The extinction versus the blow-up: Global and non-global existence of solutions of source types of degenerate parabolic equations with a singular absorption

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Abstract. We consider nonnegative solutions of degenerate parabolic equations with a singular absorption term and a source nonlinear term:

$$\partial_t u - (|u_x|^{p-2}u_x)_x + u^{-\beta}\chi_{\{u>0\}} = f(u,x,t), \quad \text{in } I \times (0,T),$$

with the homogeneous zero boundary condition on $I = (x_1, x_2)$, an open bounded interval in $\mathbb{R}$. Through this paper, we assume that $p > 2$ and $\beta \in (0,1)$. To show the local existence result, we prove first a sharp pointwise estimate for $|u_x|$. One of our main goals is to analyze conditions on which local solutions can be extended to the whole time interval $t \in (0,\infty)$, the so called global solutions, or by the contrary a finite time blow-up $\tau_0 > 0$ arises such that $\lim_{t\to\tau_0} \|u(t)\|_{L^\infty(I)} = +\infty$. Moreover, we prove that any global solution must vanish identically after a finite time if provided that either the initial data or the source term is small enough. Finally, we show that the condition $f(0,x,t) = 0$, $\forall (x,t) \in I \times (0,\infty)$ is a necessary and sufficient condition for the existence of solution of equations of this type.