

Bifibrations of model categories

vendredi 14 octobre 2016 14:00 (50 minutes)

In this talk, I will explain how to endow the total category \mathcal{E} of a well-behaved Grothendieck bifibration $\mathcal{E} \rightarrow \mathcal{B}$ with a structure of a model category when both the basis \mathcal{B} and all fibers \mathcal{E}_b of the bifibration are model categories.

The motivating example is the well-known Reedy model structure on a diagram category $[\mathcal{R}, \mathcal{M}]$. The crucial step in its construction by transfinite induction lies in the successor case, which is usually handled by reasoning on latching and matching functors. A first observation is that those functors define a Grothendieck bifibration on the restriction functor $[\mathcal{R}_{\lambda+1}, \mathcal{M}] \rightarrow [\mathcal{R}_\lambda, \mathcal{M}]$ where \mathcal{R}_λ denotes the full subcategory of \mathcal{R} whose objects have degree less than λ . Unfortunately, this bifibration fails to fulfil the conditions of application of existing theorems in the literature ([1], [2]), which would have allowed to lift the model structure from the base category $\mathcal{B} = [\mathcal{R}_\lambda, \mathcal{M}]$ to the total category $\mathcal{E} = [\mathcal{R}_{\lambda+1}, \mathcal{M}]$.

I will explain how to relax the hypotheses appearing in [1] and [2] by focusing on (co)cartesian lifts over acyclic (co)fibrations rather than over weak equivalences. This idea leads us to a simple and elegant condition for our new construction: some commutative squares in the base category are required to satisfy a homotopical version of the Beck-Chevalley condition. To conclude, I will apply the result to the Reedy construction and its generalizations ([3], [4]).

[1] Stanculescu, A.E., Bifibrations and weak factorization systems, Applied Categorical Structures, 20(1):19-30, 2012

[2] Harpaz, Y, and Prasma, M., The Grothendieck construction for model categories, Advances in Mathematics, 218:1306-1363 (August 2015)

[3] Berger, C., and Moerdijk, I., On an extension of the notion of Reedy category, Mathematische Zeitschrift, 269(3):977-1004, December 2011

[4] Shulman, M., Reedy categories and their generalizations, arXiv preprint, arXiv:1507.01065 (2015)

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