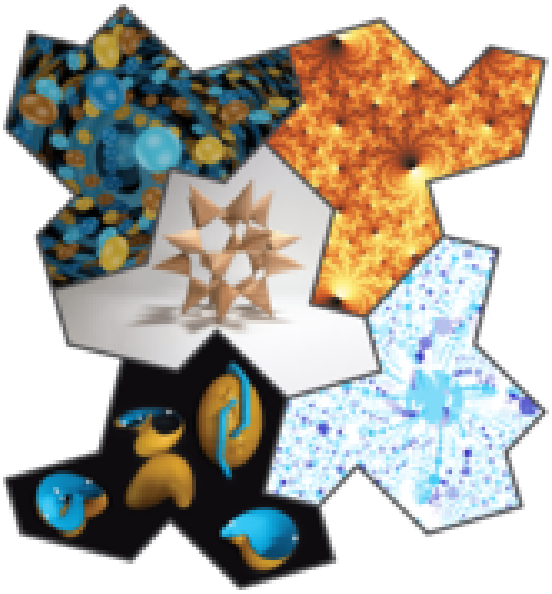
Rigorous Illustrations - Their creation and evaluation for mathematical research



Rapport sur les contributions

https://indico.math.cnrs.fr/e/13124

ID de Contribution: **1** Type: **Non spécifié**

Welcome and Introduction to the Workshop Structure

lundi 19 janvier 2026 09:30 (25 minutes)

Orateurs: BACHMAN, David (Pitzer College); HARRISS, Edmund; DORFSMAN-HOPKINS, Gabriel (St. Lawrence University); SKRODZKI, Martin

ID de Contribution: **2** Type: **Non spécifié**

Why rigorous Illustration?

lundi 19 janvier 2026 10:00 (25 minutes)

Illustration of mathematics can be thought of as a dual to modelling, instead of trying to create a mathematical model of a physical process we create a physical model of a mathematical idea. In both cases we can never have the certainty that the two systems line up perfectly, but can check some of the basic principals. For example if we assume that a flat piece of paper is a Euclidean plane, the compass makes a circle and a straight edge is indeed straight then these bridge between the drawings we make and the proofs of geometry. In this conference we hope to investigate the link between illustration and concept and ask how we can ensure that the interaction is rigorous within reason for the context.

Orateur: HARRISS, Edmund

ID de Contribution: **3** Type: **Non spécifié**

Illuminating Impossible Objects

lundi 19 janvier 2026 10:30 (25 minutes)

In 1992, Roger Penrose published a short, enigmatic paper connecting the famous impossible triangle to cohomology. The paper contains a second, stranger figure – a septagonal ring of Schroder stairs – with a cohomological interpretation. He closes with a cryptic hint: "I believe that considerations such as these may open up intriguing possibilities for further exotic types of impossible figure." Are there any genuinely novel impossible objects that can be illustrated or classified? What makes an impossible figure mathematically impossible, rather than merely a visual trick?

This talk will describe a cohomological framework that addresses these questions with novel and surprising examples involving Necker cubes, mechanical devices, tilings, and a curious set of staircases where the order you climb matters. We will also present a novel animation technique that reveals paradoxical structures that hide within both paradoxical and solvable systems. In all these, the illustrations and animations are more than visual artefacts of the impossible objects they represent: they are a form of proof.

Orateur: GHRIST, Robert (University of Pennsylvania)

ID de Contribution: **4** Type: **Non spécifié**

From connecting the dots to conjugacy of dynamical systems

lundi 19 janvier 2026 11:30 (25 minutes)

Around 1986, I was working on an interval map which was self-induced, with the Tribonacci constant (real root of $x^3 = x^2 + x + 1$) as coefficient, and I knew that a particular rotation of the 2-torus had been proved (by Gérard Rauzy) to have the same property; were these 2 systems conjugate? The conjugacy would, in that case, be a surjective map from the interval to the torus; what would it look like?

I remembered the classic children game of connecting numbered dots, and thought I could do the same to see if there was any possibility of a continuous conjugacy. I computed the 10609 first iterate of 0 by the interval map, and the 10609 first iterate of (0,0) for the rotation; I ordered the second sequence according to the order given by the first, and connected the dots, as an experiment. I expected to get a random set of segments with no structure, but got a very nicely behaved curve, which was clearly self-similar.

It was then clear that the two systems were conjugate, it just remained to prove it, which turned out to be quite easy. It also turned out along the years that followed that there were many more things to do with this game, for example a real tree, and that the illustration could go much further; there are still things I do not understand here, and questions I could not even ask 40 years ago.

