Quantum Gravitational Detectorology

Murat Koloğlu

Yale University

based on

WIP with E. Herrmann, I. Moult.

See also:

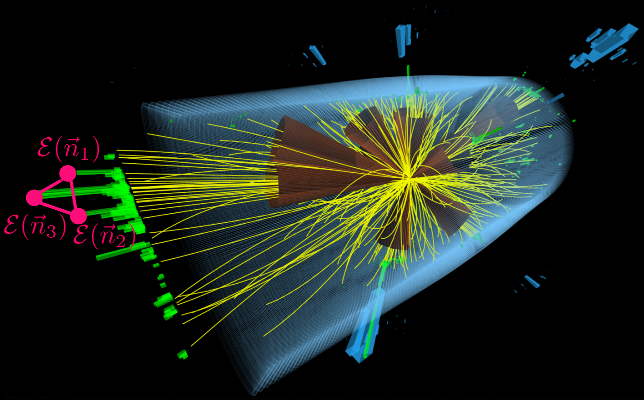
[arXiv:2209.00008]

with S. Caron-Huot, P. Kravchuk, D. Meltzer, D. Simmons-Duffin.

November 26, 2024

What are detectors?

Detectors at the LHC

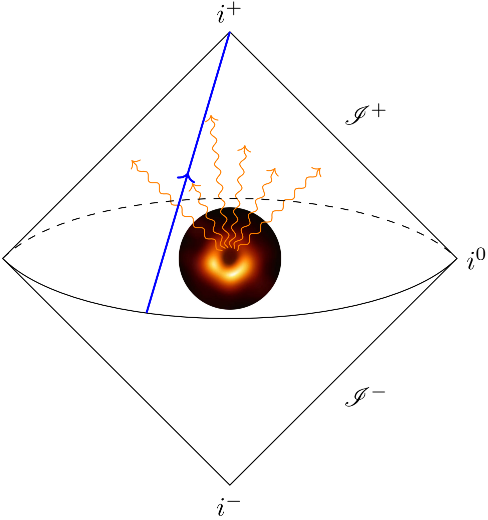


$$\mathcal{E}(\hat{n}) = \lim_{r\to\infty} \int_0^\infty dt r^2 n^i T_{0i}(t, r\hat{n}) \, .$$

[Basham Brown Ellis Love, Korchemsky Sterman]

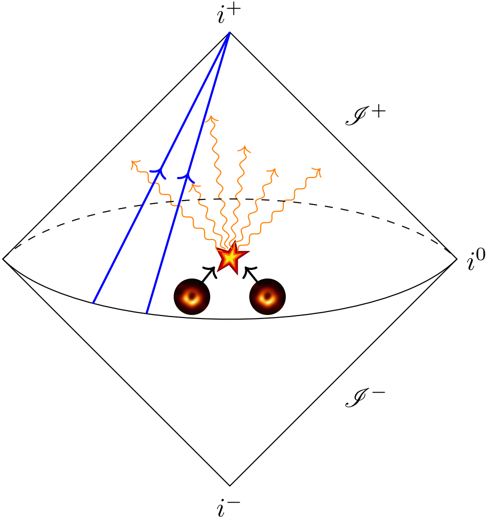
Detectors are light-ray operators

Energy detector $\mathcal{E}(\widehat{n}) = \mathbf{L}[T](\infty, z)$, $z = (1, \widehat{n})$ in a one-point event shape [EHT '19]



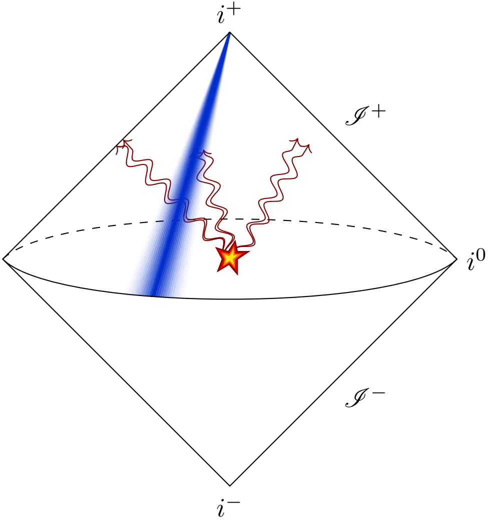
Event shapes, energy-energy correlators

$$\langle \text{BH BH} | \mathcal{E}(\widehat{n}_1)\mathcal{E}(\widehat{n}_2) | \text{BH BH} \rangle$$



General detectors

More general detectors are fuzzier, measuring nontrivial angular distributions.



What is the space of detectors?

What are allowable asymptotic measurements?

What is the “space” of detectors in a theory (CFT)?

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Detectors $\Leftrightarrow$ what we can measure in a cross section.

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Theory dependent!

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Theory dependent!

C.f. bare local operator $\mathcal{O}_0(x)$: “measure at a point”

$\Rightarrow$ renormalize: “good” operator $\quad \mathcal{O}_R(x) = Z\mathcal{O}_0(x)$.

e.g. $\phi^2(x)$ in Wilson-Fisher theory.

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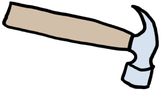
e.g. $\phi^2(x)$ in Wilson-Fisher theory.

Similarly, detectors $\mathcal{D}_0$ suffer from IR divergences

$\Rightarrow$ renormalize detectors: $\mathcal{D}_R = Z\mathcal{D}_0$

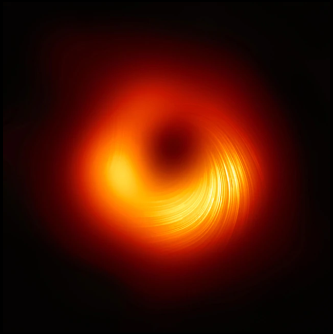
On hammers and cameras

 $= \sum_i h_i \mathcal{O}_i$

 $= \sum_j c_j \mathcal{D}_j$

experiment vs dynamics

No hammers just cameras in gravity



$\mathcal{O}$ vs $\mathcal{D}$

local operator $\mathcal{O}$	detector $\mathcal{D}$
"measure at a point"	"measure in cross-sections"
QFT: ✓ GR: X	QFT: ✓ GR: ✓
UV divergence	IR divergence
need to renormalize	need to renormalize
theory-dependent	theory-dependent
OPE	light-ray OPE
radial quantization	?

Some simple detectors

$$\mathcal{E}_J(\widehat{n}), \qquad \widehat{n} \in S^{d-2}$$

Free massless theory: counts particles propagating along $\widehat{n}$, weighted by E^{J-1}.

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Free massless theory: counts particles propagating along $\widehat{n}$, weighted by E^{J-1}.

Turn on interactions: $J \neq 2$ is not IR safe!

E is conserved. $E^{\#}$ not due to soft and collinear radiation.

Renormalize to obtain good detectors.

Detectors in free massless scalar theory

Leading twist trajectory in free theory given by "E^{J-1}" detectors:

$$\mathbf{L}[T](\widehat{n}) = \mathcal{E}_2(\widehat{n}) \propto \int_0^\infty dE\, E^{d-2} a^\dagger(E\widehat{n}) a(E\widehat{n})$$

$$\mathbb{O}_J(\widehat{n}) = \mathcal{E}_J(\widehat{n}) \propto \int_0^\infty dE\, E^{J+d-4} a^\dagger(E\widehat{n}) a(E\widehat{n})$$

For $J \in 2\mathbb{Z}_{\geq 0}$,

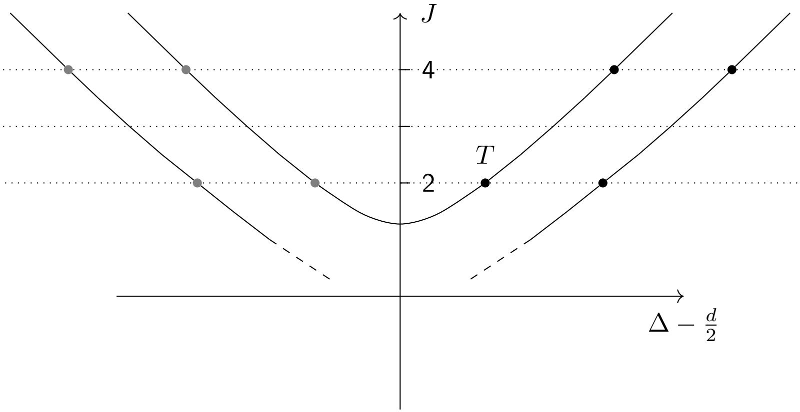
$$\mathcal{E}_J = \int_{\text{null line}} \mathcal{O}_J$$

where $\mathcal{O}_J = \phi \partial_{\mu_1} \cdots \partial_{\mu_J} \phi$.

$$\Rightarrow \gamma_{\mathcal{E}_J} \sim \gamma_{\mathcal{O}_J}$$

More generally, $\mathcal{E}_J$ makes sense as a "light-ray operator" of the leading twist Regge trajectory. In particular, not $\int_{\text{null line}} \mathcal{O}$ for some $\mathcal{O}$.

Regge trajectories and the Chew-Frautschi plot



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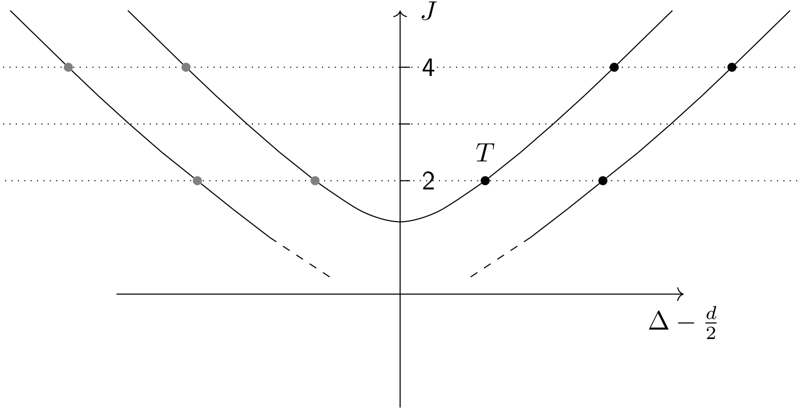

Local operators do not make sense at continuous J. Trajectories are light-ray operators $\mathbb{O}^{\pm}_{i,J}$. [Kravchuk Simmons-Duffin '18]

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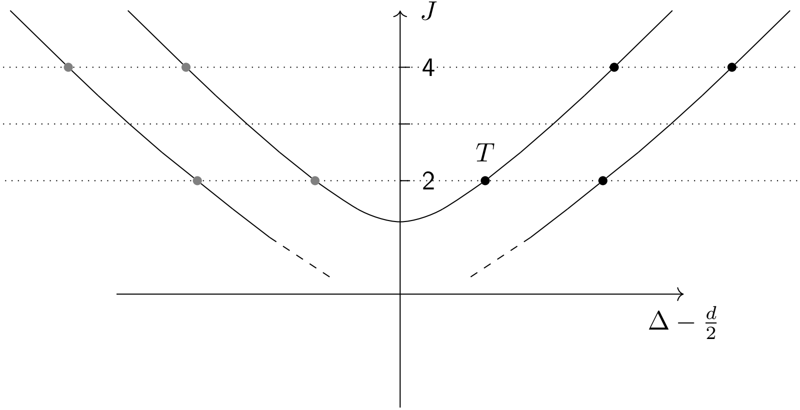

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Today: What does this plot look like in perturbative quantum gravity?

Some questions on detectors in quantum gravity

1. What is the spectrum of asymptotic detector operators in quantum gravity?
2. What are the dynamics of asymptotic states and detector operators in quantum gravity?
3. What is the detector OPE in quantum gravity?
4. What is the interplay of asymptotic symmetries with detector operators?

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We begin with a few humble first steps.

Need to compute accessibles observables to address these questions.

$\Rightarrow$ Detector event shapes.

Suitable observables: event shapes

In order to understand detector operators and extract physics, we need to study their correlation functions inside suitable states: event shapes

$$\langle \Psi | \mathcal{D} \cdots \mathcal{D} | \Psi \rangle \,.$$

Energy correlators have been very useful in extracting interesting physics from collider experiments, e.g. extraction of α_s from the EEC at LHC [CMS '23].

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What can the EEC and other event shapes teach us in gravitational theories?

Event shapes in gravity: classical and quantum

Recently, classical parts of event shapes in gravity have been studied, via soft limits [Gonzo Pokraka '20]. They have also computed the factorizing piece $\langle \mathcal{E} \rangle^2$ of the EEC in gravity. This does not include the correlations between detectors: "$\langle \mathcal{E}^2 \rangle - \langle \mathcal{E} \rangle^2$".

Detector one-point functions are very useful and natural observables. For example, LIGO measurements of gravitational waves can be thought of as one-point event shapes

$$\langle \text{LIGO detector} \rangle \sim \text{gravitational waveform} \sim \langle \int d\alpha e^{i\omega\alpha} \partial_v h_{ij} \rangle \, .$$

A lot of interesting work in computing waveforms and other classical observables [Kosower Maybee O'Connell '18, ...].

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Focus has been on properties of states rather than of detectors. We propose the reverse. Compute quantum correlations of detectors in simple states and extract properties of detectors.

Plan

- Review detectors / light-ray operators in QFT.
- Construct some simple detectors in gravity.
- Reformulate computation of detector event shapes in perturbation theory from underlying scattering amplitudes.
- Compute EEC (and more) in perturbative quantum gravity.
- More in progress!

