**Exercise 1 (Combined Predictor-Corrector Step)** Let $z := (x, s, y) \in N_2(\beta)$, $\beta \in [0, 1/2]$, $\mu := \mu(z)$. Let $\Delta z^c = (\Delta x^c, \Delta s^c, \Delta y^c)$ and $\Delta z^p = (\Delta x^p, \Delta s^p, \Delta y^p)$ denote the predictor and corrector directions at $z$. For $\alpha \in [0, 1]$, let $\Delta z^{pc} := (\Delta x^{pc}, \Delta s^{pc}, \Delta y^{pc}) := (1-\alpha) z^c + \alpha z^p$ denote the combined predictor-corrector step. Prove the following:

1. $s \Delta x^{pc} + x \Delta s^{pc} = (1-\alpha)\mu 1_n - xs$.
2. $\mu(z + \Delta z^{pc}) = (1-\alpha)\mu$.
3. $\left\| \frac{(1-\alpha)\mu 1_n - xs}{\sqrt{xs\mu}} \right\|_2 \leq (1-\alpha)\frac{\beta}{\sqrt{1-\beta}} + \alpha\sqrt{n} \leq \sqrt{\beta} + \alpha\sqrt{n}$.
   (Hint: apply the triangle inequality using the bounds we already proved for the predictor and corrector directions.)
4. $\mathrm{dist}_{\mathrm{c}}(z + \Delta z^{pc}) = \frac{1}{1-\alpha} \left\| \frac{\Delta x^{pc} \Delta s^{pc}}{\mu} \right\|_2$.
5. For $\alpha = \sqrt{\frac{\beta}{n}}/5$, $z + \Delta z^{pc} \in N_2(\beta)$.
   (Hint: Combine part 3 and 4 done in the lecture.)