Introduction to the dynamical properties of Toeplitz operators on the Hardy space of the unit disc.

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In this mini-course, we will discuss the hypercyclicity of Toeplitz operators T_φ defined on the Hardy space H^2 of the open unit disk. The first part of the course will provide a brief introduction to the notion of hypercyclicity. In particular, we will prove the Godefroy–Shapiro criterion for hypercyclicity in terms of the density of eigenvectors. We will then present some basic facts about Toeplitz operators on the Hardy space. Finally, we will prove two results concerning the hypercyclicity of Toeplitz operators.

The first result concerns anti-analytic symbols and is due to Godefroy and Shapiro (1991). The second result concerns symbols of the form $\varphi(e^{i\theta}) = ae^{-i\theta} + b + ce^{i\theta}$, with $\theta \in \mathbb{R}$ and $a, b, c \in \mathbb{C}$, and is due to Shkarin (2012). If time permits, I will briefly discuss some more recent results by Baranov–Lishanskii, Abakumov–Baranov–Charpentier–Lishanskii, and Grivaux–Ostermann and myself.