A Tropical Day

Rapport sur les contributions

https://indico.math.cnrs.fr/e/12919

ID de Contribution: **1** Type: **Non spécifié**

Beyond Boolean networks, a multi-valued approach

lundi 21 octobre 2024 10:00 (45 minutes)

Boolean networks can be viewed as functions on the set of binary strings of a given length, described via logical rules. They were introduced as dynamic models into biology, in particular as logical models of intracellular regulatory networks involving genes, proteins, and metabolites. Since genes can have several modes of action, depending on their expression levels, binary variables are often not sufficiently rich, requiring the use of multi-valued networks instead. The steady state analysis of Boolean networks is computationally complex, and increasing the number of variable values beyond 2 adds substantially to this complexity. I will report on joint work with Juliana García Galofre, Ayelén García Rial, Reinhard Laubenbacher and Mercedes Pérez Millán. We propose a representation of multi-valued networks using multi-valued logic functions (which are tropical operations), providing a biologically intuitive representation of the network. We give an algorithm to compute the steady states of a multi-valued network that has a complexity that, in many cases, is essentially the same as that for the case of binary values. We provide a basic implementation of the algorithm, that uses tools to compute lattice points in rational polytopes.

Orateur: DICKENSTEIN, Alicia (University of Buenos Aires)

ID de Contribution: **2** Type: **Non spécifié**

Polynomial systems with many real positive solutions via tropical geometry

lundi 21 octobre 2024 11:30 (45 minutes)

We use a version Viro's patchworking theorem to construct systems with number of positive solutions which is at least the size of the largest positively decorable subcomplex of a regular triangulation of a polytope. We explain a duality result about such subcomplexes which enables us to get the currently best lower bounds known on the maximal number of positive solutions of polynomial systems with fixed supports.

This is a joint work with Francisco Santos and Pierre-Jean Spaenlehauer.

Orateur: BIHAN, Frédéric (Université Savoie Mont Blanc)

ID de Contribution: **3** Type: **Non spécifié**

TBA

lundi 21 octobre 2024 14:45 (45 minutes)

Orateur: AMINI, Omid (CMLS, École Polytechnique)

ID de Contribution: **4** Type: **Non spécifié**

Eigenvalue methods for sparse tropical polynomial systems

lundi 21 octobre 2024 10:45 (45 minutes)

In this talk, we develop a tropical analogue of eigenvalue methods in order to effectively compute the solution set of tropical polynomial systems. Relying on the connection between tropical linear systems and mean payoff games, we show that this solution set can be obtained by solving parametric mean-payoff games, arising from approriate linearizations of the tropical polynomial system relying on a tropical Null- and Positivstellensatz and using tropical Macaulay matrices. We present two approaches: a first one based on a dichotomic search, which simply allows one to certify the solvability of a tropical polynomial system, and a second, more elaborate approach, based on homotopy path-following, allowing one to compute projections of the solution set onto any coordinate.

Orateur: BÉREAU, Antoine (CMAP, École Polytechnique, Inria)

ID de Contribution: **5** Type: **Non spécifié**

Non-properness set of a tropical polynomial map

lundi 21 octobre 2024 16:30 (45 minutes)

A tropical polynomial map is a piecewise-linear map between real Euclidean spaces. These maps represent a degeneration of classical polynomial maps between Euclidean spaces over valued fields. Accordingly, some of the pertinent classical topological invariants can be translated to polyhedral ones. In this talk, I will define the tropical analogue of the non-properness set of polynomial maps. This is the set of points at which the preimage has an extra solution "at infinity". Studying the non-properness set is beneficial to applications such as enhancing cylindrical algebraic decomposition algorithms from semi-algebraic geometry. The tropical non-properness set, on the other hand, is useful in describing the polyhedral geometry of piecewise-linear maps. I will present a correspondence theorem that relates the classical non-properness set to its tropical analogue, and illustrate a purely combinatorial procedure to compute it.

Orateur: EL HILANY, Boulos (TU Braunschweig)

ID de Contribution: **6** Type: **Non spécifié**

Likelihood Degenerations

lundi 21 octobre 2024 17:15 (45 minutes)

Computing all critical points of a monomial on a very affine variety is a fundamental task in algebraic statistics, particle physics and other fields. The number of critical points is known as the maximum likelihood (ML) degree. When the variety is smooth, it coincides with the Euler characteristic. We introduce degeneration techniques that are inspired by particle physics. The main objects that will appear are bounded regions in discriminantal arrangements and moduli spaces of point configurations. We present theory and practise, connecting complex geometry, tropical combinatorics, and numerical nonlinear algebra. This is based on a joint project with Daniele Agostini, Taylor Brysiewicz, Lukas Kühne, Bernd Sturmfels, and Simon Telen.

Orateur: FEVOLA, Claudia (Inria Saclay)

ID de Contribution: **7** Type: **Non spécifié**

Tropical homotopies two ways

lundi 21 octobre 2024 14:00 (45 minutes)

Polyhedral homotopies were originally introduced by Huber and Sturmfels nearly 30 years ago, and have since become a staple strategy for solving polynomial systems. Main topic of the talk is a generalisation thereof. Building on ideas of Jensen, Leykin, and Yu, we will discuss two distinct types of tropical homotopies: First, we will discuss how to use tropical points to construct homotopies for solving systems of polynomial equations. Second, we will discuss how to compute tropical points using homotopies for intersecting systems of balanced polyhedral complexes.

Centerpiece of the talk are systems of parametrized polynomial equations, and we will focus on two main cases: Vertically parametrized polynomial systems are systems in which parameters are shared between equations but always bound to the same monomial. These are for example the steady state equations of chemical reaction networks or they arise in the computation of ED or ML degrees. Horizontally parametrized polynomial systems are systems in which parameters are shared between monomials but always bound to the same equation. These are prominently studied using the theory of Khovanskii bases and Newton Okounkov bodies.

Orateur: REN, Yue (Durham University)