Orateur: M. ARNOUX, Pierre (Université d'Aix-Marseille)

ID de Contribution: **5** Type: **Non spécifié**

Illustrating the development of wavelets and beyond

lundi 19 janvier 2026 12:00 (25 minutes)

Orateur: DAUBECHIES, Ingrid (Duke University)

ID de Contribution: **6** Type: **Non spécifié**

Critique Session

Orateur: EVERYONE

ID de Contribution: **7** Type: **Non spécifié**

The Body of Proof: How Inscriptions Shape Mathematical Discovery

mardi 20 janvier 2026 10:00 (25 minutes)

Where do mathematical insights come from? According to classic accounts, creativity is a multi-stage process that involves combining ideas in novel ways. Evidence for these accounts, however, is drawn from artificial lab-based settings or is zoomed out from the messy, moment-to-moment details of discovery. Here, I examine a video corpus of expert mathematicians generating proofs in an ecologically valid setting. I find that mathematicians begin by creating a variety of inscriptions. They then interact with these inscriptions through gaze, speech, gesture, and writing. When they experience an insight, however, their interactions become unpredictable, and they begin to connect inscriptions in novel ways (quantified by an information-theoretic measure, surprisal). Expert mathematical discovery, I conclude, exhibits the combinatorial processing that has been proposed to characterize creativity. Even at the pinnacle of abstraction, at the highest levels of expertise, new ideas are born when the body discovers unexpected affinities among ideas. In light of these findings, I consider what it means for mathematical illustrations to be rigorous—not merely accurate representations, but active participants in discovery.

Orateur: TABATABAEIAN, Shadab (Postdoctoral researcher at Georgetown University)

ID de Contribution: **8** Type: **Non spécifié**

Is math big or small?

mardi 20 janvier 2026 10:30 (25 minutes)

When Illustrating a mathematical idea, the first thing you need to decide is the scale. Is this concept something you can hold in your hand, or something to wander around in? I will reflect on the scale of various analogies used by research mathematicians, such as Thurston's train tracks and pictures of symplectic manifolds. Topologists use the metaphors of "geography" and "botany" to organize problems in their field. I will argue that geography and botany are flexible analogies, which give a natural scale for mathematical illustrations.

Orateur: KIENZLE, Elliot (UC Berkeley)

ID de Contribution: **9** Type: **Non spécifié**

The flat and the relief : geometric material models in 19th century France

mardi 20 janvier 2026 11:30 (25 minutes)

This talk investigates several historical collections of geometric material models, drawings, photographs and other types of graphical representations in Paris at École polytechnique, Institut Henri Poincaré and Musée des arts et métiers.

The practice of model drawing played a key role in the development of descriptive geometry and, more generally, of mathematics from the mid 18th century to the beginning of the 20th century. For a long time, material models were competitors of professors and model drawing an alternative to lectures. Historical sources such as drawings (and later photographs and films), models (of all kinds, including relief drawings) and instruments, highlight the key role played by specific forms of non-textual knowledge in mathematics. These collections raise many questions questions concerning the role played by instruments in mathematics and its relation to other sciences, techniques and industry, but also to the fine arts. To be sure several instruments originated from the practice of geometric model drawing. More importantly, models and drawings themselves came to be considered as specific types of instruments by the end of the 19th century: graphical instruments.

Orateur: BRECHENMACHER, Frédéric

ID de Contribution: **10** Type: **Non spécifié**

Embodying Mathematics Without Sight: Investigating Perception–Action Loops in the Instrumented Activity of Visually Impaired Learners

mercredi 21 janvier 2026 09:30 (25 minutes)

This study investigates how perception–action loops manifest within the instrumented mathematical activity of visually impaired learners assisted by digital technologies. Grounded in the Body–Artifact Functional System (BAFS) and Gibson's ecological theory of perception, the study adopts an embodied and multisensory perspective to explore alternative ways of comprehending a mathematical concept and problem–solving strategies of mathematical exercises in the absence of sight. By shifting the focus beyond a purely visual conception of mathematics, the study contributes to a more holistic conceptualization of mathematics. A secondary qualitative analysis is conducted on video–recorded task–based interviews involving two expert learners in mathematics with different degrees of visual impairment, one with partial blindness and reduced motoric skills, and one who is totally blind. Focusing on the solution of algebraic equations, the perception–action loops identified within the BAFS framework serve as the analytical lens for data examination. Thus, a two phase–analytical process is developed: (1) examining the dynamics within the three fundamental perception–action loops emerging between each pair of components of the BAFS framework, and (2) examining the dynamics within the fourth perception–action loop formed between the body–artifact entity and the environment. To operationalize this approach, for each loop an analytical instrument in the form of two–column tables is introduced to classify the characteristics and definitions associated with the components involved in the respective perception–action loop. The findings illustrate how digital artifacts mediate access to mathematical structures, support the emergence and evolution of utilization schemes, and become functionally embodied within the learner's perceptual and cognitive systems. In addition, they show that for visually impaired learners, many mathematical actions cannot be carried out without the presence of an artifact. Digital tools therefore do not merely enhance pre-existing actions; they constitute the perceptual boundary required for accessing the mathematical environment. The results highlight the critical transition from exploratory actions (Loop 3) to the emergence of a problem- solving strategy (Loop 4), marking the point at which the artifact becomes an extension of both the perceptual and cognitive system, allowing the learner to reason through the artifact. Results highlight the central role of multisensory coordination, artifact transparency, and multilevel intentionality in shaping visually impaired learners'mathematical activity. The study contributes both methodologically by proposing an applicable analytical tool for further research and teaching, and theoretically by BAFS, Instrumental Genesis, and Gibson's ecological theory toward a more inclusive and multisensory framework, with implications for accessible mathematics education and inclusive design.
Keywords: Instrumental genesis, Body–Artifact Functional System, perception–action loops, visual impairment, video magnifier, algebraic equations.

