
Mellin Transform and Exponential Polynomial Methods in the Study of the Square Root Problem for Positive Measures

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Abstract

For recursively generated shifts, we provide definitive answers to two outstanding problems in the theory of unilateral weighted shifts: the Subnormality Problem (**SP**) (related to the Aluthge transform) and the Square Root Problem (**SRP**) (which deals with Berger measures of subnormal shifts).

In joint work with H. El Azhar, Y. Omari and E.H. Zerouali, we use the Mellin Transform and the theory of exponential polynomials to establish that (**SP**) and (**SRP**) are equivalent if and only if a natural functional equation holds for the canonically associated Mellin transform. For p -atomic measures with $p \leq 6$, our main result provides a new and simple proof of the above-mentioned equivalence.

Subsequently, we obtain an example of a 7-atomic measure for which the equivalence fails. This provides a negative answer to a problem posed by G.R. Exner in 2009, and to a recent conjecture formulated by R.E. Curto et al in 2019.

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References

[1] R.E. Curto, H. El Azhar, Y. Omari and E.H. Zerouali, The Square Root Problem and subnormal Aluthge transforms of recursively generated weighted shifts, *Integral Equations Operator Theory* 96(2024), art. 33; 17 pp.

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