Results

- EEC in gravity to leading nontrivial order in G_N.
- Collinear DDC in gravity to leading nontrivial order in G_N.

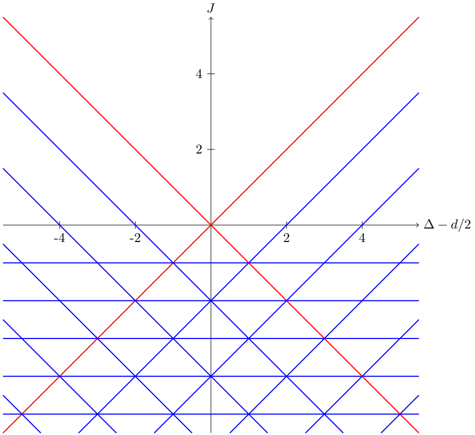
In progress:

- Collinear EEEC in gravity to leading nontrivial order in G_N.
- Energy correlators in supergravity.
- Loop corrections and detector renormalization in gravity/supergravity.

Review: Detectors in QFT

Detectors in free (perturbative) massless scalar theory

Expected structure:



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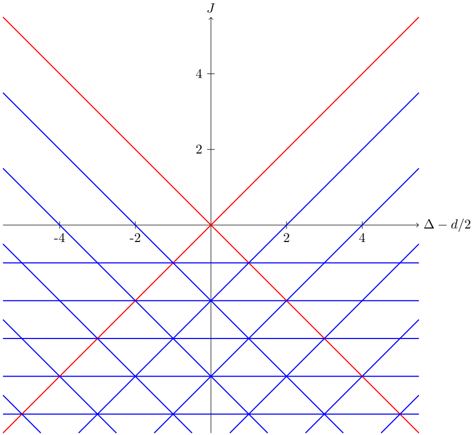


Features: Diagonal trajectories, shadow symmetry, intersections, and horizontal trajectories.

Review: Light-transform

Introduce polarizations,

$$\mathcal{O}(x,z) \equiv \mathcal{O}^{\mu_1\cdots\mu_J}(x) z_{\mu_1}\cdots z_{\mu_J}.$$

Spin is homogeneity degree in z:

$$\mathcal{O}(x,\lambda z) = \lambda^J \mathcal{O}(x,z).$$

Define light-transform of a local operator

$$\mathbf{L}[\mathcal{O}](x,z) = \int_{-\infty}^{\infty} d\alpha\,(-\alpha)^{-\Delta-J}\mathcal{O}\left(x-\frac{z}{\alpha},z\right).$$

Quantum numbers are

$$\mathbf{L} : (\Delta, J) \to (1-J, 1-\Delta).$$

Continuous spin representation of the Lorentz group.

$$\mathbf{L}[\mathcal{O}](x,z)|\Omega\rangle = 0.$$

Review: general light-ray operators [Kravchuk Simmons-Duffin '18]

More general light-ray operators $\mathbb{O}(x, z)$ are not $\mathbf{L}[\mathcal{O}]$ for some $\mathcal{O}$.

Convenient to denote the quantum numbers of $\mathbb{O}$ as

$$(\Delta_L, J_L) \equiv (1 - J, 1 - \Delta).$$

Non-perturbatively, they are best thought as more general representations appearing in the OPE of local operators.

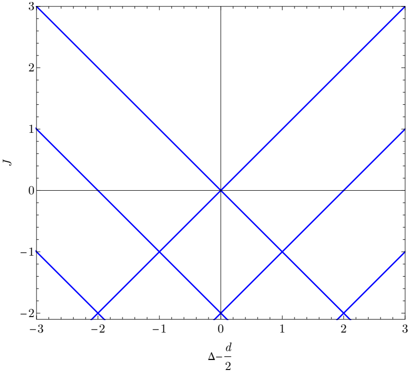
They can be constructed by projecting via a suitable kernel

$$\mathbb{O}_{\Delta,J}(x, z) = \int d^d x_1 d^d x_2 \mathcal{K}_{\Delta,J}(x_1, x_2; x, z) \mathcal{O}_1(x_1) \mathcal{O}_2(x_2)\,.$$

The residues (coming from lightcone singularities $\Leftrightarrow$ OPE) are the light-ray operators

$$\mathbb{O}_{\Delta,J}(x, z) \sim \frac{\mathbb{O}_{i,J}(x, z)}{\Delta - \Delta_i(J)}\,.$$

Shadow trajectory

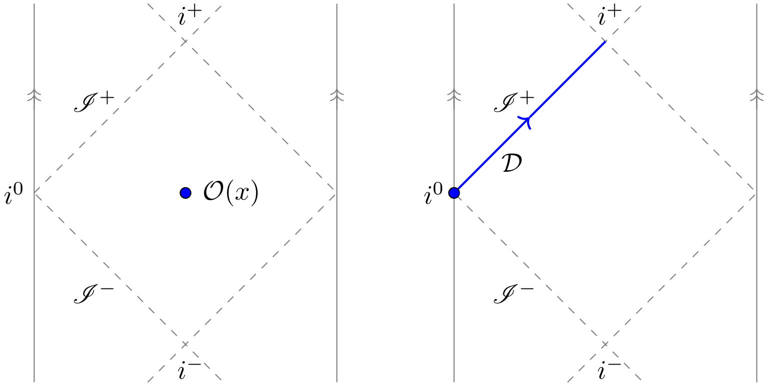


$$\mathbf{S}_J[\mathbb{O}](x,z) = \int D^{d-2}z'(-2z\cdot z')^{2-d-J_L}\,\mathbb{O}(x,z')$$

$$\text{where} \qquad D^{d-2}z = \frac{2d^dz\delta(z^2)\theta(z^0)}{\mathrm{vol}\,\mathbb{R}}$$

Note homogeneity in z: $1-d/2-\nu \mapsto 1-d/2+\nu$.
Shadow transform implements $\nu \leftrightarrow -\nu$ symmetry of Chew-Frautschi plot.

Light-ray operators as detectors: the detector frame



Detectors in free scalar theory

Sample primary detectors:

$$\mathcal{D}_\psi(z) = \int d\alpha_1 \dots d\alpha_n \, \psi(\alpha_1, \dots, \alpha_n) : \phi(\alpha_1, z) \cdots \phi(\alpha_n, z) :$$

where

$$\phi(\alpha; z) = \lim_{L \to \infty} L^{\Delta_\phi} \phi(x + Lz), \qquad \alpha = 2x \cdot z \text{ and } \Delta_\phi = \frac{d-2}{2}.$$

Primary if $\psi(\alpha_i)$ is homogenous and translationally invariant, since

$$[P^\mu, \phi(\alpha, z)] = -2z^\mu \partial_\alpha \phi(\alpha, z).$$

Detector spin:

$$J_L[\mathcal{D}_\psi] = n(1 - \Delta_\phi) + \deg_\alpha \psi \, .$$

Twist-2 detectors in free scalar theory

Leading trajectory detectors measuring "E^{J-1}" given by

$$\mathcal{D}_{J_L}(z) \equiv \frac{1}{C_{J_L}} \int d\alpha_1 d\alpha_2 |\alpha_1 - \alpha_2|^{2(\Delta_\phi - 1) + J_L} : \phi(\alpha_1, z)\phi(\alpha_2, z) : .$$

Diagonal trajectory with quantum numbers satisfying

$$\Delta_{L,0} = J_L + d - 2\,.$$

Shadow trajectory $\widetilde{\mathcal{D}}_{J_L}(z) = \mathbf{S}_J[\mathcal{D}_{2-d-J_L}](z)$ with $\Delta_{L,0} = -J_L$.
Together, they are the $\nu = \pm J$ trajectories.

Compute matrix elements in perturbation theory and renormalize!

Detector smorgasbord: results in Wilson-Fisher theory

- Construct and renormalize the leading twist-2 Regge trajectory $\mathcal{D}_{J_L}$ from detector renormalization.
- Construct and renormalize the Pomeron by resolving the intersection of the leading trajectory $\mathcal{D}_{J_L}$ with its shadow $\widetilde{\mathcal{D}}_{J_L} \equiv \mathbf{S}_J[\mathcal{D}_{2-d-J_L}]$.
- Construct and renormalize horizontal trajectories, explicitly at $J = -1$:

$$\mathcal{H}_{J_L}(z) = \int D^{d-2}z_1 D^{d-2}z_2 K_{J_L}(z_1, z_2; z) :\mathbf{L}[\phi](z_1)\mathbf{L}[\phi](z_2):$$

- Compute and renormalize higher-point event shapes: $\langle\phi(p)|\mathcal{D}_{J_{L1}}(z_1)\mathcal{D}_{J_{L2}}(z_2)|\phi(p)\rangle, \ldots$ [Gonzalez MK Moult, WIP]
- Compute the most general light-ray OPE in perturbation theory. [WIP]
- Investigate detectors in similar theories such as $O(N)$ models, including "$E^{J-1} \times Q$" detectors, $\mathcal{D}^-_{J_L}$. [Gonzalez MK Korchemsky Moult Zhiboedov, WIP]
- Even more general theories!

Detectors in perturbative gravity

Graviton energy detectors in perturbative gravity

Take $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G_N} h_{\mu\nu}$. Useful to define the asymptotic metric field

$$h_{\mu\nu}(\alpha, z) = \lim_{L\to\infty} L^{\Delta_h}\, h_{\mu\nu}(x + Lz)\,.$$

where $\Delta_h = \frac{d-2}{2}$. Graviton energy detector is

$$\mathcal{E}_h(z) = 2\int_{-\infty}^{\infty} d\alpha :(\partial_\alpha h_{\mu\nu}(\alpha, z))(\partial_\alpha h^{\mu\nu}(\alpha, z)) :$$

Originally introduced by [Gonzo Pokraka '20] in $d = 4$, Bondi gauge:

$$\begin{aligned}\mathcal{E}_h(z) &= \frac{1}{16\pi G_N}\int_{-\infty}^{\infty} dv (\partial_v C_{ww})(\partial_v C^{ww}) \\ &= 2\int_{-\infty}^{\infty} dv \lim_{r\to\infty} \frac{1}{r^2}(\gamma^{w\overline{w}})^2 \left(\partial_v h_{ww}(r, v, w, \overline{w})\right)\left(\partial_v h_{\overline{ww}}(r, v, w, \overline{w})\right).\end{aligned}$$

Graviton energy detectors: mode expansion

In terms of the mode expansion

$$h_{\mu\nu}(x) = \int \widehat{d}^d p \, \widehat{\delta}^+(p^2) \sum_s \left[\epsilon^{s,*}_{\mu\nu}(p) a_s(p) e^{-ip\cdot x} + \epsilon^s_{\mu\nu}(p) a^\dagger_s(p) e^{ip\cdot x}\right] \,,$$

Here, $\epsilon^s_{\mu\nu}$ are polarization tensors, where s runs over physical polarizations.

Evaluating $h_{\mu\nu}(\alpha, z)$ in terms of the mode expansion using a stationary phase method yields

$$h_{\mu\nu}(\alpha, z) \propto \int_0^\infty d\beta \beta^{\Delta_h - 1} \sum_s \left[i e^{-i\frac{d\pi}{4}} \epsilon^{s,*}_{\mu\nu}(\beta z) a_s(\beta z) e^{-i\frac{\beta\alpha}{2}} - i e^{i\frac{d\pi}{4}} \epsilon^s_{\mu\nu}(\beta z) a^\dagger_s(\beta z) e^{i\frac{\beta\alpha}{2}}\right].$$

Yielding

$$\mathcal{E}_h(z) \propto \int_0^\infty d\beta \beta^{d-2} \sum_s a^\dagger_s(\beta z) a_s(\beta z)\,.$$

A Regge trajectory of detectors in gravity

Graviton E^{J-1} detector trajectory is

$$\mathcal{D}_{J_L}(z) = \frac{1}{C_{J_L}} \int_{-\infty}^{\infty} d\alpha_1 d\alpha_2 \, \psi_{J_L}(\alpha_1, \alpha_2) : h_{\mu\nu}(\alpha_1, z) h^{\mu\nu}(\alpha_2, z) : \, .$$

The wavefunction is

$$\psi_{J_L}(\alpha_1, \alpha_2) = |\alpha_1 - \alpha_2|^{2(\Delta_h - 1) + J_L} \, .$$

Acts on asymptotic graviton states $|X\rangle = |\{p_1, s_1\}, \cdots, \{p_n, s_n\}\rangle$ as

$$\mathcal{D}_{J_L}(z)|X\rangle = \sum_{i \in X} \int_0^{\infty} d\beta \beta^{J+d-2} \frac{\delta^d(k_i - \beta z)}{2\delta(z^2)} |X\rangle \, .$$

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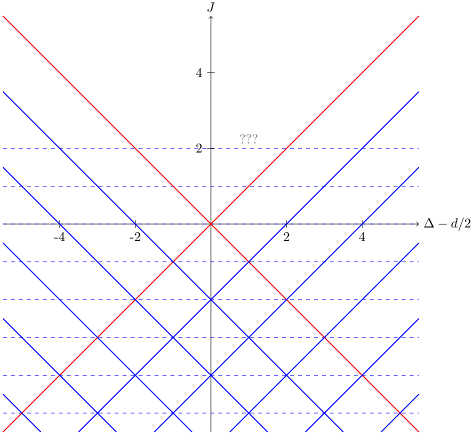
One can also construct higher-twist trajectories, and all of their shadows just as in QFT.

The start of a “Chew-Frautschi plot” in perturbative gravity



Horizontal trajectories in GR? Analogs of “reggeized gravitons” as detector trajectories? What else is missing?

