Random geometry and quantum gravity

Rapport sur les contributions

ID de Contribution: 1

Type: Non spécifié

Hamiltonian Paths on Random Planar Maps

mercredi 12 février 2025 13:30 (1 heure)

Hamiltonian paths are self-avoiding random walks that visit all sites of a given lattice. We consider various configuration exponents of Hamiltonian walks drawn on random planar maps. Estimates from exact enumerations are compared with predictions based on the Knizhnik-Polyakov-Zamolodchikov (KPZ) relations, as applied to exponents on the regular hexagonal lattice. Astonishingly, when the maps are bipartite, a naive use of KPZ does not reproduce all the measured exponents, but an Ansatz may possibly account for the observed discrepancies. We further study Hamiltonian cycles on various families of bipartite planar maps, which fall into two universality classes, with respective central charges c = -1 or c = -2. The first group comprises maps of fixed vertex valency p larger than 3, whereas the second group involves maps with mixed vertex valencies, as well as a so-called rigid case. For each class, a universal configuration exponent and a novel critical exponent associated with long-distance contacts along a Hamiltonian cycle are predicted from KPZ and the corresponding exponent on regular (hexagonal or square) lattices. This time, the KPZ predictions are numerically confirmed by exact enumeration results for p-regular maps, with p = 3, 4, 5, 6, 7, and for maps with mixed valencies (2,3) and (2,4). The scaling limit of fully-packed systems thus poses intriguing unresolved questions from both the Liouville Quantum Gravity and the Schramm-Loewner Evolution perspectives. Based on joint works with Ph. Di Francesco, O. Golinelli and E. Guitter.

Orateur: DUPLANTIER, Bertrand (Institut de Physique Théorique, Université Paris-Saclay)

Interface scaling limit for the critic ...

ID de Contribution: 2

Type: Non spécifié

Interface scaling limit for the critical planar Ising model perturbed by a magnetic field

mercredi 12 février 2025 16:15 (1 heure)

In this talk, I will consider the interface separating +1 and -1 spins in the critical planar Ising model with Dobrushin boundary conditions perturbed by an external magnetic field. I will prove that this interface has a scaling limit. This result holds when the Ising model is defined on a bounded and simply connected subgraph of $\delta \mathbb{Z}^2$, with $\delta > 0$. I will show that if the scaling of the external field is of order $\delta^{15/8}$, then, as $\delta \to 0$, the interface converges in law to a random curve whose law is conformally covariant and absolutely continuous with respect to SLE₃. This limiting law is a massive version of SLE₃ in the sense of Makarov and Smirnov and I will give an explicit expression for its Radon-Nikodym derivative with respect to SLE₃. I will also prove that if the scaling of the external field is of order $\delta^{15/8}g(\delta)$ with $g(\delta) \to 0$, then the interface converges in law to SLE₃. In contrast, I will show that if the scaling of the external field is of order $\delta^{15/8}g(\delta) \to 0$, then the interface degenerates to a boundary arc.

Orateur: PAPON, Léonie (Durham University, UK)

The chemical distance metric for ...

ID de Contribution: 3

Type: Non spécifié

The chemical distance metric for non-simple CLE

mercredi 12 février 2025 15:00 (1 heure)

We construct the continuum analogue of the chemical distance metric in lattice models such as percolation. The chemical distance metric is the graph distance induced by the percolation clusters. It is known that for critical percolation, the lengths have non-trivial scaling behaviour, however it is very difficult to find the exact scaling exponent. (This is one of the questions from Schramm's ICM 2006 article that remains unsolved.)

In a joint work with Valeria Ambrosio and Jason Miller, we construct a chemical distance metric on the CLE gasket for each $\kappa \in]4, 8[$. We show that it is unique metric that is geodesic, Markovian, and conformally covariant. The characterisation is reminiscent of the LQG metric, but our objects behave very differently, and hence our techniques also differ significantly from those used in LQG. For $\kappa = 6$, we conjecture that our random metric space is the scaling limit of critical percolation.

Orateur: YUAN, Yizheng (Statistical Laboratory, Cambridge University, UK)