Poincaré-type Series for the Arc Space of a Fat Point

Fat point is a scheme defined by an ideal whose solution set is a single point (but the ideal is not necessarily maximal, so it may have multiplicity).

For an algebraic variety, the arc scheme can be thought of as the scheme of all possible formal trajectories on the variety (in other words, power series solutions of the corresponding equations). This scheme is defined by an ideal in an infinite dimensional polynomial ring obtained by the original equations by formal differentiation. The original multiplicity structure of a fat point "propagates" to its arc scheme in a nontrivial and intriguing way (for example, it is capable of encoding nontrivial partition identities).

One way to describe this is multiplicity structure is to build a Poincaré-type series from the multiplicities of the truncations of varying orders. In the talk I will describe some recent results showing that these series are geometric series for a fat point on a line and some higher-dimensional cases and report results of computational experiments exploring more complicated fat points.

The talk is based on joint works with Rida Ait El Manssour.

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