Orateur: PALAJ, Loris (PhD student at the University of Bologna)

ID de Contribution: **11** Type: **Non spécifié**

Telling Math Stories With Graphics

mercredi 21 janvier 2026 10:00 (25 minutes)

With the goal of prompting researchers and science communicators to think critically about conveying mathematical concepts to broad audiences, Jen will demonstrate how illustrated explanatory diagrams can support text-driven narratives, or encapsulate full concepts as self-contained image-based modules.

Orateur: CHRISTIANSEN, Jen

ID de Contribution: **12** Type: **Non spécifié**

What is visualization literacy and how should we measure it

mercredi 21 janvier 2026 10:30 (25 minutes)

Researchers have proposed many definitions of visualization literacy, targeting various aspects of the term. But we have yet to fully capture what it really means to be literate in visualizations, which has important downstream implications, such as how to effectively teach visualization skills to younger generations. Despite not having a clear sense of what it is, we must design tests that measure this ability in order to run empirical studies and evaluate interventions to make progress within the visualization community. This tension between not fully understanding visualization literacy as a construct while still needing to measure it is what makes the study of it so challenging. We ran a one-day workshop at CHI 2024 to facilitate critical conversations around understanding, measuring, and improving visualization literacy. This workshop led to a multi-institutional collaborative autoethnography on the topic of visualization literacy measurements. Based on our reflections on the complexity and fluidity of visualization literacy, we propose several calls to action from the conceptual, operational, and methodological perspectives such as broadening test scopes and modalities, improving test ecological validity, and seeking interdisciplinary collaboration.

Orateur: GE, Lily Wanqian (Northwestern University)

ID de Contribution: **13** Type: **Non spécifié**

The Visual, The Poetic, the Mathematical Connecting at the Foundations: Responsive Practice as Research Methodology

mercredi 21 janvier 2026 11:30 (25 minutes)

What can mathematical illustration learn from studio art research methodologies, and how canvisual principles and poetics be developed together to illustrate abstract concepts? Thispresentation bridges two worlds that share more than is immediately visible: both mathematicalresearch and studio art practice pursue forms that do not yet exist, working iteratively throughobservation, experimentation, and responsive adjustment to reveal patterns that resist verbaldescription.Drawing from fifteen years teaching Art Foundations alongside studio research tracing invisiblephenomena, I propose that rigorous illustration shares core methodological ground with what TimIngold calls "thinking-through-making." Both practices involve following rather than imposing,allowing form to emerge through correspondence with the subject rather than throughpredetermined design.In studio practice, we do not begin knowing what the work will look like. We enter throughobservation, material exploration, or formal questioning, then develop understanding throughiterative mark-making, allowing materials and subjects to guide us toward unimagined forms.Mathematicians visualizing abstract concepts engage in parallel process: testing configurations,following emergent patterns, discovering which formal elements—color, scale, spatialorganization, rhythm—reveal structure most effectively.Through my collaboration with mathematician/artist Edmund Harriss and artist/technologistVincent Edwards visualizing chaos theory—translating the logistic map's bifurcation into physicalvessels from equations to bitmaps to clay—we demonstrate how foundations' emphasis on formal,poetic exploration of color, pattern, and form develops the reasoning required for rigorousmathematical illustration. The collaboration brings strength through interplay of mathematicalprecision, material correspondence, and poetic formal inquiry. The presentation reframesillustration not as communicating pre-existing knowledge but as inquiry itself—one both artists andmathematicians employ when pushing into territories "not yet understood well by anyone."

Orateur: SCHMITT, Jean (University of Arkansas)

ID de Contribution: **14** Type: **Non spécifié**

How Not to Draw a Sphere

mercredi 21 janvier 2026 12:00 (25 minutes)

This talk will describe some of the challenges of communicating mathematical experiences, including the value of rigor and its opposites.

