Journée Analyse Appliquée Hauts-de-France

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Université Polytechnique Hauts-de-France

Programme Scientifique

Exposé de Patrick Ciarlet :

T-coercivity: a practical tool for the study of variational formulations. Application to the magnetostatic model

Variational formulations are a popular tool to analyse linear PDEs (eg. neutron diffusion, Maxwell equations, Stokes equations …), and it also provides a convenient basis to design numerical methods to solve them. Of paramount importance is the inf-sup condition, designed by Ladyzhenskaya, Necas, Babuska and Brezzi in the 1960s and 1970s. As is well-known, it provides sharp conditions to prove well-posedness of the problem, namely existence and uniqueness of the solution, and continuous dependence with respect to the data. Then, to solve the approximate, or discrete, problems, there is the (uniform) discrete inf-sup condition, to ensure existence of the approximate solutions, and convergence of those solutions to the exact solution. Often, the two sides of this problem (exact and approximate) are handled separately, or at least no explicit connection is made between the two.

In this talk, I will focus on an approach that is completely equivalent to the inf-sup condition for problems set in Hilbert spaces: T-coercivity. It relies on the design of an explicit operator to realize the inf-sup condition (basic Tcoercivity). If the operator is carefully chosen, it can provide useful insight for a “natural” definition of the approximation of the exact problem. Two kinds of results can be derived. On one hand, the derivation of the discrete inf-sup condition often becomes elementary (discrete T-coercivity), at least when one considers conforming methods, that is when the discrete spaces are subspaces of the exact Hilbert spaces : both the exact and the approximate problems are considered, analysed and solved at once. On the other hand, the knowledge of the operator may lead to a new variational formulation (explicit T-coercivity).

In itself, T-coercivity is not a new theory, however it seems that some of its strengths have been overlooked (discrete or explicit T-coercivity), and that, if used properly, it can be a simple, yet powerful tool to analyse and solve linear PDEs. This claim will be illustrated on the magnetostatic model and, time permitting, on eddy current models.

Exposé d'Aline Lefebvre-Lepot :

Numerical simulation of suspensions: taking close interactions into account

We address the problem of direct numerical simulation of suspensions of rigid particles in a Stokes flow. We focus on the singular effects due to short-range interactions (lubrication effects) as the particles approach each other. Taking these lubrication effects into account in numerical simulations is a difficult problem: capturing the singularity requires, for example, the use of very fine meshes in the gap between the particles.

We describe in this presentation two methods for taking account of the effects of lubrication in numerical simulations. The first is based on an asymptotic development of the solution in the gap between the particles. It provides accurate results using coarse meshes and without adding new assumptions or models. We then describe a second method, based on a "viscous" contact model. From a numerical point of view, it boils down to solving a convex optimisation problem at each time step. It will be shown that dry contacts, with or without friction, can be treated within the same theoretical framework. Finally, the coupling problem between these different contact models will be discussed.

Exposé de Florian de Vuyst :

Emerging issues in machine learning-accelerated fluid-structure interaction

Une tendance actuelle en calcul scientifique est de tirer avantage des techniques de Machine Learning (ML) pour accélérer la résolution de problèmes aux dérivées partielles. En couplage multiphysique par exemple, les techniques de réduction de dimensionalité permettent de réduire la dimension des variables d'interactions et/ou de fabriquer des modèles réduits pour chaque composant. Néanmoins, la stabilité globale en temps long n'est pas garantie sans effort supplémentaire. Dans le cas particulier de l'interaction fluide-structure, il convient d'analyser les transfert d'énergie pour une conservation ou une décroissance de l'énergie du système couplé. Cette question rentre dans le cadre du "Physics-informed ML".
Dans cet exposé, on présentera 3 modèles originaux de couplage fluide-structure 1d-1d où l'on réécrira les équations sous forme conservative et l'on exhibera une entropie pour chaque modèle. On précisera également les termes de transfert d'énergie. L'exposé se terminera par des perspectives en ML informé par la physique.

Exposé de Clara Patriarca :

Homogenization of Leray’s flux problem for the steady-state Navier-Stokes equations in a multiply-connected planar domain

The steady motion of a viscous incompressible fluid in a multiply-connected, planar, bounded domain (perforated with a large number of small holes) is modeled through the Navier-Stokes equations with non-homogeneous Dirichlet boundary data satisfying the general outflow condition. Under either a symmetry assumption on the data or under a smallness condition on each of the boundary fluxes (therefore, no constraints on the magnitude of the boundary velocity are imposed), we apply the classical energy method in homogenization theory and study the asymptotic behavior of the solutions to this system as the size of the perforations goes to zero: it is shown that the effective equations remain unmodified in the limit. The main novelty of the present work lies in the obtainment of the required uniform bounds, which are achieved by a contradiction argument based on Bernoulli’s law for solutions of the stationary Euler equations.