. Indeed, he exploited the combinatorics of an infinite Coxeter group to build surfaces with an arbitrarily large genus and a small number of systoles that split the surface into polygons.

A few words should also be said about Olivier's qualities as a teacher. His ability to illustrate profound ideas in a simple way. For example, describe a blackboard session where, after, say, 30 minutes, G was written on the board 15 times, 5 or 6 of which were crossed out and replaced with K…

To sum up, Olivier combines advanced results with a series of highly original and deep ideas, allowing him to apply sophisticated techniques in unexpected contexts!

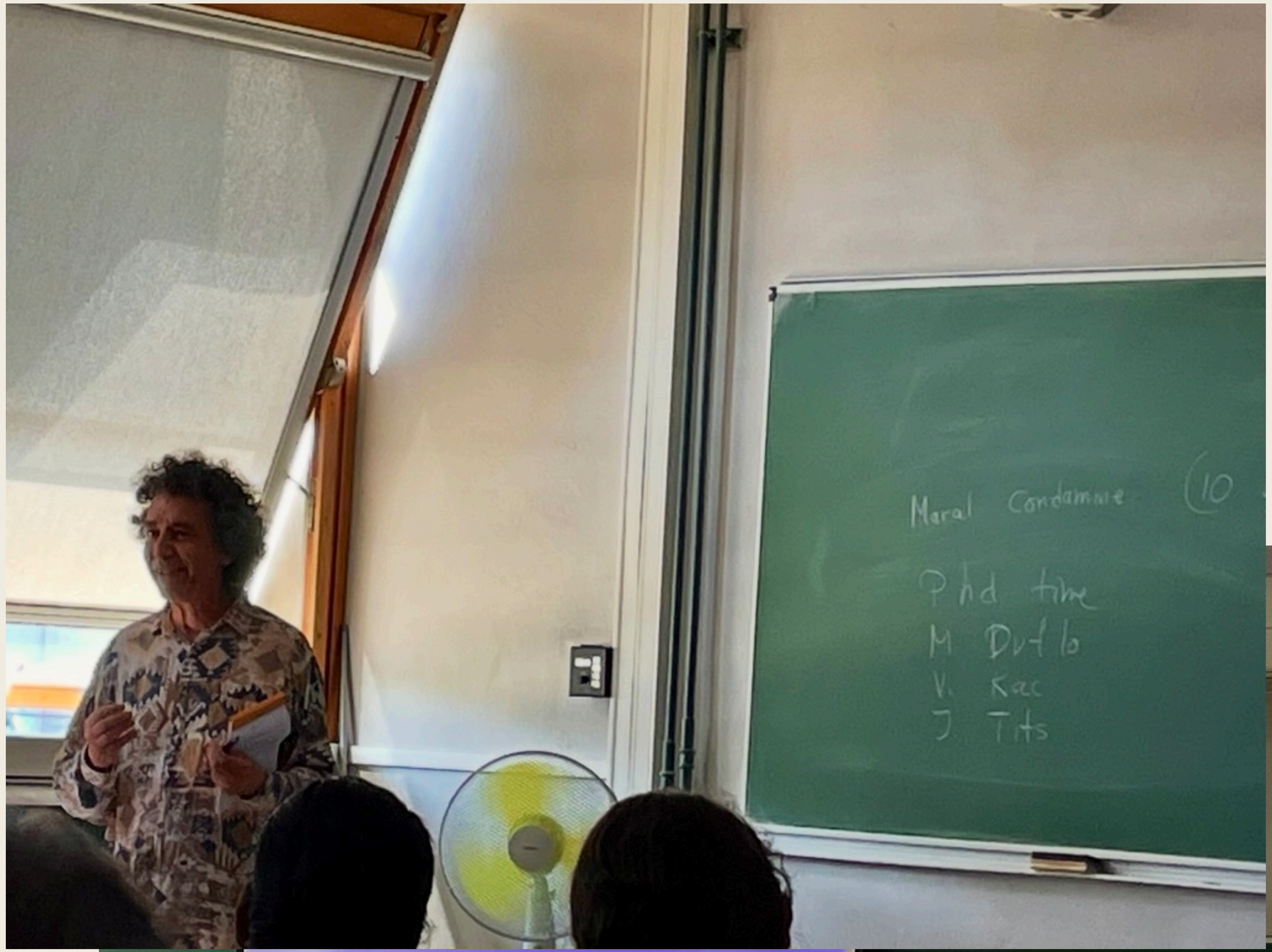

9 12 '96

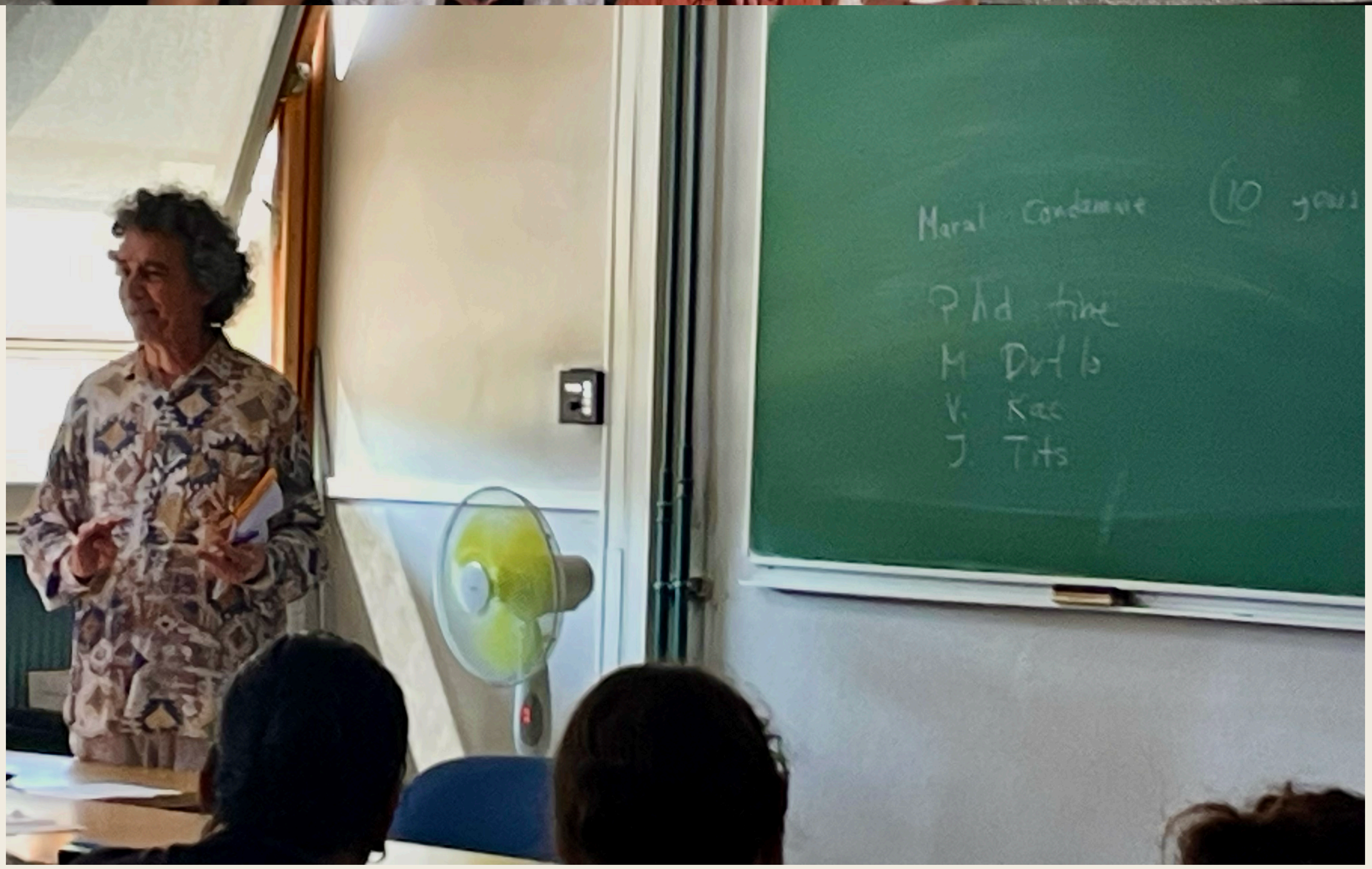


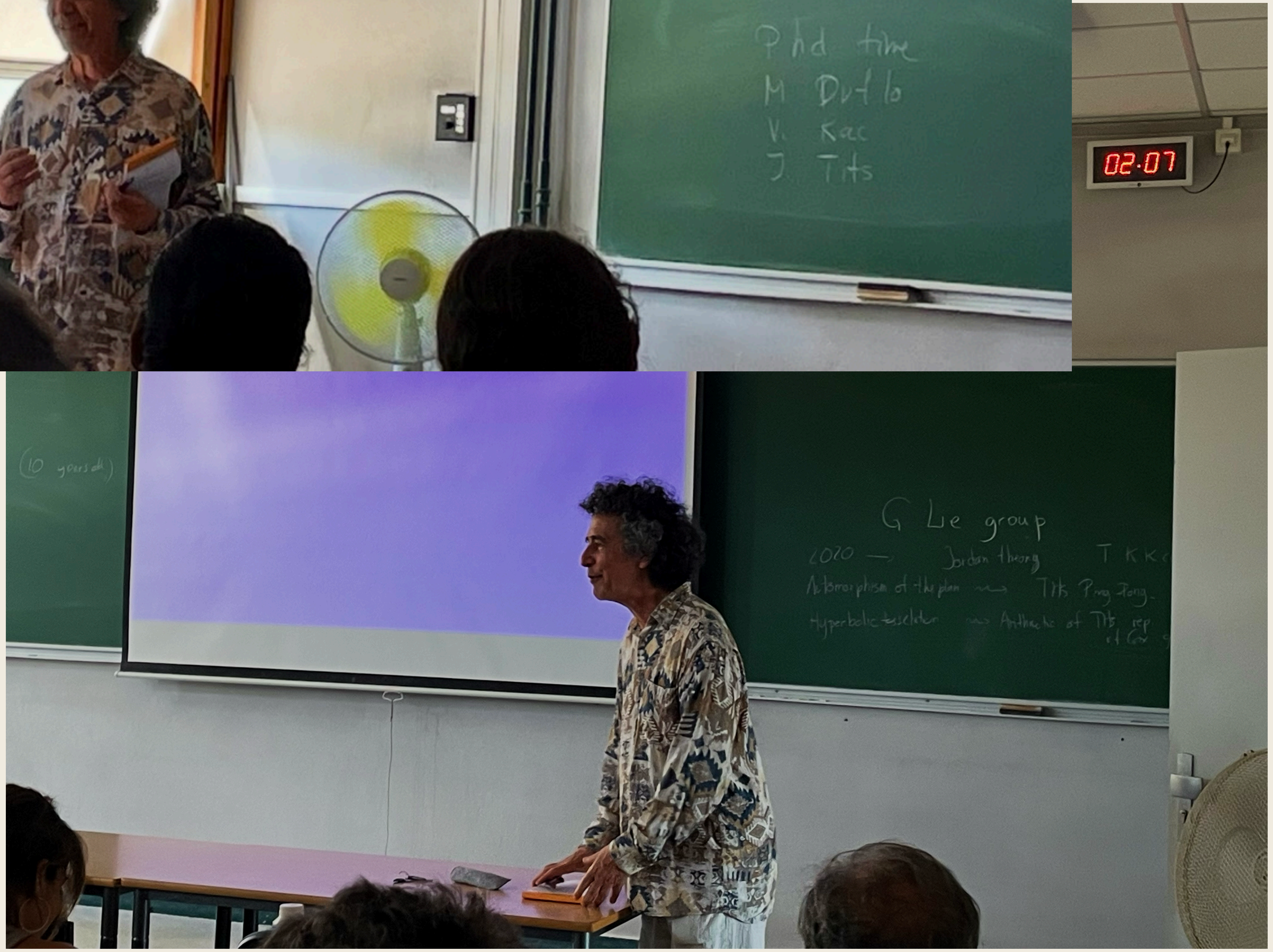


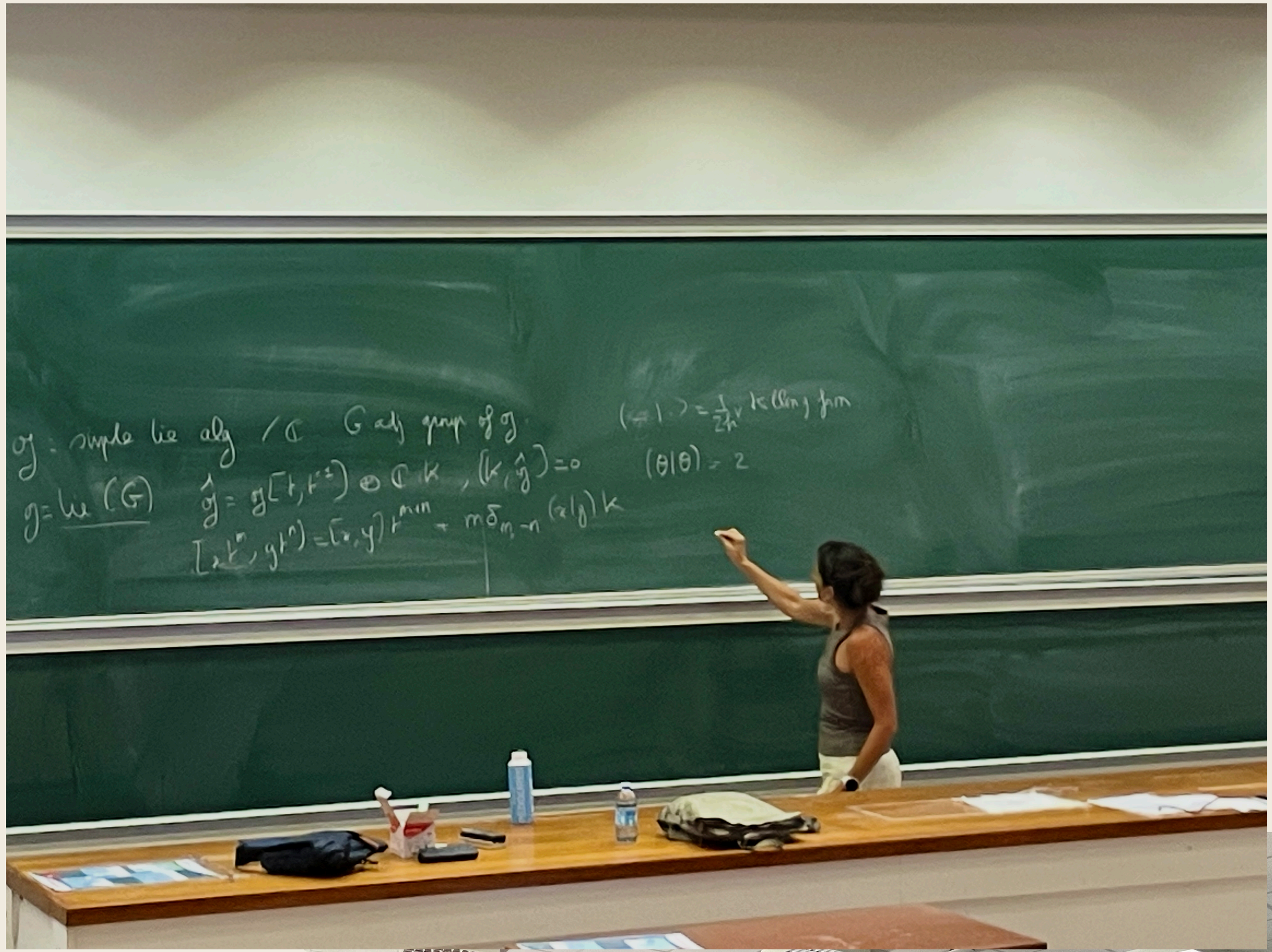


