Kinetically constrained models and boostrap percolation

Monday, June 16, 2025 - Friday, June 20, 2025

IHP

Scientific Program

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\mbox{ 10h-12h} &
I. Hartarsky & I. Hartarsky & I. Hartarsky & I. Hartarsky & I. Hartarsky\\
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Bootstrap percolation (BP) is a class of monotone cellular automata that can be viewed as models of nucleation or spread of infections (or news) in a population. To give an example, consider the square lattice and declare each vertex infected independently with probability p. At each time step, infect each vertex with at least two infected neighbours and never remove infections. How long does it typically take for the origin to become infected? This model and related ones are strongly linked to other lattice spin models in statistical physics such as the stochastic Ising model, kinetically constrained models (KCM) and percolation. They are also explored from the viewpoint of social sciences, computer science, extremal combinatorics and others.

One of the goals of this course is to provide an introduction to BP with an emphasis on the classical lattice statistical physics viewpoint. Some of the highlights include a very precise answer to the question asked above, as well as the universality theory when varying the choice of the local rule inducing infection spread. The simplicity of the models calls for no technical prerequisites beyond basic notions in probability, but results turn out to be surprisingly deep and challenging. In order to keep the presentation more visual and less technical, we focus on models in one and two dimensions, which provide ample opportunity for showcasing the most important ideas in the field.

In addition to its own interest, BP is instrumental for the study of KCM, a class of interacting particle systems which can be viewed as stochastic non-monotone counterparts of BP. Sites are again either infected or healthy but the evolution, instead of being deterministic, follows a Markovian dynamics of Glauber type. Each site waits an independent, mean one, exponential time and, provided the current configuration satisfies a constraint which depends on the infections in a local neighbourhood (e.g.
there are at least two infected nearest neighbours), it refreshes its state to healthy with probability 1-q and infected with probability q. KCM's were introduced in the physics literature in the 1980's with the aim to model the liquid-glass transition, a longstanding open problem in condensed matter physics. Indeed the kinetic constraints can mimic the mechanism of local caging which slows the liquid motion down at low temperature/high density. Extensive numerical simulations indicate that, despite their simplicity, KCM display a remarkable glassy behavior. On the other hand, KCM pose very challenging and interesting mathematical problems. In fact, the presence of the constraints induce non-
attractiveness, the occurrence of several invariant measures, and the failure of many powerful tools to analyze relaxation to equilibrium (coercive inequalities, coupling, censoring ...).

Major progress has been made in the last fifteen years towards a full and rigorous understanding of the large time behavior of the stationary process. We will present these results, illustrating both the high level ideas and some novel technical tools necessary to deal with the presence of constraints

and the lack of attractiveness. We will conclude by presenting a choice of open problems concerning the out of equilibrium dynamics. Indeed, despite some achievements, robust tools to analyse KCM in this regime are still lacking and several beautiful questions remain open, even for the simpler choices of constraints.