Exercise 1 (Rescaling the Central Path) Let $z(\mu) = (x(\mu), s(\mu), y(\mu))$ be the central path for the primal-dual LP with data $(\mathbf{A}, b, c)$, $\mathbf{A} \in \mathbb{R}^{m \times n}, \text{rank}(\mathbf{A}) = m, b \in \mathbb{R}^m, c \in \mathbb{R}^n$. Let $w, z > 0_n$ satisfy $WZ = \alpha \mathbf{I}_n$, where $W = \text{diag}(w), Z = \text{diag}(z)$. Examine the rescaled LP with data $(\bar{\mathbf{A}}, \bar{b}, \bar{c}) := (\mathbf{A}W, b, Z^{-1}c)$, and let $\bar{z}(\mu) = (\bar{x}(\mu), \bar{s}(\mu), \bar{y}(\mu))$ denote its central path.

1. Prove that $\bar{z}(\mu) = (W^{-1}x(\alpha\mu), Z^{-1}s(\alpha\mu), \frac{y(\alpha\mu)}{\alpha})$.
   (Hint: Show that the right hand side satisfies the central path equations.)
2. Let $(w, z) = (x(\alpha), s(\alpha))$. What is $(\bar{x}(1), \bar{s}(1))$?

Exercise 2 (Interpretation of Step Equations)
Let $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\text{rank}(\mathbf{A}) = m$, $t \in \mathbb{R}^n$, $r > 0_n$. Examine the linear system:

$$r\Delta x + r^{-1}\Delta s = t \tag{1}$$

$$\mathbf{A}\Delta x = b \tag{2}$$

$$\mathbf{A}^\top \Delta y + \Delta s = c. \tag{3}$$

Prove that Δx above is a minimizer of:

$$\min_{\mathbf{A}z=b} \|t - rz\|_2^2 /2 + \langle c, z \rangle .$$

(Hint: Use Δy as Lagrange multiplier for $\mathbf{A}z = b$.) **Remark:** It can similarly be shown that $(\Delta s, \Delta y)$ is a minimizer (in fact, unique) of $\min_{\mathbf{A}^\top v + w = c} \left\|t - r^{-1}w\right\|_2^2 - \langle b, v \rangle$.