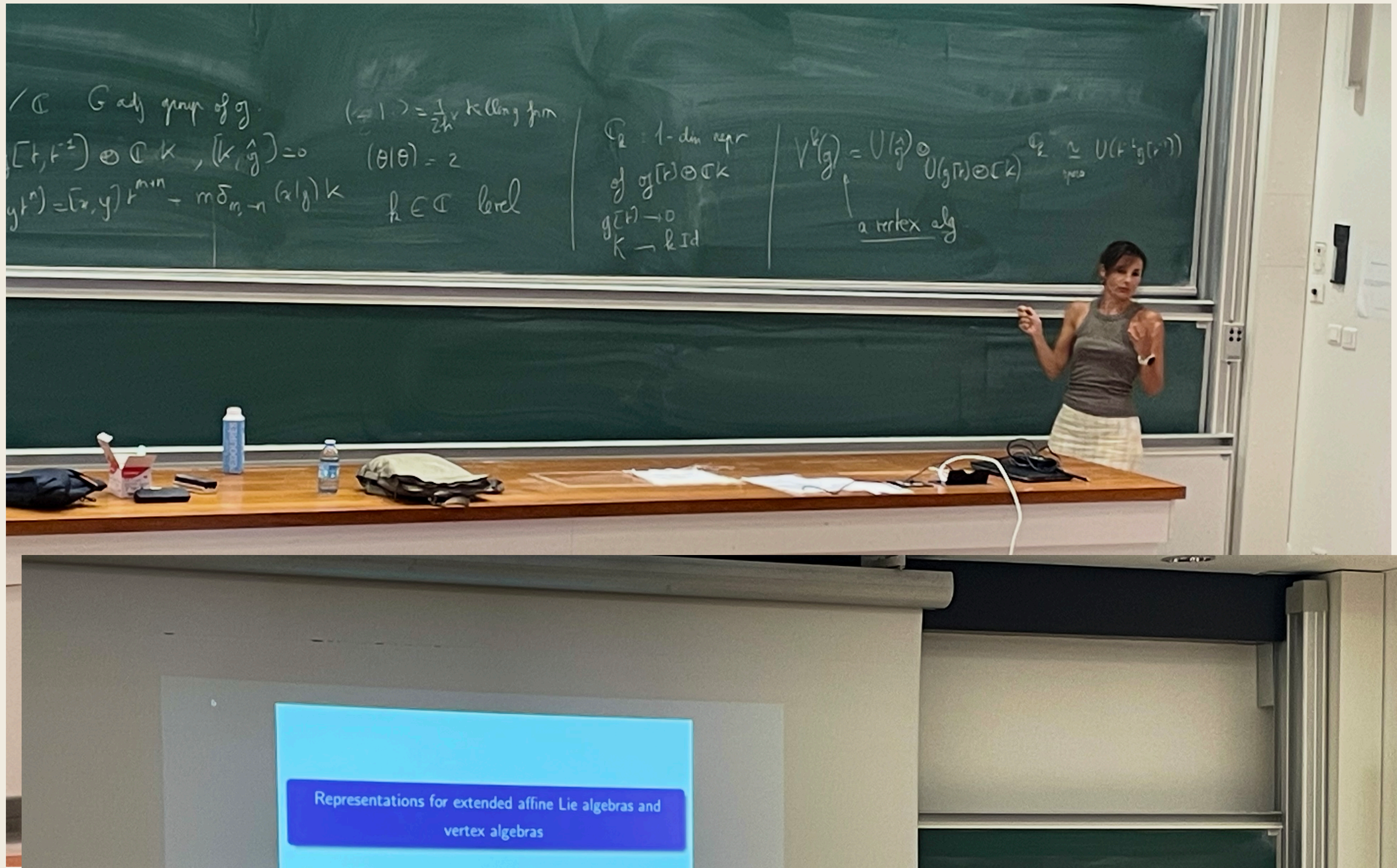



Maral Condamine (10
Phd time
M Duflo
V. Kac
J. Tits

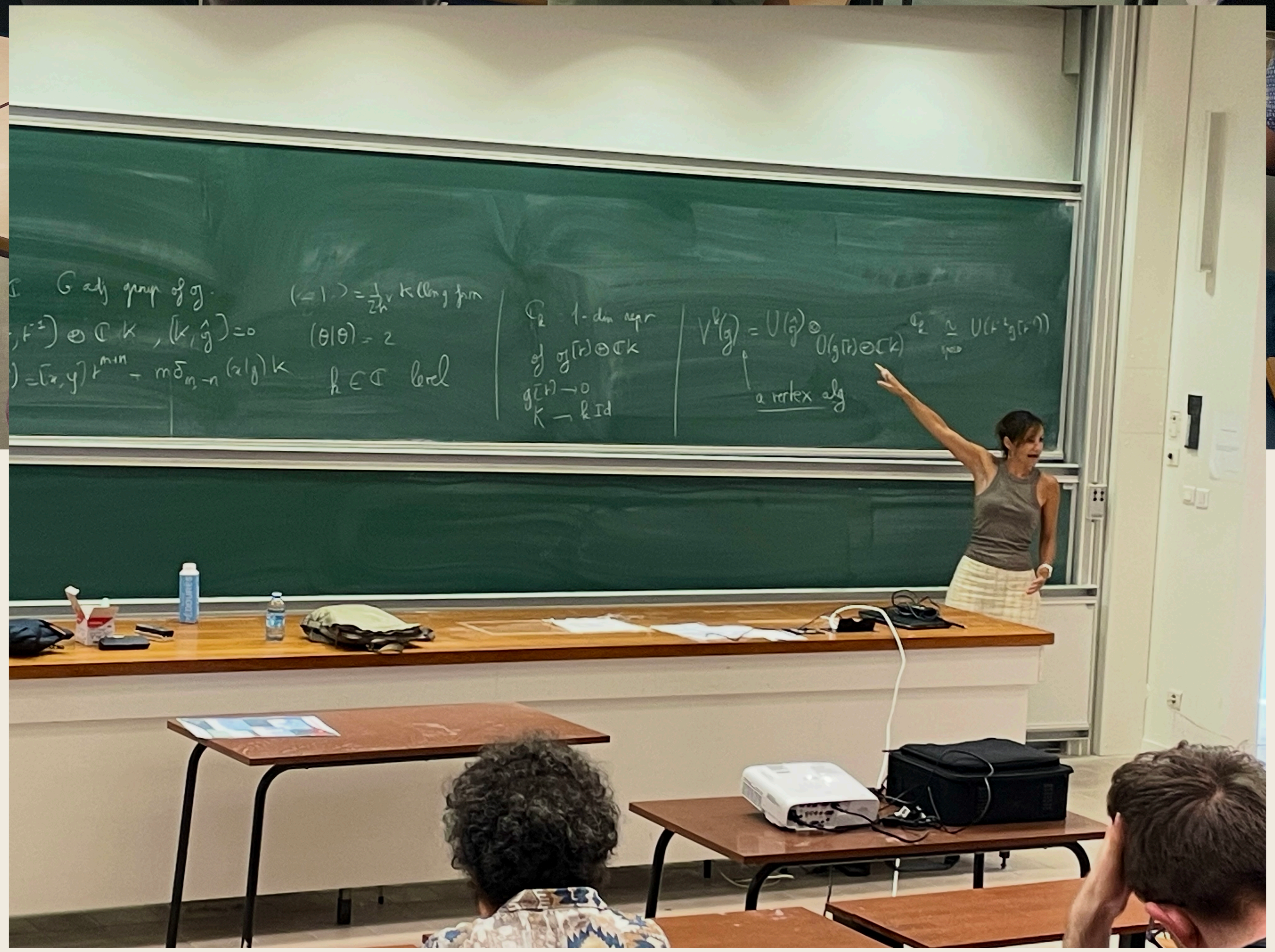

Maral Condamine (10 years
Phd time
M Duflo
V. Kac
J. Tits