Event shapes from amplitudes

Perturbation theory in the detector frame

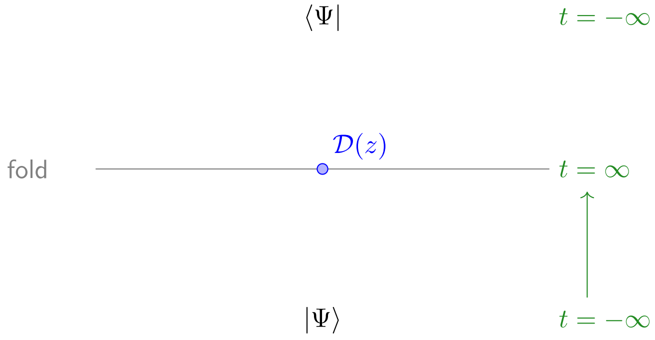
Recall, detectors annihilate the vacuum:

$$\mathcal{D}(z)|\Omega\rangle = 0\,.$$

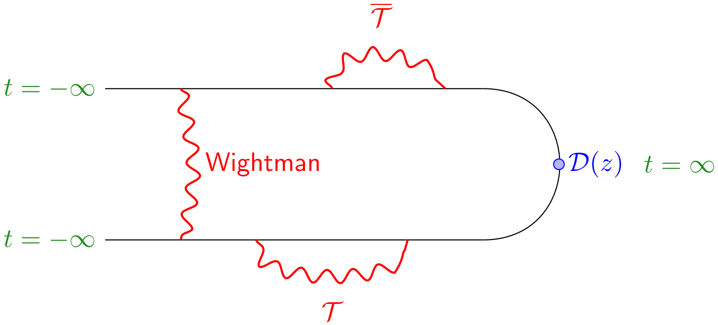
So, compute observables:

$$\langle\Psi_1|\mathcal{D}(z)|\Psi_2\rangle.$$

Use in-in formalism with Schwinger-Keldysh contour:



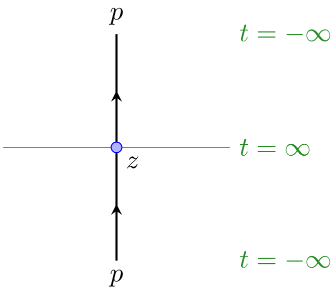
Schwinger-Keldysh contour and Feynman rules



Tree level matrix element: scalar detector

Tree level matrix element

$$\langle 0|\phi(-q)\mathcal{D}_{J_L}(z)\phi(p)|0\rangle = (2\pi)^d \delta^d(p-q) V_{J_L}(z;p)$$



One computes

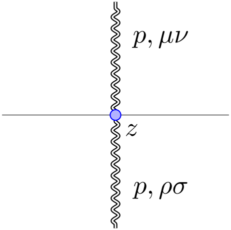
$$V_{J_L}(z;p) = \int_0^\infty d\beta \beta^{-J_L-1} \delta^d(p - \beta z).$$

Tree level matrix element: graviton detector

Tree level matrix element

$$\langle 0|h_{\mu\nu}(-q)\mathcal{D}_{J_L}(z)h_{\rho\sigma}(p)|0\rangle = (2\pi)^d\delta^d(p-q)\Pi_{\mu\nu\rho\sigma}(p)V_{J_L}(z;p)$$

where Π is the projector onto physical states.

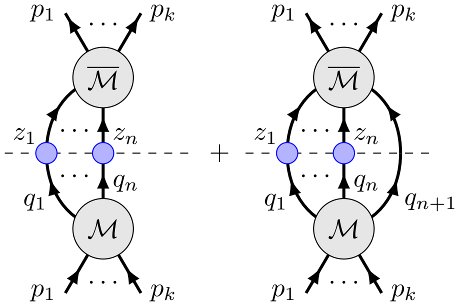


Once again, universal factor:

$$V_{J_L}(z;p) = \int_0^\infty d\beta\beta^{-J_L-1}\delta^d(p-\beta z).$$

Event shapes from perturbative amplitudes

$\mathbb{E}_n =$



$+ \quad \cdots$

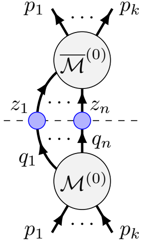
$$= \frac{1}{N_k} \sum_X \int \left[\prod_{a=1}^{n} \widehat{d}^d q_a \, V_{J_{La}}(q_a, z_a) \right] \left[\prod_{b \in X} \widehat{d}^d q_b \, W(q_b) \right] \times$$

$$\widehat{\delta}^d(q_1 + \ldots + q_n + Q_X - P) \left| \mathcal{M}_{k \to n+|X|}(p_i, q_j) \right|^2 .$$

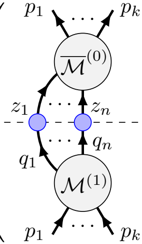
Loop expansion of event shapes

$$\mathbb{E}_n = g^{2(k+n-2)} \left[\mathbb{E}_n^{(0)} + g^2 \widetilde{\mathbb{E}}_n^{(1)} + \cdots \right]$$

$$= g^{2(k+n-2)} \Bigg[$$



$$+ g^2 \Bigg($$



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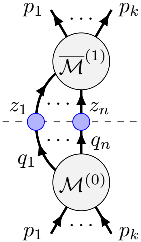

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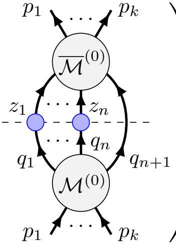
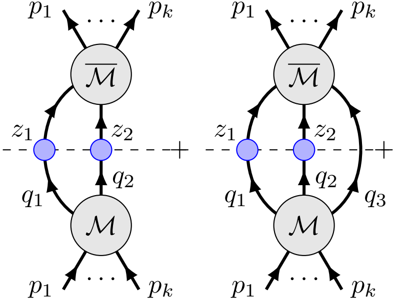


$$\Bigg) + \cdots \Bigg]$$

Two point event shape from perturbative amplitudes

$$\mathbb{E}_2 = \mathbb{E}_2^{(0)} + \mathbb{E}_2^{(1)} + \cdots =$$



$$\cdots .$$

$$\mathbb{E}_2^{(1)}(p_i; z_1, z_2) = \int \prod_{a=1}^{3} d^d q_a \, V_{J_{L1}}(q_1, z_1) \, V_{J_{L2}}(q_2, z_2) \, \delta^+(q_3^2) \times \left|\mathcal{M}_{k\to 3}(p_i, q_1, q_2, q_3)\right|^2 \delta^d(q_1 + q_2 + q_3 - P)$$

$$= \int_0^\infty d\beta_1 d\beta_2 \beta_1^{-J_{L1}-1} \beta_2^{-J_{L2}-1} \delta^+((P - \beta_1 z_1 - \beta_2 z_2)^2) \times \left|\mathcal{M}_{k\to 3}(p_i, q_1 = \beta_1 z_1, q_2 = \beta_2 z_2, q_3 = P - \beta_1 z_1 - \beta_2 z_2)\right|^2 .$$

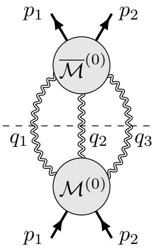
Gravity amplitudes

A simple state in a simple theory of gravity

Consider Einstein gravity with minimally coupled massive scalar field:

$$S_{\mathrm{EH}+\Phi} = \int d^d x \sqrt{-\mathfrak{g}} \left(\frac{1}{16\pi G_N} R + \frac{1}{2} \Phi \left(\Box - m^2 \right) \Phi \right) .$$

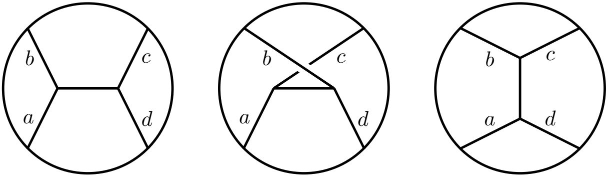
We are interested in the EEC in a simple state; we pick two annihilating scalars. To compute the EEC we need the following underlying amplitude squared:



where $\mathcal{M}^{(0)}_{2\to 3}$ is the tree level amplitude for $\Phi\Phi \to hhh$.

Color-kinematics duality and double copy

Using cubic representation of tree level amplitudes, in manifestly color-kinematic dual form:



$$\Leftrightarrow \; f^{abe}f^{ecd} - f^{ace}f^{ebd} - f^{aed}f^{ebc} = 0.$$

Double copy gauge theory amplitudes to gravity amplitudes:

$$i\mathcal{A}_m^{\text{tree}}(1,2,3,\ldots,m) = g^{m-2}\sum_i \frac{n_i c_i}{\prod_{\alpha_i} p_{\alpha_i}^2}\,,$$

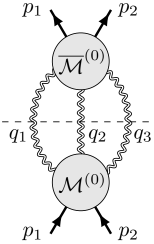
replace $c_i \to \tilde{n}_i$:

$$i\mathcal{M}_m^{\text{tree}}(1,2,\ldots,m) = \left(\frac{\kappa}{2}\right)^{m-2}\sum_i \frac{n_i\,\widetilde{n}_i}{\prod_{\alpha_i} p_{\alpha_i}^2}\,.$$

Reverse unitarity

Can compute amplitude and sew and square it, or use reverse unitarity on a suitable loop amplitude:

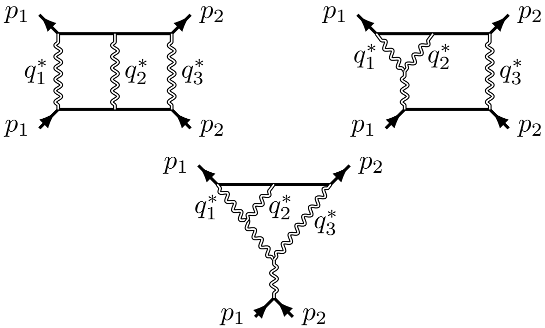
$$\text{Cut}\left[\mathcal{M}^{(2)}_{2\to 2}(p_1,p_2\to p_1,p_2)\right] =$$



$$= \sum_{\text{states}} \mathcal{M}^{(0)}_{2\to 3}(p_1,p_2;q_i)\overline{\mathcal{M}}^{(0)}_{2\to 3}(p_1,p_2;q_i).$$

Some of the diagrams that enter into the computation

$\mathcal{M}^{(0)}_{2\to 3}$ is a sum of 15 (cubic) diagrams, $|\mathcal{M}^{(0)}_{2\to 3}|^2$ is $15 \times 15 = 225$ diagrams, e.g.:



Gravitational Detectorology

Gravitational energy correlators

Want to compute EEC, EEEC in asymptotically-flat GR.

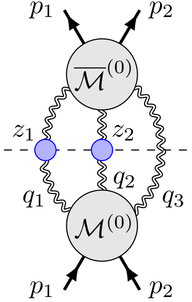
Easiest to compute in scattering massive scalars:

$$\text{EEC} = \langle \Phi(p_1)\Phi(p_2)|\mathcal{E}(z_1)\mathcal{E}(z_2)|\Phi(p_1)\Phi(p_2)\rangle \,,$$
$$\text{EEEC} = \langle \Phi(p_1)\Phi(p_2)|\mathcal{E}(z_1)\mathcal{E}(z_2)\mathcal{E}(z_3)|\Phi(p_1)\Phi(p_2)\rangle \,.$$

We have computed the EEC to leading nontrivial order in G_N. This is beyond the classical result, exhibiting quantum correlations between detectors in GR.

We are finishing computing the EEEC in the collinear limit.

Leading (finite-angle) G_N correction

$\mathbb{E}_2^{(1)} =$  .

