Bifibrations of model categories

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In this talk, I will explain how to endow the total category $\mathcal{E}$ of a well-behaved Grothendieck bifibration $\mathcal{E} \to \mathcal{B}$ with a structure of a model category when both the basis $\mathcal{B}$ and all fibers $\mathcal{E}_b$ of the bifibration are model categories.

The motivating example is the well-known Reedy model structure on a diagram category $[\mathcal{R}, \mathcal{M}]$. The crucial step in its construction by transfinite induction lies in the successor case, which is usually handled by reasoning on latching and matching functors. A first observation is that those functors define a Grothendieck bifibration on the restriction functor $[\mathcal{R}_{\lambda+1}, \mathcal{M}] \to [\mathcal{R}_{\lambda}, \mathcal{M}]$ where $\mathcal{R}_{\lambda}$ denotes the full subcategory of $\mathcal{R}$ whose objects have degree less than $\lambda$. Unfortunately, this bifibration fails to fulfill the conditions of application of existing theorems in the literature ([1], [2]), which would have allowed to lift the model structure from the base category $\mathcal{B} = [\mathcal{R}, \mathcal{M}]$ to the total category $\mathcal{E} = [\mathcal{R}_{\lambda+1}, \mathcal{M}]$.

I will explain how to relax the hypotheses appearing in [1] and [2] by focusing on (co)cartesian lifts over acyclic (co)fibrations rather than over weak equivalences. This idea leads us to a simple and elegant condition for our new construction: some commutative squares in the base category are required to satisfy a homotopical version of the Beck-Chevalley condition. To conclude, I will apply the result to the Reedy construction and its generalizations ([3], [4]).


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