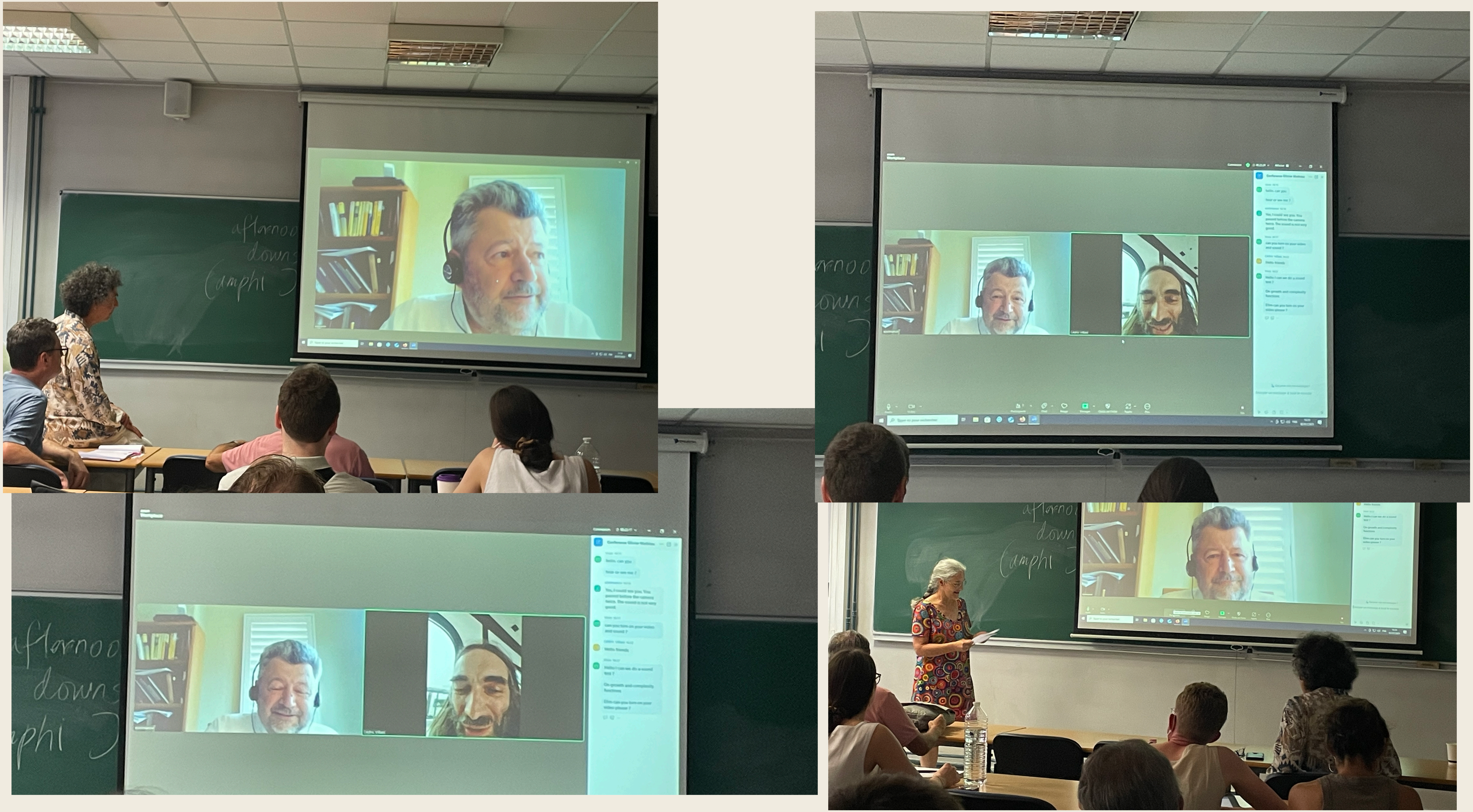

G Lie group





Representations for extended affine Lie algebras and vertex algebras
Shaobin Tan
School of Mathematical Sciences, Xiamen University
July 02, 2024