The detector integrals

This leads to the following result, after performing the trivial integrals due to delta functions:

$$\mathbb{E}_2^{(1)}(p_i, z_j) = \frac{1}{(2\pi)^{2d-1}} (2P \cdot z_1)^{J_{L1}} (2P \cdot z_2)^{J_{L2}} (P^2)^{-J_{L1}-J_{L2}-1}$$
$$\times \int_0^1 d\alpha_1 \alpha_1^{-J_{L1}-1} (1-\alpha_1)^{-J_{L2}-1} (1-\alpha_1\zeta)^{J_{L2}} |\mathcal{M}|^2(p_i, q_j^*),$$

where $P = p_1 + p_2$, the detector separation cross ratio is

$$\zeta \equiv \zeta_{12} = \frac{(2z_1 \cdot z_2)(P^2)}{(2P \cdot z_1)(2P \cdot z_2)} = \frac{1 - \widehat{n}_1 \cdot \widehat{n}_2}{2}\,.$$

and the amplitude squared is evaluated at special kinematics for the gravitons

$$q_1^* = \frac{P^2}{2P \cdot z_1} \alpha_1 z_1, \quad q_2^* = \frac{P^2}{2P \cdot z_2} \frac{1-\alpha_1}{1-\alpha_1\zeta} z_2, \quad q_3^* = P - q_1^* - q_2^*\,.$$

The EEC integral

EEC is determined by the dimensionless integral:

$$\mathcal{G}^{(1)}_{\mathsf{EEC}}(\zeta,\chi_1,\chi_2,x) = \int_0^1 d\alpha\,\alpha^2\,(1-\alpha)^2\,(1-\alpha\,\zeta)^{-3}\left|\mathcal{M}_{2\to3}\left(\frac{p_i}{\sqrt{s}},\frac{q_j^*}{\sqrt{s}}\right)\right|^2 .$$

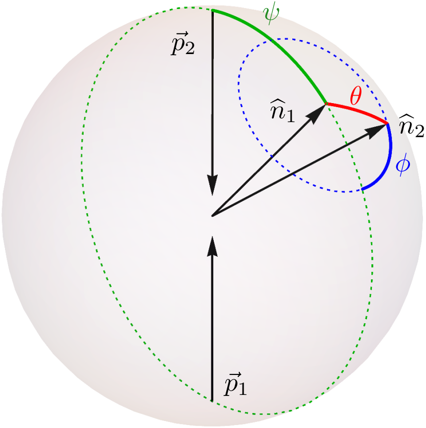
We have defined $P = p_1 + p_2$, $Q = p_1 - p_2$ and the following cross ratios,

$$\chi_a = -\frac{Q\cdot z_a}{P\cdot z_a}, \qquad x = \frac{\sqrt{s-4m^2}}{\sqrt{s}}.$$

In CoM frame,

$$\begin{aligned} \zeta &= \frac{1-\cos\theta}{2}, \\ \chi_1 &= x\cos\psi, \\ \chi_2 &= x\cos\theta\cos\psi - x\sin\theta\sin\psi\cos\phi\,. \end{aligned}$$

Kinematics in the center of mass (CoM) frame



$$s = -(p_1 + p_2)^2, \quad p_{1,2} = \frac{1}{2}(\sqrt{s}, \pm\widehat{z}\sqrt{s - 4m^2}), \quad z_i = (1, \widehat{n}_i)$$

EEC in GR: full angular dependence

EEC in gravity in its full glory:

$$\mathcal{G}^{(1)}_{\mathsf{EEC}}(\zeta,\chi_1,\chi_2,x) = r^{(0)}(\zeta,\chi_1,\chi_2,x)+\sum_{i=1}^{7} r^{(i)}(\zeta,\chi_1,\chi_2,x)\times f^{(i)}(\zeta,\chi_1,\chi_2)\,.$$

Rational coefficients $r^{(i)}$ of the basis of transcendental functions

$$f^{(1)}(\zeta,\chi_1,\chi_2) = \frac{\text{arctan}\left[\frac{\chi_2-\chi_1+2\zeta\,\chi_1}{\sqrt{\Delta}}\right]-\text{arctan}\left[\frac{\chi_2-\chi_1-2\zeta}{\sqrt{\Delta}}\right]}{\sqrt{\Delta}}\,,$$

$$f^{(2)}(\zeta,\chi_1,\chi_2) = \frac{\text{arctan}\left[\frac{\chi_2-\chi_1+2\zeta}{\sqrt{\Delta}}\right]-\text{arctan}\left[\frac{\chi_2-\chi_1-2\zeta}{\sqrt{\Delta}}\right]}{\sqrt{\Delta}}\,,$$

$$f^{(3)}(\zeta,\chi_1,\chi_2) = \log\left(1-\chi_1\right)\,, \qquad f^{(4)}(\zeta,\chi_1,\chi_2) = \log\left(1+\chi_1\right)\,,$$

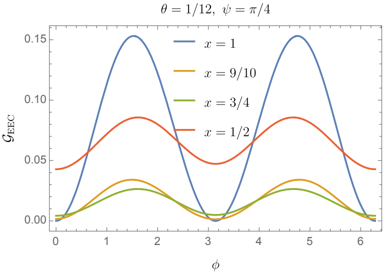
$$f^{(5)}(\zeta,\chi_1,\chi_2) = \log\left(1+\chi_2\right)\,, \qquad f^{(6)}(\zeta,\chi_1,\chi_2) = \log\left(1-\chi_2\right)\,,$$

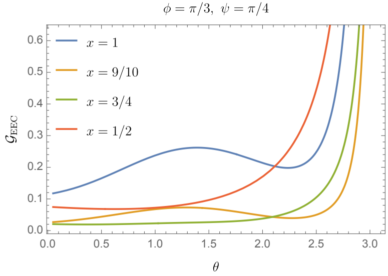
$$f^{(7)}(\zeta,\chi_1,\chi_2) = \log\left(1-\zeta\right)\,.$$

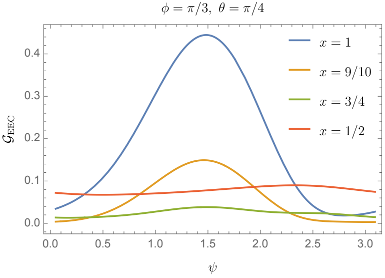
The discriminant Δ is given by

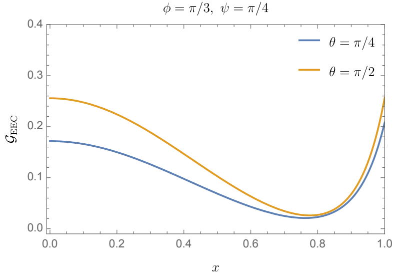
$$\Delta = 4\zeta(1-\zeta)+2\chi_1\chi_2(1-2\zeta)-\chi_1^2-\chi_2^2\,.$$

EEC in GR: 2d plots

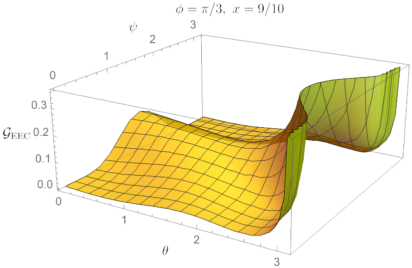


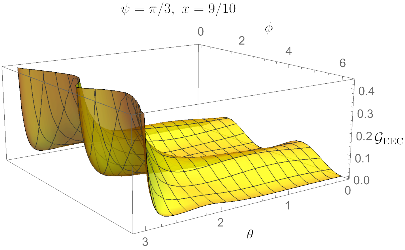






EEC in GR: 3d plots





EEC in GR: Collinear limit

The EEC in GR is finite in the collinear limit $\zeta \to 0$.

$$\mathcal{G}^{(1)}_{\mathsf{EEC}}(\theta \to 0, \psi, \phi, x) = g_0(\psi, x) + g_2(\psi, x)\cos(2\phi) + g_4(\psi, x)\cos(4\phi) + \mathcal{O}(\theta)\,,$$

where we define the coefficient functions

$$g_0(\psi, x) = \left(1 - x^2\right)^2 \left(\frac{11y^2}{36} + \frac{14y}{9} + \frac{2839}{840} + \frac{1037}{252y} + \frac{361}{105y^2} + \frac{9}{5y^3} + \frac{1}{2y^4}\right)$$

$$g_2(\psi, x) = \left(1 - x^2\right)^2 \left(-\frac{11y^2}{36} - \frac{14y}{9} - \frac{1003}{315} - \frac{459}{140y} - \frac{92}{45y^2} - \frac{7}{10y^3}\right)$$

$$g_4(\psi, x) = \left(1 - x^2\right)^2 \frac{(1+y)^2}{1260y^2}$$

and $y = \frac{(x^2\cos^2\psi - 1)}{(1-x^2)}$. In the massless limit $(x = 1)$,

$$\mathcal{G}^{(1)}_{\mathsf{EEC}}(\theta \to 0, \psi, \phi, x = 1) = \frac{11}{18}\sin^4\psi\sin^2\phi + \mathcal{O}(\theta)\,.$$

EEC in GR: back-to-back limit

Another interesting limit to study is in the back-to-back region, where $\theta \to \pi$. In comparison to the collinear limit, we find a power divergence as we approach $\theta \to \pi$:

$$\mathcal{G}^{(1)}_{\mathsf{EEC}}(\theta \to \pi, \psi, \phi, x) \sim \frac{b(\psi, \phi, x)}{(\theta - \pi)^2} + \mathcal{O}\left((\theta - \pi)^{-1}\right) \,,$$

where $b(\psi, \phi, x)$ is a transcendental function of the form

$$b(\psi, \phi, x) = b_0 + b_1 \operatorname{arctanh}\left(x \cos\psi\right) + b_2 \frac{\arctan\left(\frac{x \cos\phi \sin\psi}{\sqrt{\widetilde{\Delta}(\psi,\phi,x)}}\right)}{\sqrt{\widetilde{\Delta}(\psi, \phi, x)}}$$

where $\widetilde{\Delta}(\psi, \phi, x) = 1 - x^2\left(\cos^2\psi + \cos^2\phi \sin^2\psi\right)$.

Collinear DDC in GR

One can also compute the event shapes of more general E^{J-1} type detectors, $\mathcal{D}_{J_L}$, in the collinear limit.

$$\mathcal{G}^{(1)}_{J_{L1},J_{L2},\text{col.}}(\psi,\phi,x) = \int_0^1 d\alpha_1 \alpha_1^{-J_{L1}-1}\,(1-\alpha_1)^{-J_{L2}-1}\,|\mathcal{M}^{\text{col.}}_{2\to3}|^2\left(\frac{p_i}{\sqrt{s}},\frac{q_j^c}{\sqrt{s}}\right).$$

$$\mathcal{G}^{(1)}_{J_{L1},J_{L2},\text{col.}}(\psi,\phi,x) = \frac{\Gamma\left(2-J_{L1}\right)\Gamma\left(2-J_{L2}\right)}{\Gamma\left(4-J_{L1}-J_{L2}\right)}(1-x^2)^2\frac{(1+y)^2}{2y^2}\cos 4\phi + h_2(\chi,x)\cos 2\phi + h_0(\chi,x),$$

where h_2 and h_0 are some known but unwieldy polynomials in χ, x (meromorphic in J_{L1},J_{L2}).

The result has interesting pole structure in J_{L1} and J_{L2}. What are the implications for the spectrum of detectors in GR?

An IHES special

Fresh off the bakery: EEC in $\mathcal{N} = 8$ supergravity

Visiting IHES has already paid off! Thanks to Julio for his warm hospitality and for computing $|\mathcal{M}_{2\to 3}^{(0),\,\mathcal{N}=8}|$, I present...

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...drumroll ...

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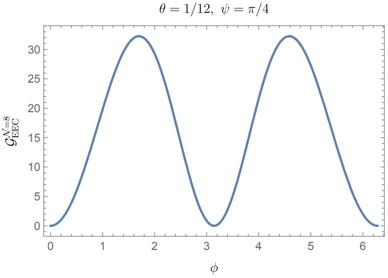
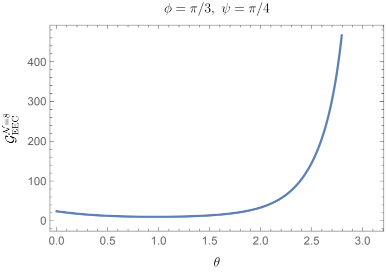
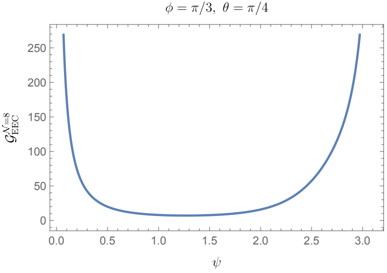
...drumroll ...

$$\mathcal{G}_{\mathsf{EEC}}^{(1),\mathcal{N}=8} = \frac{4\zeta^2 + 4\zeta\left(\chi_1\chi_2 - 1\right) + \left(\chi_1 - \chi_2\right)^2}{(1-\zeta)\zeta\left(1-\chi_1^2\right)\left(1-\chi_2^2\right)\left(2(2\zeta-1)\chi_2\chi_1 + \chi_1^2 + \chi_2^2\right)}$$
$$\times \Bigg[4\zeta\left(\chi_1+\chi_2\right)\frac{\arctan\left(\frac{(2\zeta-1)\chi_1+\chi_2}{\sqrt{\Delta}}\right)}{\sqrt{\Delta}}$$
$$+\left(\left(\chi_1-\chi_2\right)^2 + 2\zeta\left(2\chi_2\chi_1 + \chi_1 + \chi_2\right)\right)\frac{\arctan\left(\frac{2\zeta+\chi_1-\chi_2}{\sqrt{\Delta}}\right)}{\sqrt{\Delta}}$$
$$+\left(\left(\chi_1-\chi_2\right)^2 + 2\zeta\left(2\chi_2\chi_1 - \chi_1 - \chi_2\right)\right)\frac{\arctan\left(\frac{2\zeta-\chi_1+\chi_2}{\sqrt{\Delta}}\right)}{\sqrt{\Delta}}$$
$$+\left(\chi_1-\chi_2\right)\left(\operatorname{arctanh}\left(\chi_1\right) - \operatorname{arctanh}\left(\chi_2\right)\right)\Bigg].$$

Fresh off the bakery: EEC in $\mathcal{N} = 8$ supergravity plots







Fresh off the bakery: EEC in $\mathcal{N} = 8$ supergravity limits

Collinear limit:

$$\mathcal{G}_{\mathsf{EEC}}^{(1),\mathcal{N}=8}(\theta \to 0, \psi, \phi) = 8 \csc^4 \psi \sin^2 \phi + \mathcal{O}(\theta)\,.$$

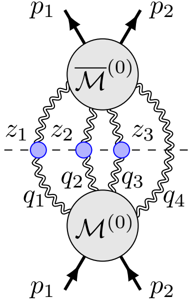
Back-to-back limit:

$$\begin{aligned}\mathcal{G}_{\mathsf{EEC}}^{(1),\mathcal{N}=8}(\theta \to \pi, \psi, \phi) = \frac{1}{(\theta - \pi)^2} \frac{4 \csc^4 \psi \sin \phi}{\left(\cot^2 \psi + \cos^2 \phi\right)} \\ \times \Big[4 \cos \psi \sin \phi \,\mathrm{arctanh}(\cos \psi) \\ - 2 \cos \phi \left(\pi - 2 \arctan(\cot \phi)\right) \Big] \\ + \mathcal{O}\left(\frac{1}{\theta - \pi}\right)\,.\end{aligned}$$

EEEC in gravity and supergravity

EEEC leading order

$$\mathbb{E}_3^{(1)}(p_i; z) \supset$$



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EEEC in the collinear limit

While the full EEEC is computationally challenging, the triple-collinear limit, where all three detectors approach each other with fixed ratios is achievable.

