HIGHER ORDER HARMONIC SCHWARZIAN DERIVATIVES

MARÍA J. MARTÍN

ABSTRACT. Let Ω be a domain in the complex plane. A *harmonic mapping* in Ω is a complex valued function $f = u + iv$ whose real and imaginary parts are (real) harmonic in Ω. Analytic functions are a special case where the real and imaginary parts are conjugate harmonic functions, satisfying the Cauchy–Riemann equations. In those cases when the domain Ω is simply connected, the harmonic mapping f can be written as $f = h + \overline{g}$, where both h and g are analytic in Ω.

The landmark paper by James Clunie and Terry Sheil-Small in 1984, points out that many of the classical results for conformal functions have clear analogues for univalent harmonic mappings. However, as Peter Duren mentions in his book on harmonic mappings in the plane, "as soon as analyticity is abandoned, serious obstacles arise".

The purpose of this talk is to show a generalization of some particular objects introduced for analytic functions to those cases when the functions considered are merely harmonic. More concretely, we propose a definition of higher order Schwarzian derivatives for locally univalent harmonic mappings. The definition will be based on the relation between the classical formulas of the pre-Schwarzian and Schwarzian derivatives of locally univalent analytic functions and the derivatives of the generating functions of the methods due to Newton and Halley, respectively, for approximating zeros. We will prove that the higher order harmonic Schwarzian derivatives we obtain coincide, in those cases when the functions considered are holomorphic, with the Aharonov invariants, introduced by Don Aharonov in 1969.

This is a joint work with F. Pérez-González and Alexis Quintero-Díaz.

DEPARTAMENTO DE ANÁLISIS MATEMÁTICO & IMAULL, UNIVERSIDAD DE LA LAGUNA. AV. ASTROFÍSICO FRANCISCO SÁNCHEZ, S/N. FACULTAD DE MATEMÁTICAS. 38200, LA LAGUNA, TENERIFE, SPAIN.

Email address: `maria.martin@ull.es`