

# Crystal Structure of Localized Quantum Unipotent Coordinate Category

jeudi 21 novembre 2024 14:00 (50 minutes)

For a monoidal category  $\mathcal{T}$ , if there exists a “real commuting family  $(C_i, R_{C_i}, \phi_i)_{i \in I}$ ”, we can define a localization  $\tilde{\mathcal{T}}$  of  $\mathcal{T}$  by  $(C_i, R_{C_i}, \phi_i)_{i \in I}$ .

Let  $R = R(\mathfrak{g})$  be the quiver Hecke algebra (=KLR algebra) associated with a symmetrizable Kac-Moody Lie algebra  $\mathfrak{g}$  and  $C_w$  the subcategory of  $R$ -gmod (=the category of graded finite-dimensional  $R$ -modules) associated with a Weyl group element  $w$ , which is a monoidal category with a real commuting family  $(C_i, R_{C_i}, \phi_i)_{i \in I}$ . Thus, we get its localization  $\tilde{C}_w$ , which is called a “localized quantum unipotent coordinate category” associated with  $w$ . In the former half of the talk, we shall present that for a (semi-)simple  $\mathfrak{g}$  and the longest element  $w_0$ , the family of self-dual simple modules in  $\tilde{C}_{w_0} = \widetilde{R\text{-gmod}}$  holds a crystal structure and is isomorphic to the cellular crystal  $\mathbb{B}_{i_1 \dots i_N}$  where  $i_1 \dots i_N$  is an arbitrary reduced word of  $w_0$ . Furthermore, in the latter half of the talk, the latest result, which is a joint work with M. Kashiwara, will be presented that for a general symmetrizable Kac-Moody Lie algebra  $\mathfrak{g}$  and a general Weyl group element  $w$ , the family of self-dual simple modules in the localized category  $\tilde{C}_w$  also holds a crystal structure, and it is isomorphic to the cellular crystal  $\mathbb{B}_{i_1 \dots i_m}$  associated with a reduced word  $i_1 \dots i_m$  of  $w$ .

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