The computation is achieved by first computing the triple-collinear limit of the 2 scalar $\to 4$ graviton amplitude, squaring (sewing) the collinear amplitude, then performing the detector integrals:

$$\text{EEEC}\big|_{\text{coll}} \sim \int_0^1 d\alpha_1 d\alpha_2 \theta(1-\alpha_1-\alpha_2)\alpha_1^2\alpha_2^2(1-\alpha_1-\alpha_2)^2 \times |\mathcal{M}|^2_{2\to 4,\text{coll}}(p_i, q_j^*),$$

where

$$q_j^* = \frac{s}{2P\cdot z_j}\alpha_j z_j, \quad \text{for } 1 \le j \le 3, \qquad q_4^* = P - \sum_{j=1}^{3} q_i^*,$$

$$\alpha_3 = 1-\alpha_1-\alpha_2.$$

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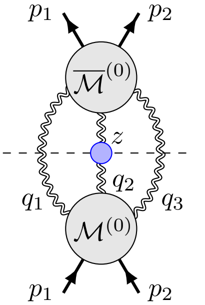
$$\alpha_3 = 1 - \alpha_1 - \alpha_2.$$

Interesting crossing equation and detector OPE encoded within. . .

Getting ahead of ourselves:
Renormalization and mixing of detectors in GR

Loop corrections to detector one-point functions in GR

$$\mathbb{E}_1^{(2)}(p_i; z) \supset$$



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Loop integrals

The qualitatively new piece to compute is the following loop integral

$$\mathbb{E}_1^{(2)}(p_i; z) \supset \int_0^\infty d\beta \beta^{-J_L-1} \int d^d q_2 \delta^+(q_2^2)\delta^+((P-\beta z - q_2)^2) \\ \times \left|\mathcal{M}_{k\to 3}(p_i, q_1{=}\beta z, q_2, q_3{=}P{-}\beta z{-}q_2)\right|^2 .$$

We expect interesting divergences to appear in this observable which indicate (additive) renormalization / mixing of the detectors.

No collinear divergences in gravity, but soft divergences!

Expected mixing

Since the interactions carry mass dimension, the expected renormalization is schematically:

$$[\mathcal{D}_{J_L}]_R = \mathcal{D}_{J_L} + G_N \mathbb{O}^{(1)}_{J_L} + G_N^2 \mathbb{O}^{(2)}_{J_L} + \cdots \ .$$

Onset of the vertical collapse of the Chew-Frautschi plot in a theory with scale.

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Onset of the vertical collapse of the Chew-Frautschi plot in a theory with scale.

One could detect the presence of horizontal trajectories as they should appear as poles in J in the loop corrections,

$\Rightarrow$ reggeized graviton trajectories?

Concluding remarks

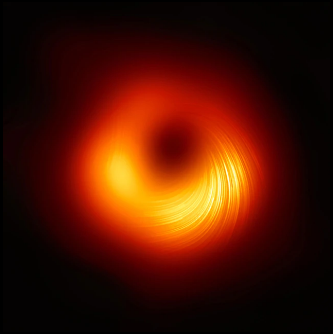
Conclusions

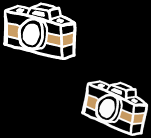
- Introduced new families of detector operators in gravity.
- Modernized computation of detector event shapes from perturbative amplitudes.
- Computed new observables in gravity and supergravity: EEC, EEEC...
- A foray into new quantum observables in gravity!

Gravitational hopes and dreams

- The spectrum of asymptotic observables (detectors) in GR.
- A notion of “Regge trajectories” of detectors in gravity? Analyticity?
- Descendent detectors, $\mathcal{K}$ and $\mathcal{N}_i$, and such.
- The light-ray OPE in gravitational theories.
- A unified language for scattering states and detectors.
- An improved understanding of asymptotic symmetries and the organization of detectors in representations.
- Coupling a CFT to gravity: deformation of a CFT Chew-Frautschi plot.

Thanks!





Appendix: Detectorology in WF

Detectors in Wilson-Fisher theory

Wilson-Fisher spectra

Recall, leading Regge trajectory

$$\mathcal{O}_J(x) = [\phi\phi]_{0,J}(x)$$
$$\mathbb{O}_J^+(x,z) = \mathbf{L}[\mathcal{O}_J](x,z) \qquad \text{for } J \in 2\mathbb{Z}_{\geq 0}$$

Analytic curve

$$\Delta(J) = 2\Delta_\phi + J + \gamma(J)$$

where

$$\Delta_\phi = 1 - \frac{1}{2}\epsilon + \frac{1}{108}\epsilon^2 + \frac{109}{11664}\epsilon^3 + \left(\frac{7217}{1259712} - \frac{2\zeta(3)}{243}\right)\epsilon^4 + O(\epsilon^5),$$
$$\gamma(J) = -\frac{1}{9J(J+1)}\epsilon^2 + \left(\frac{22J^2 - 32J - 27}{486J^2(J+1)^2} - \frac{2H(J)}{27J(J+1)}\right)\epsilon^3 + O(\epsilon^4),$$

with $H(J) = \frac{\Gamma'(J+1)}{\Gamma(J+1)} + \gamma_E$.

Note $(J_L, \Delta_L) = (1-\Delta, 1-J)$.

A toy model for intersecting trajectories

Consider diagonal shadow trajectories

$$\nu_{\pm}(J) = \pm J$$

Combine trajectory and its shadow into single complex curve

$$\nu^2 - J^2 = 0.$$

Turn on interactions. A toy model:

$$\begin{aligned} \nu^2 - J^2 + \epsilon^2 &= 0 \\ \Rightarrow \Delta &= \tfrac{d}{2} \pm \sqrt{J^2 - \epsilon^2} \\ &= \tfrac{d}{2} \pm \left(J - \frac{\epsilon^2}{2J} \right) + \cdots \end{aligned}$$

However, the curve $\nu^2 - J^2 + \epsilon^2 = 0$ is smooth!

Resolving the WF intercept

Leading trajectory and its shadow given by

$$\nu_\pm(J) = \pm\left(2\Delta_\phi + J + \gamma(J) - \tfrac{d}{2}\right) \ .$$

Combine into single complex curve:

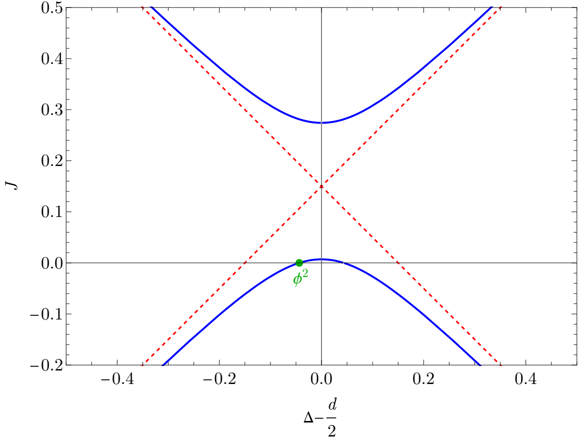
$$\begin{aligned}\nu^2 &= (2\Delta_\phi - d/2 + J + \gamma(J))^2 \\ &= J^2 - J\epsilon + \left(\frac{J}{27} + \frac{1}{4} - \frac{2}{9(J+1)}\right)\epsilon^2 \\ &\quad + \left(\frac{109J^3 + 164J^2 + 265J - 114}{2916(J+1)^2} - \frac{4H(J)}{27(J+1)}\right)\epsilon^3 + O(\epsilon^4).\end{aligned}$$

Resolved the $J = 0$ poles!

Further poles at $J = -1, -2, \ldots$ indicate other intersections that are expected to be similarly resolved.

Resolved leading trajectory in WF

Leading Regge trajectory in Wilson-Fisher fixed point in $4-\epsilon$ dimensions. Order ϵ^4 interactions, $\epsilon = 0.3$.



Regge intercept

$$
\begin{aligned}
J_0(\epsilon) &= \left(\frac{1}{2} + \frac{\sqrt{2}}{3}\right)\epsilon - \frac{11\sqrt{2}+21}{162}\epsilon^2 \\
&\quad + \frac{465 + 421\sqrt{2} + 54(4+3\sqrt{2})\pi^2 - 648\sqrt{2}\zeta(3)}{17496}\epsilon^3 \\
&\quad + c_4\epsilon^4 + O(\epsilon^5) \\
&= 0.971405\epsilon - 0.225656\epsilon^2 + 0.248731\epsilon^3 - 0.631547\epsilon^4 + O(\epsilon^5).
\end{aligned}
$$

The ϕ^2 operator

Setting $J = 0$,

$$|\nu| = \frac{\epsilon}{6} - \frac{19\epsilon^2}{162} + \left(-\frac{937}{17496} + \frac{4\zeta(3)}{27}\right)\epsilon^3 + O(\epsilon^4)$$

matches with

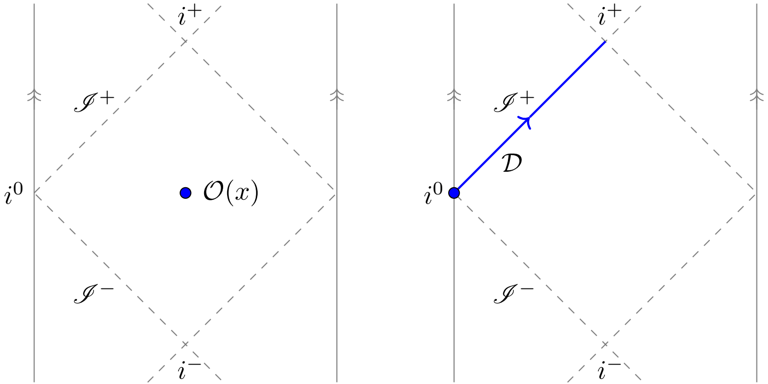
$$\Delta_{\phi^2} = \Delta_-(0) = \frac{d}{2} - |\nu|.$$

[Alday Henriksson van Loon '17, Caron-Huot Gobeil Zahraee '20]

In fact, adding the known $O(\epsilon^4)$ term in Δ_{ϕ^2} + analyticity allowed us to predict the ϵ^4 term in J_0.

Renormalization of detectors

The detector frame



Generators in renormalization

Lorentz generators are exact in perturbation theory.

Local operator $\mathcal{O}_0(x=0)$:

$$\left.\text{primariness } [K^\mu, \mathcal{O}(0)] = 0,\right\} \text{corrected in perturbation theory,}$$

$$\left.\text{spin } J,\right\} \text{exact in perturbation theory,}$$

$$\left.\begin{array}{l}\text{dimension } \Delta \\ [D, \mathcal{O}_0(x)] = \Delta_0 \mathcal{O}_0(x).\end{array}\right\} \text{corrected to } \Delta = \Delta_0 + \gamma(J).$$

Detector $\mathcal{D}(x=\infty, z)$:

$$\left.\begin{array}{l}\text{primariness } [P^\mu, \mathcal{D}(z)] = 0, \\ \text{detector spin } J_L,\end{array}\right\} \text{exact in perturbation theory,}$$

$$\left.\begin{array}{l}\text{detector dimension} \\ [D, \mathcal{D}(\infty, z)] = -\Delta_L \mathcal{D}(\infty, z).\end{array}\right\} \text{corrected } \Delta_L = \Delta_{L,0}(J_L) + \gamma_L(J_L).$$

Anomalous dimension or anomalous spin

Traditional frame:

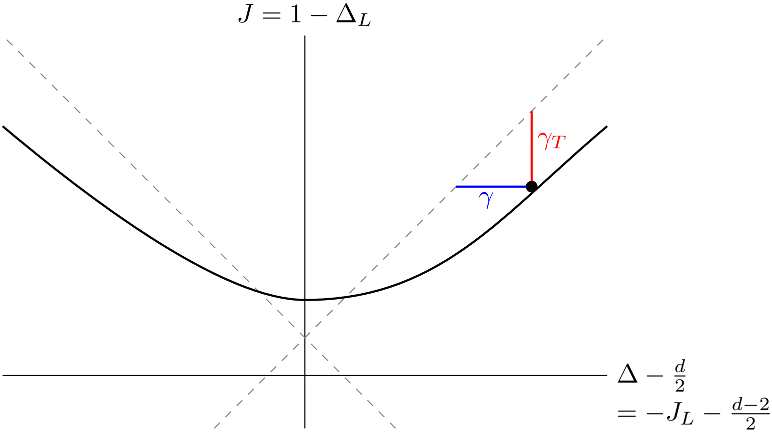
$$\text{fix } J, \text{ renormalize } \Delta$$
$$\Rightarrow \gamma(J)$$

Detector frame:

$$\text{fix } J_L = 1 - \Delta, \text{ renormalize } \Delta_L = 1 - J$$
$$\Rightarrow \gamma_L(J_L) \equiv \gamma_T(\Delta - \tau_0).$$

Recall, $(J_L, \Delta_L) = (1 - \Delta, 1 - J)$.

