

## Short talk: On $L^2$ posterior contraction rates in Bayesian nonparametric regression models (Chair Olivier Roustant))

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The nonparametric regression model with normal errors has been extensively studied, both from the frequentist and Bayesian viewpoint. A central result in Bayesian nonparametrics is that under assumptions on the prior, the data-generating distribution (assuming a true frequentist model) and a semi-metric  $d(\cdot, \cdot)$  on the space of regression functions that satisfy the so called testing condition, the posterior contracts around the true distribution with respect to  $d(\cdot, \cdot)$ , and the rate of contraction can be estimated. In the regression setting, the semi-metric  $d(\cdot, \cdot)$  is often taken to be the Hellinger distance or the empirical  $L^2$  norm (i.e., the  $L^2$  norm with respect to the empirical distribution of the design) in the present regression context. Typical examples of priors include Gaussian processes for which the theory can be elegantly simplified. However, extending contraction rates to the “integrated”  $L^2$  norm usually requires more work, and has previously been done for instance under sufficient smoothness or boundedness assumptions, which may not necessarily hold. In this work we show that, for priors based on truncated random basis expansions and in the random design setting, a high probability two sided inequality between the empirical  $L^2$  norm and the integrated  $L^2$  norm holds in appropriate spaces of functions of low frequencies, under mild assumptions on the underlying basis (which can be for instance a Fourier, wavelet or Laplace eigenfunction basis), allowing us to directly deduce an  $L^2$  contraction rate from an empirical  $L^2$  one without further assumption on the true regression function. We also discuss extensions to semi supervised learning on graphs, where the basis is estimated from the data itself.

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