STABILIZATION TO TRAJECTORIES FOR PARABOLIC EQUATIONS

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Joint work with Sérgio S. Rodrigues.

We consider the controlled parabolic equations

$$\partial_t y - \nu\Delta y + f(t, y, \nabla y) + \eta = 0, \qquad\qquad y|_\Gamma = g + \zeta; \tag{1}$$

In the variables $(t, x, \bar{x}) \in (0, +\infty) \times \Omega \times \Gamma$, the unknown in the equation is the function $y = y(t, x) \in \mathbb{R}$. For the internal-control case, we set $\eta = \sum_{i=1}^{M} u_i\Phi_i,\ \zeta = 0$ in (1) where the given functions $\Phi_i = \Phi_i(x)$ will play the role of internal actuators. For the boundary-control case, we set $\eta = 0,\ \zeta = \sum_{i=1}^{M} u_i\Psi_i$ where the given functions $\Psi_i = \Psi_i(\bar{x})$ will play the role of boundary actuators. Finally, M is a positive integer, and in both systems, $u = u(t) \in \mathbb{R}^M$ is a (control) vector function at our disposal.

The exponential feedback stabilization to (nonstationary) trajectories for a general class of semilinear parabolic equations in a given bounded domain is addressed. Both internal and boundary actuators are supported in small region. In [1], We prove that under suitable conditions on the family of actuators, they allow us to stabilize the system. Moreover, some estimates on the number of actuators that we need to stabilize the system are established. Using the Dynamical Programming Principle, the stabilizing controls can be taken in feedback form and be computed by solving a suitable differential Riccati equation. Some simulations are presented by using Finite Element Method (FEM) and performed with MATLAB.

References

[1] D. Phan and S. S. Rodrigues. Stabilization to trajectories for parabolic equations. arXiv:1608.02412 [math.OC], August 2016. URL: https://arxiv.org/abs/1608.02412.