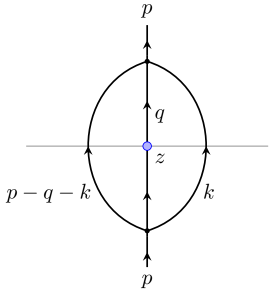
Reciprocity



$$\gamma(J) = \gamma_L(J_L)$$

Two loop diagram

$$\mathcal{F}^{(2)}_{J_L}(z;p) =$$



One computes

$$\begin{aligned}
&\mathcal{F}^{(2)}_{J_L}(z;p) \\
&= \frac{(-i\lambda\widetilde{\mu}^{\epsilon})(+i\lambda\widetilde{\mu}^{\epsilon})}{2} \int \frac{d^dq}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{i}{p^2+i0}\frac{-i}{p^2-i0} V_{J_L}(z;q) \\
&\qquad\qquad\qquad\qquad \times (2\pi)^2\delta^+(k^2)\delta^+((p-q-k)^2) \\
&= (\lambda\widetilde{\mu}^{\epsilon})^2 \frac{\operatorname{vol} S^{d-2}}{2^d(2\pi)^{2d-2}} \frac{\Gamma(\frac{d-2}{2})\Gamma(-J_L)}{\Gamma(-J_L+\frac{d-2}{2})} (-2z\cdot p)^{J_L}(-p^2)^{\frac{d-4}{2}-J_L-2}\theta(-p^2).
\end{aligned}$$

Two loop divergence

The result looks regular, but has a hidden divergence:

$$(-2z\cdot p)^{J_L}(-p^2)^{\frac{d-4}{2}-J_L-2} = \frac{\pi}{2\epsilon(J_L+1)}\int d\beta\beta^{-J_L-1}\delta^d(p-\beta z)+O(\epsilon^0)$$
$$= \frac{\pi}{2\epsilon(J_L+1)}V_{J_L}(z;p)+O(\epsilon^0).$$

Therefore, the two loop event shape has a divergence proportional to the tree level vertex:

$$\langle\mathcal{D}_{J_L}(z)\rangle_{\text{2-loop}} = -\frac{1}{\epsilon}\frac{\lambda^2}{2(4\pi)^4}\frac{1}{J_L(J_L+1)}\langle\mathcal{D}_{J_L}(z)\rangle_{\text{tree}}+O(\epsilon^0).$$

So, the detectors are multiplicatively renormalized.

Two loop anomalous dimensions

The bare detector has

$$\langle\Omega|\phi_R(-p)\mathcal{D}_{J_L}(z)\phi_R(p)|\Omega\rangle = Z_\phi^{-1}\left(V_{J_L}(z;p) + \mathcal{F}^{(2)}_{J_L}(z,p)\right) + O(\lambda^3),$$

Define the renormalized detector

$$[\mathcal{D}_{J_L}(z)]_R = Z_{J_L}^{-1}\mathcal{D}_{J_L}(z)$$
$$Z_{J_L} \equiv Z_\phi^{-1}\left(1 - \frac{1}{\epsilon}\frac{\lambda^2}{2(4\pi)^4}\frac{1}{J_L(J_L+1)}\right) + O(\lambda^3).$$

The anomalous dimension in the detector frame is then

$$-\gamma_L(J_L) = \frac{\partial \log Z_{J_L}}{\partial\lambda}\beta(\lambda) = \frac{\lambda^2}{(4\pi)^4}\left(\frac{1}{J_L(J_L+1)} - \frac{1}{6}\right) + O(\lambda^3)$$

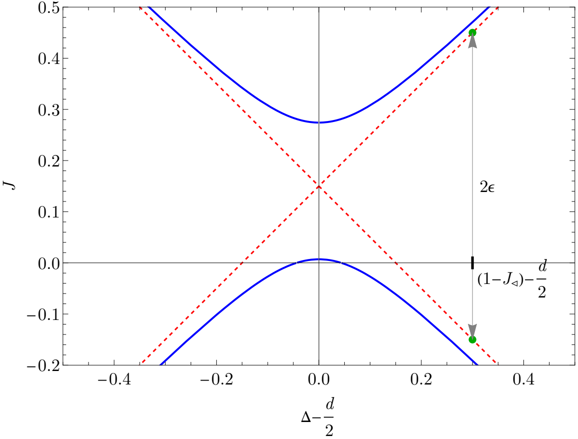
Finally, we land on the known curve (agrees with $\Delta(J)$)

$$J(\Delta) = \Delta - (d-2) - \gamma_L(1-\Delta)$$
$$= \Delta - 2 + \epsilon + \frac{\epsilon^2}{9}\left(\frac{1}{(\Delta-1)(\Delta-2)} - \frac{1}{6}\right) + O(\epsilon^3).$$

The Pomeron of Wilson-Fisher

Resolving the intersection

Want to recover



Study $\mathcal{D}_{J_L}$ and $\widetilde{\mathcal{D}}_{J_L} \equiv \widehat{\mathbf{S}}_J[\mathcal{D}_{2-d-J_L}]$ near the intercept.

The two loop divergence, again

Turns out, to be expected, the two loop diagram has another divergence:

$$\mathcal{F}^{(2)}_{J_L}(z;p) \sim \frac{1}{J_L - J_\triangleleft} \frac{\lambda^2 \mu^{2\epsilon}}{2(4\pi)^4} \mathcal{R}(\epsilon) \widehat{\mathbf{S}}_J [V_{2-d-J_\triangleleft}](z;p)$$

where $J_\triangleleft \equiv \frac{d-6}{2}$, coming from

$$(-p^2)^{\frac{d-4}{2} - J_L - 2} \theta(-p^2) \sim \frac{1}{J_\triangleleft - J_L} \delta(p^2)\,.$$

The coefficient has the form

$$\mathcal{R}(\epsilon) = \frac{1}{\epsilon} + 1 + O(\epsilon)\,.$$

With this we have

$$\langle \mathcal{D}_{J_L} \rangle \sim \frac{1}{J_L - J_\triangleleft} \frac{\lambda^2 \mu^{2\epsilon}}{2(4\pi)^4} \mathcal{R}(\epsilon) \langle \widetilde{\mathcal{D}}_{J_\triangleleft} \rangle_{\text{tree}} + (\text{regular at } J_L = J_\triangleleft).$$

Mixing of the leading trajectory and its shadow

Schematically, we have the divergences of the form

$$\langle \mathcal{D}_{J_L} \rangle \sim \frac{1}{(J - J_{\triangleleft})\epsilon} + \frac{1}{J - J_{\triangleleft}} + \frac{1}{\epsilon} \, .$$

We seek to renormalize all of these divergences.

Basis of operators at free theory

A convenient, nondegenerate basis is given by

$$\mathbb{D}_{J_L} = \begin{pmatrix} \mathcal{D}_{J_L} \\ \mathcal{D}'_{J_L} \end{pmatrix}, \qquad \mathcal{D}'_{J_L} \equiv \frac{\mu^{2-d-2J_L}\widetilde{\mathcal{D}}_{J_L} - \mathcal{D}_{J_L}}{J_L - \frac{2-d}{2}}$$

These operators have the same mass dimensions and Lorentz spin J_L. However, $\mathcal{D}'_{J_L}$ do not have definite scaling dimensions.

In fact, $\mathcal{D}'_{\frac{2-d}{2}}$ and $\mathcal{D}_{\frac{2-d}{2}}$ form a log-multiplet in the free theory. Interactions break this log-multiplet into two independent operators.

Summarize divergences as

$$\langle \mathcal{D}_{J_L} \rangle = \frac{\lambda^2}{(4\pi)^4} \left(\left[\frac{1}{12\epsilon} - \frac{1}{2\epsilon J_L} + \frac{\mathcal{R}(\epsilon)}{2(J_L - J_\triangleleft)} \right] \langle \mathcal{D}_{J_L} \rangle_{\text{tree}} - \frac{\epsilon \mathcal{R}(\epsilon)}{2(J_L - J_\triangleleft)} \langle \mathcal{D}'_{J_L} \rangle_{\text{tree}} \right) + \text{(regular)}$$

Dilatation operator at loop level

Define renormalized operators

$$[\mathbb{D}_{J_L}]_R \equiv \mathcal{Z}_{J_L}^{-1}\mathbb{D}_{J_L}$$

with suitable 2×2 matrix $\mathcal{Z}_{J_L}$.

Dilatation operator D acts on this 2-dim basis as the 2×2 matrix

$$\mathscr{D} = \begin{pmatrix} 2-d-J_L & 0 \\ 2 & J_L \end{pmatrix} + \frac{\lambda^2}{(4\pi)^4}\begin{pmatrix} \frac{1}{J_L}-\frac{1}{6} & \epsilon\mathcal{R}(\epsilon) \\ \frac{2}{J_L(2-d-J_L)} & \frac{1}{2-d-J_L}-\frac{1}{6} \end{pmatrix} + O(\lambda^3).$$

The Pomeron and the Regge intercept

The (left) eigenvectors $\mathscr{D}$ give the Pomeron and subleading Reggeon

$$v_{\pm}[\mathbb{D}_{J_L}]_R; \qquad v_{\pm} = \left(1, \pm\frac{\lambda}{\sqrt{2}(4\pi)^2}(1+O(\epsilon)) + O(\lambda^2)\right)$$

and the eigenvalues give the Regge intercepts

$$J_{\pm} = \left(\frac{1}{2} \pm \frac{\sqrt{2}}{3}\right)\epsilon + O(\epsilon^2).$$

Reggeized scalars in Wilson-Fisher

Horizontal trajectory

Consider the product of operators

$$\mathcal{H}_{J_{L1},J_{L2}}(z_1,z_2) \equiv \mathcal{D}_{J_{L1}}(z_1)\mathcal{D}_{J_{L2}}(z_2).$$

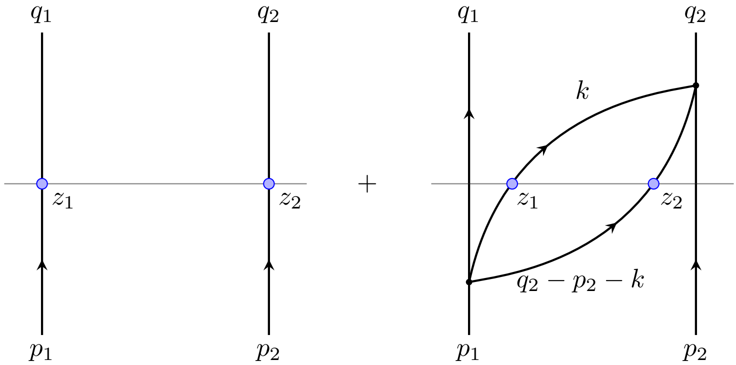
The most important one will be

$$\mathcal{H}_{3-d,3-d}(z_1,z_2) \propto : \mathbf{L}[\phi^2](\infty,z_1)\mathbf{L}[\phi^2](\infty,z_2) :$$

Decompose into Lorentz irreps:

$$\mathcal{H}_{J_{L1},J_{L2};J_L}(z) \equiv \int D^{d-2}z_1 D^{d-2}z_2 K_{J_L}(z_1,z_2;z)\mathcal{H}_{J_{L1},J_{L2}}(z_1,z_2).$$

Horizontal trajectory renormalization



Horizontal trajectory divergence

$$\langle \mathcal{H}_{J_{L1},J_{L2}}(z_1,z_2)\rangle_{\substack{\text{2 loop,}\\ \text{conn.}}} \sim \frac{(\lambda\widetilde{\mu}^{\epsilon})^2}{4(2\pi)^{2d-2}}\frac{-1}{J_{L1}+J_{L2}+2} \times \int D^{d-2}z_3 D^{d-2}z_4 K_\alpha(z_1,z_2;z_3,z_4)\langle \mathcal{H}_{3-d,3-d}(z_3,z_4)\rangle_{\text{tree}}$$

with kernel

$$K_\alpha(z_1,z_2;z_3,z_4) \equiv \frac{\pi}{\sin\pi\alpha}\frac{\left(\frac{z_{13}}{z_{23}}\right)^\alpha - \left(\frac{z_{14}}{z_{24}}\right)^\alpha}{z_{24}z_{13}-z_{14}z_{23}}, \qquad \alpha = \frac{J_{L1}-J_{L2}}{2}.$$

Diagonalizing to Lorentz irreps, we have

$$\langle \mathcal{H}_{J_{L1},J_{L2};J_L}(z)\rangle_{\substack{\text{2 loop,}\\ \text{conn.}}} \sim \frac{(\lambda\widetilde{\mu}^{\epsilon})^2}{(4\pi)^d}\frac{-1}{J_{L1}+J_{L2}+2}\kappa_\alpha(J_L)\langle \mathcal{H}_{3-d,3-d;J_L}(z)\rangle_{\text{tree}},$$

where

$$\kappa_\alpha(J_L) = \frac{2}{J_L+1}\cos(\tfrac{\pi J_L}{2})\Gamma(-\tfrac{J_L}{2})^2\Gamma(\tfrac{J_L+2}{2}-\alpha)\Gamma(\tfrac{J_L+2}{2}+\alpha)+O(\epsilon).$$

Horizontal trajectory anomalous spin?

The full picture is a lot more complicated.

Disconnected diagrams give rise to mixing within infinitely many horizontal trajectories. Computing the dilatation matrix and extracting anomalous spins is an interesting future directions.