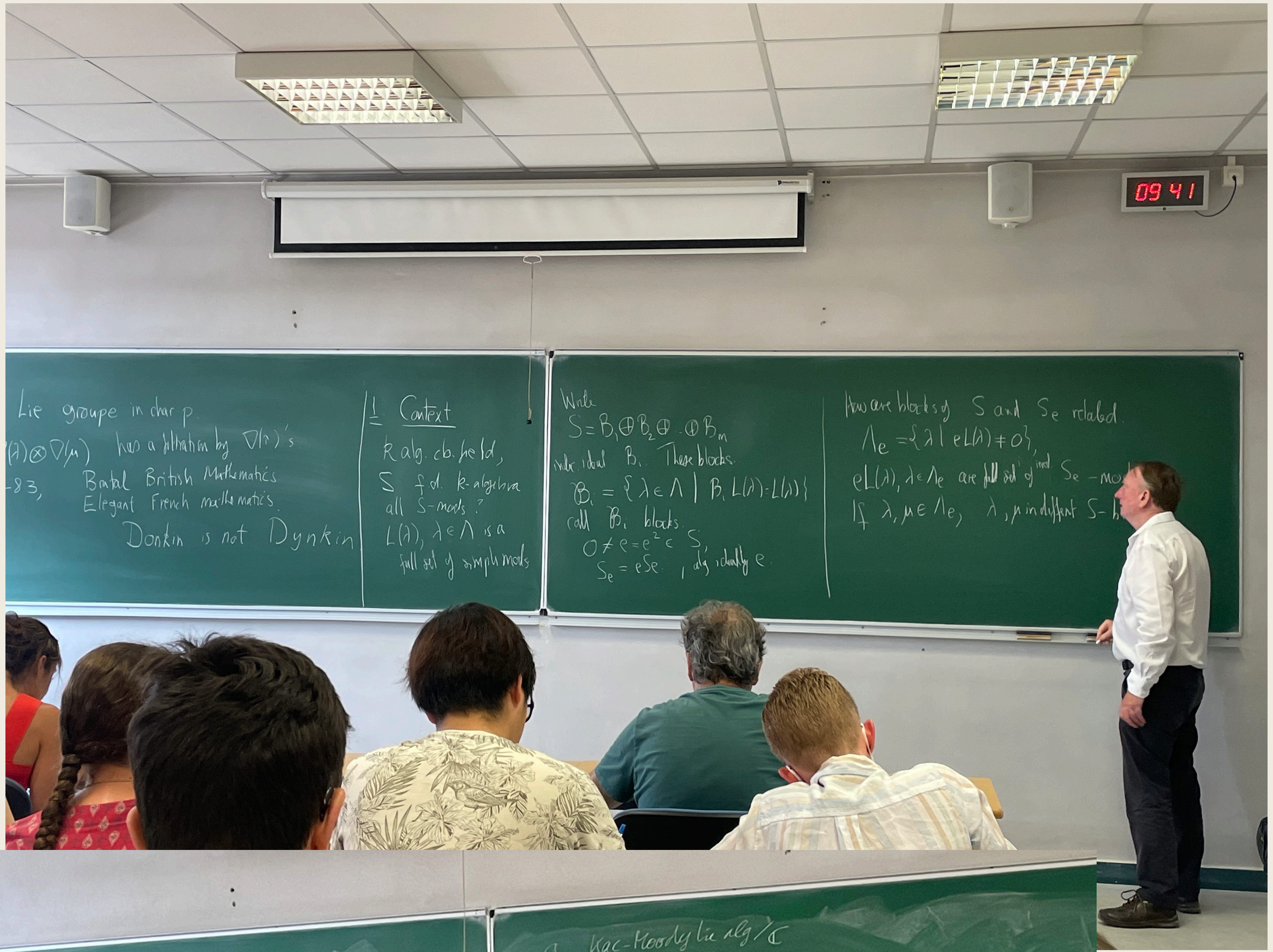



afternoo
downs
(amphi

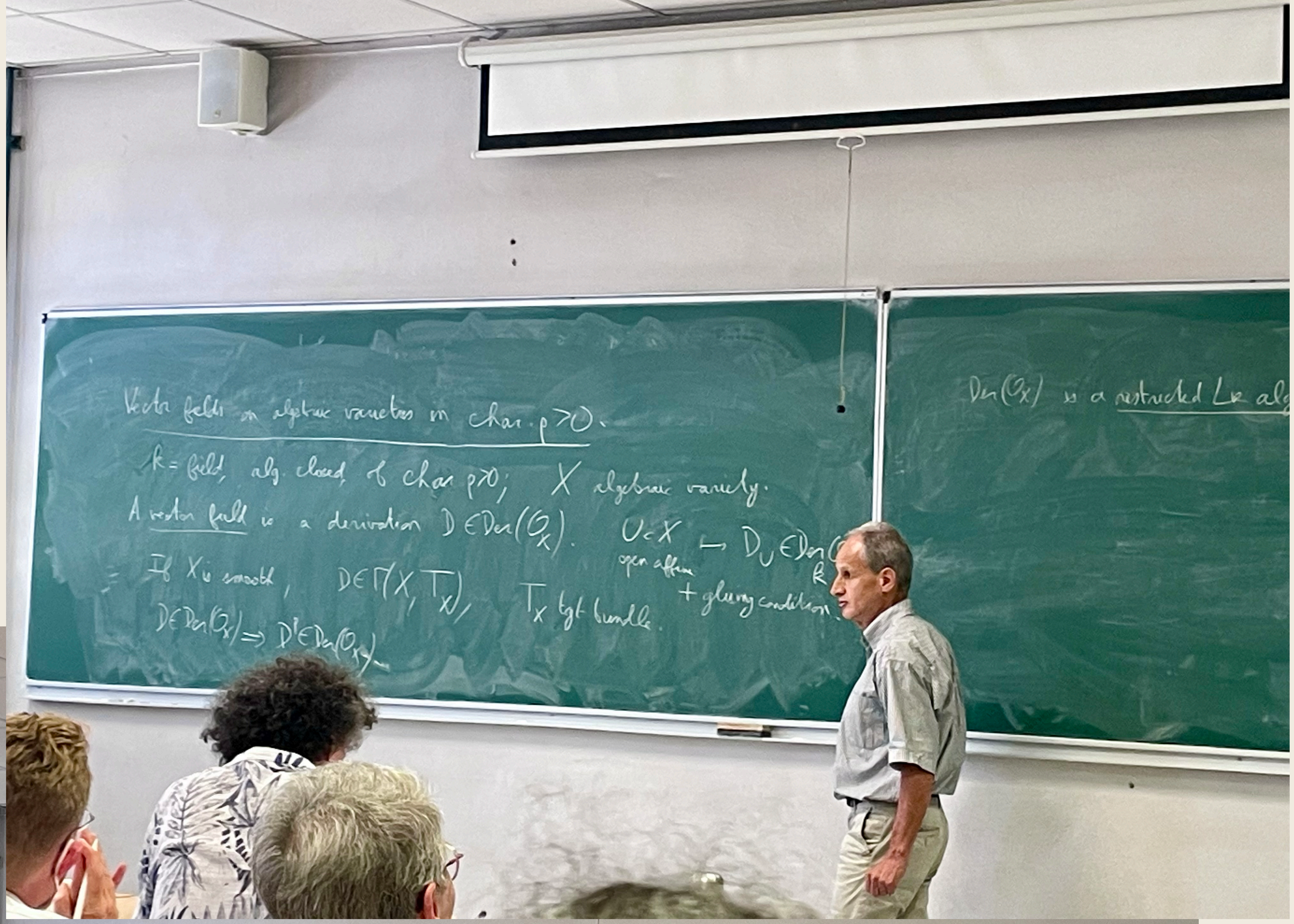








Lie groupe in char p
Context
How are blocks of S and Se related
Donkin is not Dynkin


Soit G un groupe
de Kac-Moody
1989
Kac-Moody Lie alg / ℂ
Goal understand
V(λ) ⊗ V(μ)


Vector fields on algebraic varieties in char. p>0.
k = field, alg. closed of char p>0; X algebraic variety.
Tx tgt bundle
+ gluing condition


Some universal constructions
Universal property

Merci beaucoup !