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Algebraicity Beyond Beukers-Heckman and Bober: Emerging Patterns

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We consider positive integer sequences $\rho(n)$, $n = 0, 1 \cdots$, expressible through the ratios of products of factorials, or of ratios of products of factorials along with Gamma functions. Admitting certain forms of these ratios, the generating functions (gf) of $\rho(n)$'s become generalized hypergeometric functions (gf), which turn out to be algebraic. Detailed conditions for the aforementioned algebraicity were given in [1]. In the spirit of [1], in [2] three parametrized families of, as well 52 parameter-free factorial ratios were given, all having algebraic gf's. In this work we initiate the study of factorial ratios beyond the classification of [1] and [2], for which we demonstrate by construction the algebraicity of their gf's. They include several families of shifted binomials, as well as sequences derived from recent integrality criteria put forward in [3]. For certain parametrized $\rho(n)$'s we obtained closed-form parametric expressions for their algebraic equations (algeqs) for gf's. In a parallel approach we conceived the $\rho(n)$'s in question as power moments of certain weight functions with finite support. The weights are solutions of the Hausdorff moment problem, obtained via inverse Mellin transform. In many instances the weights obey explicit algeqs that mirror those of gf's.

This correspondence can be made neatly explicit, especially when elementary functions in form of radicals are involved.

Joint work with G. H. E. Duchamp, M. Kontsevich, and G. Koshevoy.

[1] F. Beukers and G. Heckman, Invent. Math. 95, 325-354 (1989);

[2] J. W. Bober, arXiv: 0709.1977v1;

[3] A. Adolphson and S. Sperber, arXiv: 2001.03296, and private communication (2024).

Orateur: PENSON, Karol (LPTMC, Sorbonne Université)