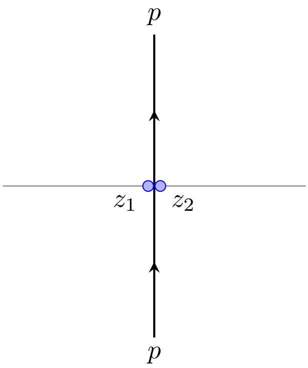
In the meantime, we can consider a modified theory with only the diagrams we considered, in which case we have

$$\begin{aligned} J(\Delta) = 1 - \Delta_L(J_L) &= -1 + \frac{\lambda^2}{(4\pi)^4}\kappa_0(1-\Delta) + O(\lambda^3) \\ &= -1 + \frac{\lambda^2}{(4\pi)^4}\frac{2\pi^2\sin(\frac{\pi\Delta}{2})}{(2-\Delta)\cos(\frac{\pi\Delta}{2})^2} + O(\lambda^3). \end{aligned}$$

Renormalizing two-point event shapes

[Gonzalez MK Moult, WIP]

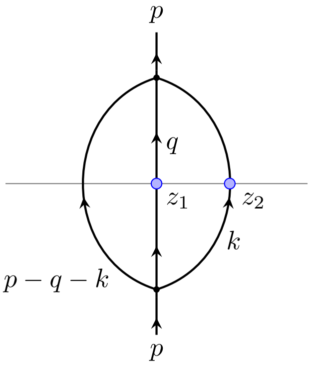
$\langle \mathcal{D}_{J_{L1}}(z_1)\mathcal{D}_{J_{L2}}(z_2)\rangle$: tree level contact term



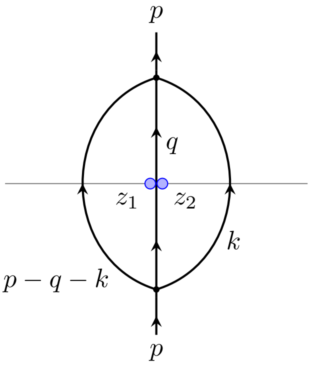
$$\langle \phi(p)|\mathcal{D}_{J_{L1}}(z_1)\mathcal{D}_{J_{L2}}(z_2)|\phi(p)\rangle \sim \delta^{d-2}(z_1-z_2)\langle \phi(p)|\mathcal{D}_{J_{L1}+J_{L2}+d-2}(z_2)|\phi(p)\rangle$$

$$\text{“}E^n\text{”} \times \text{“}E^m\text{”} \sim \text{“}E^{n+m}\text{”}$$

$\langle \mathcal{D}_{J_{L1}}(z_1)\mathcal{D}_{J_{L2}}(z_2)\rangle$: two-loop



$\equiv F_1(p, z_1, z_2)$



$\equiv F_2(p, z_1, z_2)$

Regular piece and the general light-ray OPE

Event shapes of "fuzzier" detectors (beyond $\mathbf{L}[\mathcal{O}]$):

$$F_1(p, z_1, z_2) = \frac{(\lambda\widetilde{\mu})^2}{2(2\pi)^{2d-1}}(-2p\cdot z_1)^{J_{L1}}(-2p\cdot z_2)^{J_{L2}}(-p^2)^{-3-J_{L1}-J_{L2}}\theta(p^0) \times \mathcal{G}(\zeta)$$

where $z_i = (1, \widehat{n}_i)$,

$$\mathcal{G}(\zeta) = \frac{\Gamma(-J_{L1})\,\Gamma(-J_{L2})}{\Gamma(-J_{L1}-J_{L2})}\,{}_2F_1(-J_{L1}, -J_{L2}; -J_{L1}-J_{L2}; \zeta),$$

$$\zeta = \frac{(-2z_1\cdot z_2)(-p^2)}{(-2p\cdot z_1)(-2p\cdot z_2)} = \frac{1-\widehat{n}_1\cdot\widehat{n}_2}{2}.$$

Decomposition into celestial blocks computes $\mathcal{D}_{J_{L1}}(z_1)\times\mathcal{D}_{J_{L2}}(z_2)$ light-ray OPE in perturbation theory. Celestial MFT-like structure; $\mathcal{D}_{J_L}(z)\sim\mathcal{P}_{-J_L}(y)$, OPE has celestial "double twist" operators: $[\mathcal{P}_{\delta_1}\mathcal{P}_{\delta_2}]_{n,0}\sim$ "ϕ^4 type detectors".

Contact term divergence

$F_1(p, z_1, z_2)$ secretly has a contact term divergence at $z_1 = z_2$!

Renormalization of individual detectors $\mathcal{D}_{J_{L1}}(z_1)$ and $\mathcal{D}_{J_{L2}}(z_2)$ does not remove this divergence. Need to renormalize the product. (Think $\phi^2 \times \phi^2 \sim \phi^4 + \dots$.)

These observables are related to multi-hadron fragmentation functions in QCD. New insights for the renormalization of such objects.

Computing contact term contributions to the light-ray OPE. Novel contributions?

Generalization to $O(N)$ models and $\mathcal{E}_Q$ detectors

[together with G. Korchemsky and A. Zhiboedov]

We can generalize to $O(N)$ models and to CRT-odd trajectories.

Leading "odd spin" trajectory $\mathcal{D}^-_{J_L} \supset \mathcal{Q} = \int dv J_v$.

Charge detectors $\mathcal{Q}$ notoriously have pathologies, whereby multiple insertions are ill defined.

One can get around these issues by considering energy$\times$charge detectors $\mathcal{E}_Q$ — or more generally $E^{J-1} \times Q$ detectors $\mathcal{D}^-_{J_L}$ — which have milder singularities due to the extra power of energy.

We are able to compute the two point event shapes of $\mathcal{D}^\pm_{J_L}$ operators in the $O(N)$ models, for example, and study their renormalization.
$\Rightarrow$ odderon,
$\Rightarrow$ QCD, collider physics...

Appendix: Some Lorentzian dynamics in CFTs

OPE Inversion formula

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \sum_{J=0}^{\infty} \oint_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i} C(\Delta, J)\Psi_{\Delta,J}(x_i)$$

where

$$C(\Delta, J_{\mathcal{O}}) \sim -\frac{f_{12\mathcal{O}} f_{34\mathcal{O}^\dagger}}{\Delta - \Delta_{\mathcal{O}}}.$$

OPE Inversion formula

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \sum_{J=0}^{\infty} \oint_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i} C(\Delta, J)\Psi_{\Delta,J}(x_i)$$

where

$$C(\Delta, J_{\mathcal{O}}) \sim -\frac{f_{12\mathcal{O}} f_{34\mathcal{O}^\dagger}}{\Delta - \Delta_{\mathcal{O}}}.$$

There is a "Lorentzian" inverse, [Caron-Huot '17]

$$C(\Delta, J) \propto \int d^d x_1 d^d x_2 d^d x_3 d^d x_4 \langle [\mathcal{O}_1(x_1), \mathcal{O}_4(x_4)][\mathcal{O}_3(x_3), \mathcal{O}_2(x_2)]\rangle \times \widetilde{G}_{J+d-1,\Delta-d+1}(x_1, x_2, x_3, x_4).$$

OPE Inversion formula

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \sum_{J=0}^{\infty} \oint_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i} C(\Delta, J)\Psi_{\Delta,J}(x_i)$$

where

$$C(\Delta, J_{\mathcal{O}}) \sim -\frac{f_{12\mathcal{O}} f_{34\mathcal{O}^\dagger}}{\Delta - \Delta_{\mathcal{O}}}.$$

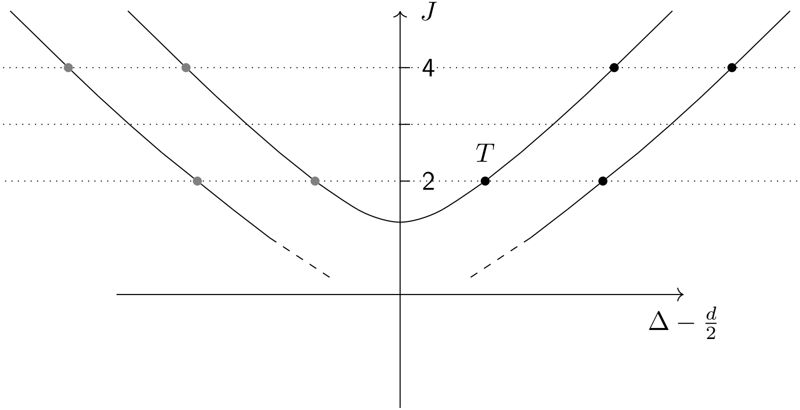
There is a "Lorentzian" inverse, [Caron-Huot '17]

$$C(\Delta, J) \propto \int d^d x_1 d^d x_2 d^d x_3 d^d x_4 \langle [\mathcal{O}_1(x_1), \mathcal{O}_4(x_4)][\mathcal{O}_3(x_3), \mathcal{O}_2(x_2)]\rangle \times \widetilde{G}_{J+d-1,\Delta-d+1}(x_1, x_2, x_3, x_4).$$

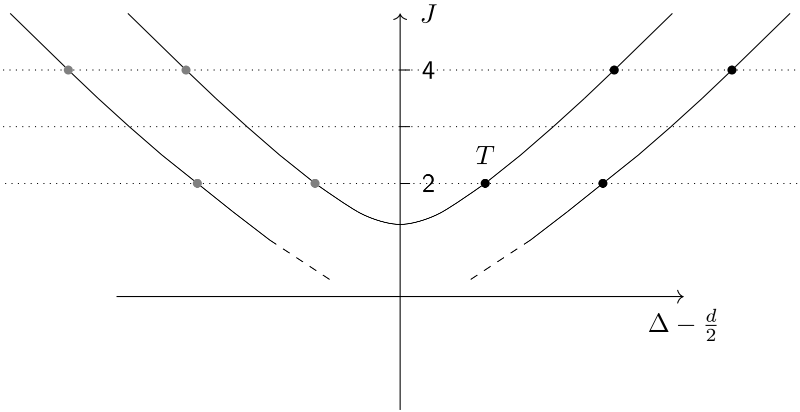
CFT data is analytic in J. Poles at Regge trajectories

$$\Delta = \Delta_i(J).$$

Regge trajectories and the Chew-Frautschi plot

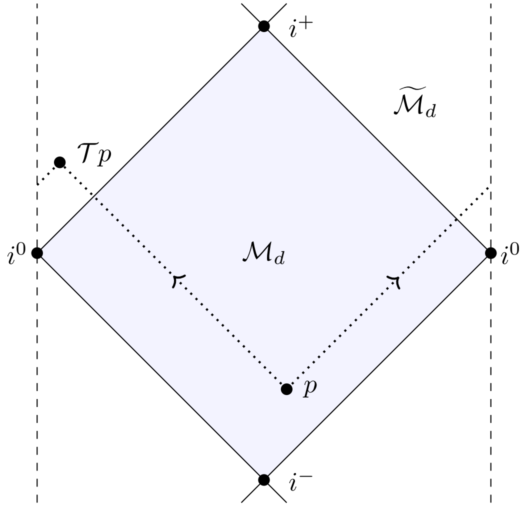


Regge trajectories and the Chew-Frautschi plot



Local operators do not make sense at continuous J. Trajectories are light-ray operators $\mathbb{O}^{\pm}_{i,J}$. [Kravchuk and Simmons-Duffin '18]

Lorentzian geometry



Light-transform

Introduce polarizations,

$$\mathcal{O}(x,z) \equiv \mathcal{O}^{\mu_1\cdots\mu_J}(x) z_{\mu_1}\cdots z_{\mu_J}.$$

Define light-transform of a local operator

$$\mathbf{L}[\mathcal{O}](x,z) = \int_{-\infty}^{\infty} d\alpha\, (-\alpha)^{-\Delta-J}\mathcal{O}\left(x - \frac{z}{\alpha}, z\right).$$

Quantum numbers are

$$\mathbf{L} : (\Delta, J) \to (1-J, 1-\Delta).$$

Makes sense for continuous spin!

$$\mathbf{L}[\mathcal{O}](x,z)|\Omega\rangle = 0.$$

Light-ray operators

Construct continuous spin object from local operators,

$$\mathbb{O}^{\pm}_{\Delta,J}(x,z) = \int d^dx_1 d^dx_2 K^{\pm}_{\Delta,J}(x_1,x_2,x,z)\phi_1(x_1)\phi_2(x_2).$$

Residues are light-ray operators,

$$\mathbb{O}^{\pm}_{\Delta,J}(x,z) \sim \frac{1}{\Delta - \Delta^{\pm}_i(J)}\mathbb{O}^{\pm}_{i,J}(x,z).$$

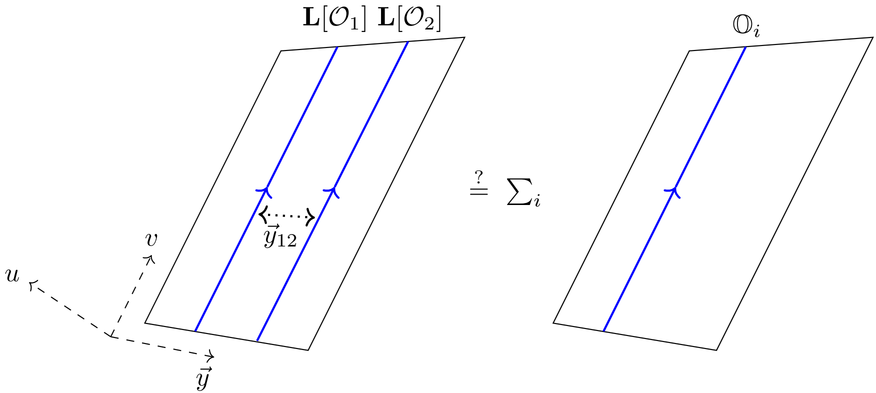
They agree with local operators when they coincide,

$$\mathbb{O}^{(-1)^J}_{i,J} = f_{12\mathcal{O}_{i,J}}\mathbf{L}[\mathcal{O}_{i,J}], \qquad J \in \mathbb{Z}_{\geq 0}.$$

Inversion formula computes matrix elements of light-ray operators,

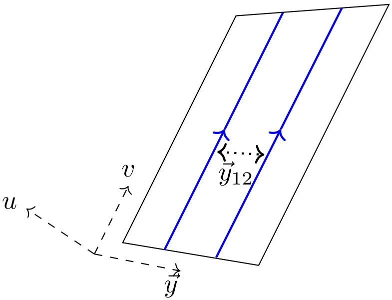
$$\langle\Omega|\phi_4\mathbb{O}^{\pm}_{\Delta,J}(x,z)\phi_3|\Omega\rangle = -C^{\pm}(\Delta,J)\langle 0|\phi_4\mathbf{L}[\mathcal{O}](x,z)\phi_3|0\rangle.$$

An OPE for light-ray operators on a null plane?



