Composition operators on weighted Hardy spaces
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Given a sequence $\beta = (\beta_n)_{n \geq 0}$ of positive real numbers (the weights) we define *the weighted Hardy space $H^2(\beta)$* as the space formed by all functions f analytic at 0 and whose power series at 0, $f(z) = \sum_{n \geq 0} a_n z^n$, satisfies

$$\|f\|^2_{H^2(\beta)} := \sum_{n \geq 0} |a_n|^2 \beta_n < +\infty \,.$$

We will always assume that the sequence of weights (β_n) satisfies the condition

$$\liminf_{n \to \infty} \beta_n^{1/n} \geq 1,$$

and so, every function $f \in H^2(\beta)$ is holomorphic on the whole open unit disc $\mathbb{D}$.

Many classical Hilbert spaces of analytic functions appear as weighted Hardy spaces. For instance, this is the case for Hardy, Bergman and Dirichlet spaces.

If $\varphi \colon \mathbb{D} \to \mathbb{D}$ is holomorphic, the composition operator C_φ of symbol φ is simply

$$C_\varphi \colon f \mapsto f \circ \varphi, \qquad \text{for all } f \in Hol(\mathbb{D}).$$

It is known that, for every symbol φ, C_φ defines a bounded operator on Hardy and Bergman spaces; but this is not true on the Dirichlet space.

In this talk I will present some results, included in [1] and [2], obtained in collaboration with P. Lefèvre, D. Li and H. Queffélec. We will deal with some problems around the following question raised by N. Zorboska in the eighties:

> Determine the sequences $\beta = (\beta_n)_{n \geq 0}$ for which every symbol $\varphi \colon \mathbb{D} \to \mathbb{D}$ defines a bounded composition operator $C_\varphi \colon H^2(\beta) \to H^2(\beta)$.

References

[1] P. Lefèvre, D. Li, H. Queffélec, L. Rodríguez-Piazza, *Characterization of weighted Hardy spaces on which all composition operators are bounded.* To appear in Analysis & PDE.

[2] P. Lefèvre, D. Li, H. Queffélec, L. Rodríguez-Piazza, *On some questions about composition operators on weighted Hardy spaces.* To appear in Pure and Applied Functional Analysis.