Orateur: ORDING, Philip (Pratt Institute)

ID de Contribution: **15** Type: **Non spécifié**

Integrating 3D Technologies and Immersive Media into Mathematics Education

jeudi 22 janvier 2026 09:30 (25 minutes)

This talk will explore the evolving research in mathematics and STEAM education at the Linz School of Education, specifically focusing on the integration of 3D Modelling and Printing (3DMP) and immersive media (AR and VR) to support mathematical learning at all levels of education. By examining the intersection of technology and pedagogy, I will highlight how these tools could foster creative thinking and bridge digital and physical learning environments. Collaborating with the Experience Workshop Movement and GeoGebra, I will showcase some practical applications of these technologies in classrooms. I will also discuss the transformative potential of 3DMP in visualising complex concepts and the role of AR/VR in creating interactive, high-engagement learning experiences. Additionally, the talk will outline how supporting data and innovative initiatives at Johannes Kepler University contribute to mathematics and STEAM education. By embracing these spatial technologies, we aim to develop transdisciplinary innovations and to advance mathematics teaching and learning.

Orateur: LAVICZA, Zsolt (Linz School of Education, Johannes Kepler University)

ID de Contribution: **16** Type: **Non spécifié**

When is enough enough? Proving in an inversive model of Euclidean space

jeudi 22 janvier 2026 10:00 (25 minutes)

Imagine you are proving a proposition in an inversive model of Euclidean space and you've end up with is a chaotic pile of circles and lines. How do you decide what to include? When is the construction sufficient? Appropriately aesthetic? Reasonably communicative? The aim of this talk is to share some of the challenges one encounters when generating mathematical illustrations for the dual purposes of communication and validation.

Orateur: BROWN, Stacy (California State Polytechnic University, Pomona)

ID de Contribution: **17** Type: **Non spécifié**

Some reflections about the role of visuals in the teaching and learning of mathematics

jeudi 22 janvier 2026 10:30 (25 minutes)

In 1990, Michael DeVilliers presented a seminal paper on the function and role of proofs in mathematics. Following the structure he suggested, I will reflect on the roles and the limitations visuals have in my life as a mathematics teacher educator, as a mathematics teacher, and as a problem solver.

Orateur: WINICKI LANDMAN, Greisy (Department of Mathematics and Statistics, California State Polytechnic University Pomona)

ID de Contribution: **18** Type: **Non spécifié**

Rigorous Sculpture?

jeudi 22 janvier 2026 11:30 (25 minutes)

Can a public participatory art installation be relevant to the boundary of current work on unsolved problems? Moreover, the question is also pertinent in the other direction: should recent research have any influence on public art? A case study of a Fall 2025 geometric construction at the University of Colorado, Pueblo will reveal the positive interactions that can occur: increased exposure and enthusiasm for mathematics, looking at an existing problem in new ways, and even developing new software tools that appear to be useful in both pursuits. Of course, as in any method exploring the boundaries of what we know to be true or untrue, pitfalls, lurk and we will conclude by examining some of these as well.

Orateur: WHITNEY, Glen (Studio Infinity)

ID de Contribution: **19** Type: **Non spécifié**

Rigorous and Flexible Illustrations: Inviting Realms of Possibilities

jeudi 22 janvier 2026 12:00 (25 minutes)

Using data from my own research on the teaching and learning of undergraduate mathematics (e.g., abstract algebra, linear algebra, complex analysis), I will share the framework of inclusive materialism (de Freitas & Sinclair, 2014), which embraces intra-actions with others and materials such as images, models, and illustrations via gesture, fictive motion, symbols, and other embodied actions. This framework argues that mathematics is both material and ideational; that is, mathematical concepts come into being from materials and simultaneously inform materials–this is the process of assemblage.

There are four major tenets of this philosophy that support and challenge the need or desire to ensure that research illustrations reveal and do not mislead mathematical structure. First, by attending to mathematics as an assemblage of human and non-human bodies, inclusive materialism sets aesthetics and affect as the impetus of mathematical activity. For example, personal feelings can inspire a mathematical idea in one person and deter another person. Second, these material practices shape and are shaped by socio-political factors suggesting that, like all human endeavors, mathematical concepts are sociocultural immersed bodies. Third, by asking questions of embodiment for whom, this framework describes phenomena which privilege differences and emphasize the changing nature, or virtuality, of matter. In other words, mathematics comes to be known in unique ways because mathematical concepts are mobile and vibrant. Finally, this framework appreciates how surprise and creativity contribute to the assemblage of a mathematical concept, because meaning is created and negotiated by (re)configuring figures, bodily movement, attitudes, conversations, and speech, in non- deterministic ways. In summary, mathematics comes to be known through an entanglement of discourse between human and non-human phenomena consisting of physical and nonphysical aspects of a situation, including objects, motions, and context. Through intra-actions with materials, realms of possibilities regarding mathematical concepts begin to emerge through sensation, action, and perception.

Orateur: SOTO, Hortensia (Colorado State University)