

# Bifibrations of model categories

*vendredi 14 octobre 2016 14:00 (50 minutes)*

In this talk, I will explain how to endow the total category  $\mathcal{E}$  of a well-behaved Grothendieck bifibration  $\mathcal{E} \rightarrow \mathcal{B}$  with a structure of a model category when both the basis  $\mathcal{B}$  and all fibers  $\mathcal{E}_b$  of the bifibration are model categories.

The motivating example is the well-known Reedy model structure on a diagram category  $[\mathcal{R}, \mathcal{M}]$ . The crucial step in its construction by transfinite induction lies in the successor case, which is usually handled by reasoning on latching and matching functors. A first observation is that those functors define a Grothendieck bifibration on the restriction functor  $[\mathcal{R}_{\lambda+1}, \mathcal{M}] \rightarrow [\mathcal{R}_{\lambda}, \mathcal{M}]$  where  $\mathcal{R}_{\lambda}$  denotes the full subcategory of  $\mathcal{R}$  whose objects have degree less than  $\lambda$ . Unfortunately, this bifibration fails to fulfil the conditions of application of existing theorems in the literature ([1], [2]), which would have allowed to lift the model structure from the base category  $\mathcal{B} = [\mathcal{R}_{\lambda}, \mathcal{M}]$  to the total category  $\mathcal{E} = [\mathcal{R}_{\lambda+1}, \mathcal{M}]$ .

I will explain how to relax the hypotheses appearing in [1] and [2] by focusing on (co)cartesian lifts over acyclic (co)fibrations rather than over weak equivalences. This idea leads us to a simple and elegant condition for our new construction: some commutative squares in the base category are required to satisfy a homotopical version of the Beck-Chevalley condition. To conclude, I will apply the result to the Reedy construction and its generalizations ([3], [4]).

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[1] Stanculescu, A.E., Bifibrations and weak factorization systems, Applied Categorical Structures, 20(1):19-30, 2012

[2] Harpaz, Y. and Prasma, M., The Grothendieck construction for model categories, Advances in Mathematics, 218:1306-1363 (August 2015)

[3] Berger, C., and Moerdijk, I., On an extension of the notion of Reedy category, Mathematische Zeitschrift, 269(3):977-1004, December 2011

[4] Shulman, M., Reedy categories and their generalizations, arXiv preprint, arXiv:1507.01065 (2015)

**Auteur principal:** M. CAGNE, Pierre (Univeristé Paris 7)

**Co-auteur:** Dr MELLIÈS, Paul-André (CNRS, Université Paris 7)

**Orateur:** M. CAGNE, Pierre (Univeristé Paris 7)

**Classification de thématique:** Topologie algébrique et applications