

Luis Chacon (talk 14): Exact local conservation of energy in fully implicit PIC algorithms

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Abstract: We consider the issue of strict, fully discrete local energy conservation for a whole class of fully implicit local-charge- and global-energy-conserving particle-in-cell (PIC) algorithms. Earlier studies [1-3] demonstrated these algorithms feature strict global energy conservation. However, whether a local energy conservation theorem exists (in which the local energy update is governed by a flux balance equation at every mesh cell) for these schemes is unclear. In this study, we show that a local energy conservation theorem indeed exists [4]. We begin our analysis with the 1D electrostatic PIC model without orbit-averaging, and then generalize our conclusions to account for orbit averaging, multiple dimensions, and electromagnetic models (Darwin). In all cases, a temporally, spatially, and particle-discrete local energy conservation theorem is shown to exist, proving that these formulations (as originally proposed in the literature), in addition to being locally charge conserving, are strictly locally energy conserving as well. In contrast to earlier proofs of local conservation in the literature [5], which only considered continuum time, our result is valid for the fully implicit time-discrete version of all models, including important features such as orbit averaging. We demonstrate the local- energy-conservation property numerically with a paradigmatic numerical example.

REFERENCES

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