The light-ray OPE

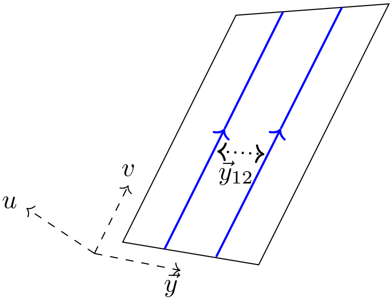
[Hofman Maldacena '08, MK Kravchuk Simmons-Duffin Zhiboedov '19 + Chang '20]



$$\int_{-\infty}^{\infty} dv_1\, \mathcal{O}_{1;v\cdots v}(u=0, v_1, \vec{y}_1) \int_{-\infty}^{\infty} dv_2\, \mathcal{O}_{2;v\cdots v}(u=0, v_2, \vec{y}_2)$$
$$\stackrel{?}{=} \sum_i |\vec{y}_{12}|^{\delta_i-(\Delta_1-1)-(\Delta_2-1)} \mathbb{O}_i.$$

The light-ray OPE

[Hofman Maldacena '08, MK Kravchuk Simmons-Duffin Zhiboedov '19 + Chang '20]



$$\int_{-\infty}^{\infty} dv_1\, \mathcal{O}_{1;v\cdots v}(u=0, v_1, \vec{y}_1) \int_{-\infty}^{\infty} dv_2\, \mathcal{O}_{2;v\cdots v}(u=0, v_2, \vec{y}_2)$$

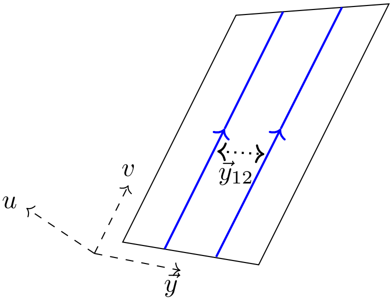
$$= \pi i \sum_{s=\pm} \sum_i \mathcal{C}_{\Delta_i - 1}(\vec{y}_{12}, \partial_{\vec{y}_2}) \mathbb{O}^s_{i, J=J_1+J_2-1}(\vec{y}_2)$$

$+$ higher transverse spin.

Dilatations around x for $\mathbf{L}[\mathcal{O}_1](x, z_1)\mathbf{L}[\mathcal{O}_2](x, z_2) \Leftrightarrow J = J_1 + J_2 - 1$.

The light-ray OPE

[Hofman Maldacena '08, MK Kravchuk Simmons-Duffin Zhiboedov '19 + Chang '20]

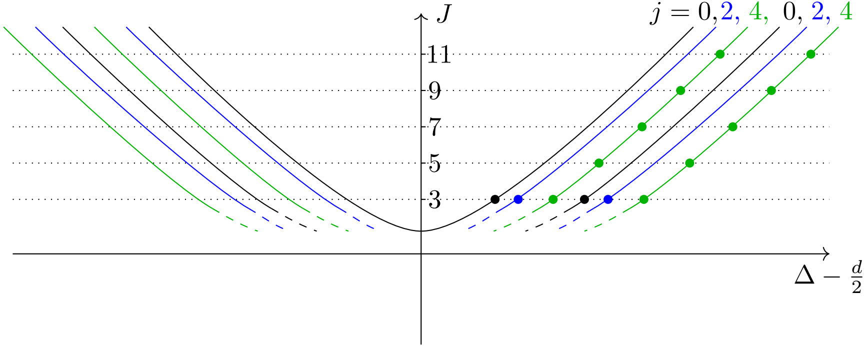


$$\int_{-\infty}^{\infty} dv_1\, \mathcal{O}_{1;v\cdots v}(u=0,v_1,\vec{y}_1) \int_{-\infty}^{\infty} dv_2\, \mathcal{O}_{2;v\cdots v}(u=0,v_2,\vec{y}_2)$$
$$= \pi i \sum_{s=\pm} \sum_i \mathcal{C}_{\Delta_i-1}(\vec{y}_{12},\partial_{\vec{y}_2}) \mathbb{O}^s_{i,J=J_1+J_2-1}(\vec{y}_2)$$
$$+ \text{higher transverse spin.}$$

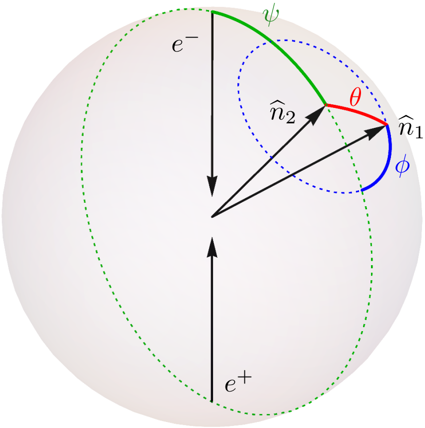
Dilatations around x for $\mathbf{L}[\mathcal{O}_1](x,z_1)\mathbf{L}[\mathcal{O}_2](x,z_2) \Leftrightarrow J = J_1 + J_2 - 1$.

Energy detector OPE

$$\mathcal{E}\times\mathcal{E}=\sum_i\left(\mathbb{O}^+_{i,J=3,j=0}+\mathbb{O}^+_{i,J=3,j=2}+\mathbb{O}^+_{i,J=3,j=4}\right)+\sum_{n,i}\mathcal{D}_{2n}\mathbb{O}^+_{i,J=3+2n,j=4}.$$



Transverse spin j conjugate to ϕ



Appendix: Deriving the light-ray OPE

Decompose product on the celestial sphere

$$\mathbf{L}[\phi_1](x,z_1)\mathbf{L}[\phi_2](x,z_2) = \sum_i \sum_{j=0}^{\infty} \mathcal{C}_{12i}(z_1,z_2,\partial_2)\mathbb{W}_{\Delta_i,j}(x,z_2)\,,$$

Appendix: Deriving the light-ray OPE

Decompose product on the celestial sphere

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if and only if

$$\begin{aligned}\mathbb{W}_{\Delta,j}(x,z) &= \int d^d z_1 d^d z_2 L_{\Delta,j}(z_1,z_2;z)\mathbf{L}[\phi_1](x,z_1)\mathbf{L}[\phi_2](x,z_2) \\ &= \int d^d x_1 d^d x_2 \mathcal{L}_{\Delta,j}(x_1,x_2;x,z)\phi_1(x_1)\phi_2(x_2)\,.\end{aligned}$$

Appendix: Deriving the light-ray OPE

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Relate the kernel $\mathcal{L}_{\Delta,j}$ to the light-ray operator kernel,

$$\mathbb{O}^{\pm}_{\Delta,J}(x,z) = \int d^d x_1 d^d x_2 K^{\pm}_{\Delta,J}(x_1,x_2,x,z)\phi_1(x_1)\phi_2(x_2).$$

Appendix: Deriving the light-ray OPE

Decompose product on the celestial sphere

$$\mathbf{L}[\phi_1](x,z_1)\mathbf{L}[\phi_2](x,z_2) = \sum_i \sum_{j=0}^{\infty} \mathcal{C}_{12i}(z_1,z_2,\partial_2)\mathbb{W}_{\Delta_i,j}(x,z_2)\,,$$

if and only if

$$\begin{aligned}\mathbb{W}_{\Delta,j}(x,z) &= \int d^dz_1 d^dz_2 L_{\Delta,j}(z_1,z_2;z)\mathbf{L}[\phi_1](x,z_1)\mathbf{L}[\phi_2](x,z_2) \\ &= \int d^dx_1 d^dx_2 \mathcal{L}_{\Delta,j}(x_1,x_2;x,z)\phi_1(x_1)\phi_2(x_2)\,.\end{aligned}$$

Relate the kernel $\mathcal{L}_{\Delta,j}$ to the light-ray operator kernel,

$$\mathbb{O}^{\pm}_{\Delta,J}(x,z) = \int d^dx_1 d^dx_2 K^{\pm}_{\Delta,J}(x_1,x_2,x,z)\phi_1(x_1)\phi_2(x_2).$$

Turns out

$$\mathcal{L}_{\Delta,j} \propto \mathcal{D}_j K^{+}_{\Delta,J=-1+j} + \mathcal{D}_j K^{-}_{\Delta,J=-1+j}\,,$$

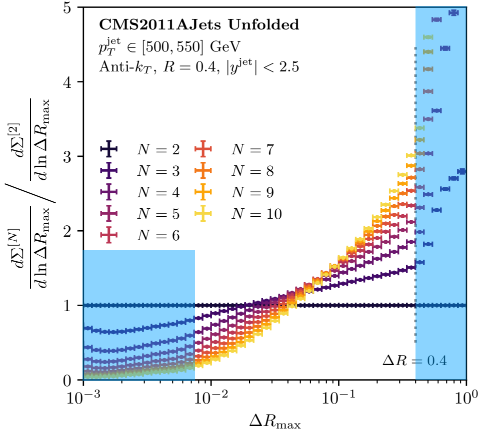
and we have

$$\mathbb{W}_{\Delta_i,j}(x,z) \propto \mathcal{D}_j \mathbb{O}^{(-1)^j}_{\Delta,J=-1+j}(x,z)$$

Appendix: LHC physics

OPE in nature: N-point energy correlator at the LHC

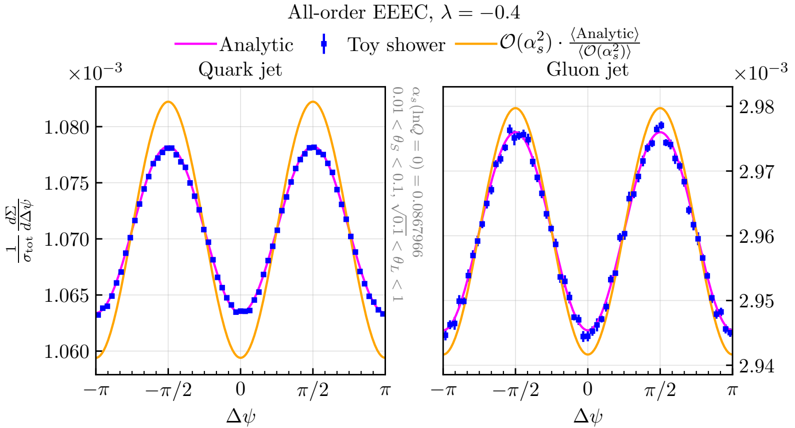
$$\mathcal{E}(\widehat{n}_1)\mathbb{O}_J(\widehat{n}_2) \sim |n_{12}|^{\gamma(J+1)}\mathbb{O}_{J+1}(\widehat{n}_2)$$



[Komiske Moult Thaler Zhou '22]

Applications: transverse spin interferometry

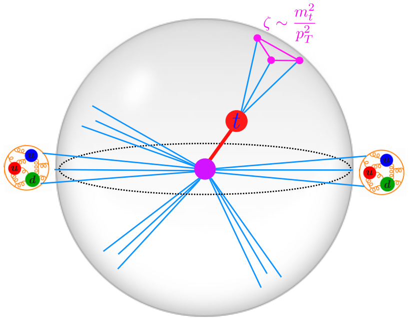
$$\mathcal{E}\times\mathcal{E} = \sum_i \left(\mathbb{O}^+_{i,J=3,j=0} + \boxed{\mathbb{O}^+_{i,J=3,j=2}} + \mathbb{O}^+_{i,J=3,j=4} \right) + \sum_{n,i} \mathcal{D}_{2n} \mathbb{O}^+_{i,J=3+2n,j=4}.$$

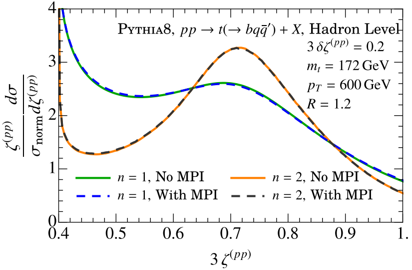


[Salam et al, Moult et al]

Applications: top quark decay

"E detectors" $\langle \mathcal{E}_2 \mathcal{E}_2 \mathcal{E}_2 \rangle$ and "E^2 detectors" $\langle \mathcal{E}_3 \mathcal{E}_3 \mathcal{E}_3 \rangle$ in top quark decay:





[Holguin Moult Pathak Procura '22]